Stiffness Analysis of Above Knee Prosthesis

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Abstract: While a healthy human walks, his or her legs mutually perform good repeatability with high accuracy. This provides an esthetical movement and balance. People with above knee prosthesis want to perform walking as esthetical as a healthy human. Therefore, to achieve a healthy walking, the above knee prosthesis must provide a good stiffness performance. Especially stiffness values are required when adding a second axis movement to the ankle for eversion and inversion. In this paper, stiffness analysis of above-knee prosthesis is presented. The translational displacement of above knee prosthesis is obtained when the prosthesis is subjected to the external forces. Knowing stiffness values of the above knee prosthesis, designers can compute prosthesis parameters such as ergonomic structure, height, and weight and energy consumption.

Keywords: Stiffness analysis, above knee prosthesis, joint stiffness, prosthesis, accuracy.

1. Introduction

It is required that serial robots have to make their tasks with a high accuracy, high precision and high stiffness [1]. The stiffness of robot manipulators generally provides to obtain the desired position and force commands with high accuracy [2, 3]. If the stiffness at the end point of robot manipulator is modified and identified accurately, it would be possible to compensate coupling and posing errors caused by the external forces [4].

(Abele et al; 2007) presented two methods to obtain the Cartesian stiffness matrix of a 5R robot. They reported that second method is better than first because of considering both joint and link stiffness. When load is applied, all deformations are considered such as links deformations and joint stiffness values.

This paper presents the stiffness analysis of above-knee prosthesis. The translational displacement of above knee prosthesis is obtained when the prosthesis is subjected to the external forces. Small displacements along x, y and z axes of the robot’s end-effector are illustrated by figures. Knowing stiffness values of the above knee prosthesis, researches can design prosthesis as optimal structure, height, and weight and energy consumption.

2. Kinematic Analysis and Jacobian Matrix

2.1. Design of the Prosthesis

Cartesian stiffness matrices due to preserving fundamental properties of the stiffness matrices [4].

(Ang and Andeen; 1995) presented variable passive compliance generated by means of topology of robot manipulators. As a conclusion they reported that a non-diagonal matrix is effective to prevent jamming and vibrations [4].

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This paper presents the stiffness analysis of above-knee prosthesis. The translational displacement of above knee prosthesis is obtained when the prosthesis is subjected to the external forces. Small displacements along x, y and z axes of the robot’s end-effector are illustrated by figures. Knowing stiffness values of the above knee prosthesis, researches can design prosthesis as optimal structure, height, and weight and energy consumption.

Figure 1. Proposed above knee prosthesis and its structure: knee joint(A); the entire prosthesis(B); ankle joint(C).
The solid model of proposed above knee prosthesis is illustrated on (Figure.1). It has three joints. The first one is knee joint which is capable of one-axis movement and the second and third one compose of ankle joint which is capable of two-axis movement.

2.2. Kinematic Model of Above Knee Prosthesis

This section deals with kinematic model of above knee prosthesis. The coordinate systems attached to each joint of above knee prosthesis is presented in (Figure.1). The D-H parameters of the above knee prosthesis is illustrated in (Table.1).

The proposed above knee prosthesis has 3 rotational joints. The first one is knee joint which composes of one-axis movement. The second and third joint compose of ankle joint which performs two-axis movement. Ankle joint provides plantar flexion, dorsiflexion eversion and inversion movements.

The overall transformation matrix of above knee prosthesis can be written as

\[
\begin{bmatrix}
\mathbf{T}_1 \\
\mathbf{T}_2 \\
\mathbf{T}_3 \\
\mathbf{T}_4
\end{bmatrix}
= \begin{bmatrix}
\mathbf{T}_1 \\
\mathbf{T}_2 \\
\mathbf{T}_3 \\
\mathbf{T}_4
\end{bmatrix}
\]

(1)

After determining D-H parameters, transformation matrices are obtained as follows.

\[
\mathbf{T}_i = \begin{bmatrix}
\mathbf{T}_1 & \mathbf{T}_2 & \mathbf{T}_3 & \mathbf{T}_4
\end{bmatrix}
\]

(2)

The multiplication of overall transformation matrices is obtained as:

\[
\mathbf{T}_4 = \begin{bmatrix}
r_{11} & r_{12} & r_{13} & p_x \\
r_{21} & r_{22} & r_{23} & p_y \\
r_{31} & r_{32} & r_{33} & p_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(3)

where

\[
\begin{align*}
r_{11} &= c\theta_3(c\theta_1 c\theta_2 - s\theta_2 s\theta_2) \\
r_{12} &= -s\theta_3(c\theta_1 c\theta_2 - s\theta_2 s\theta_2) \\
r_{13} &= c\theta_3 s\theta_2 + c\theta_2 s\theta_1 \\
p_x &= l_2(c\theta_1 c\theta_2 - s\theta_2 s\theta_2) + l_4(c\theta_1 s\theta_2 + c\theta_2 s\theta_4) + l_1 c\theta_1 \\
r_{21} &= c\theta_3(c\theta_1 s\theta_2 + c\theta_2 s\theta_1) \\
r_{22} &= -s\theta_3(c\theta_1 s\theta_2 + c\theta_2 s\theta_1) \\
r_{23} &= s\theta_1 s\theta_2 - c\theta_1 c\theta_2 \\
p_y &= l_2(c\theta_2 s\theta_2 + c\theta_2 s\theta_1) - l_4(c\theta_1 c\theta_2 - s\theta_1 s\theta_2) + l_1 s\theta_1 \\
r_{31} &= s\theta_3 \\
r_{32} &= c\theta_3 \\
r_{33} &= 0 \\
p_z &= 0
\end{align*}
\]

(4)

The Jacobian matrix of prosthesis is obtained as follows

\[
\mathbf{J} = \begin{bmatrix}
-l_3 s\theta_1 - l_3 s(\theta_1 + \theta_2) + l_4 c(\theta_1 + \theta_2) \\
l_1 c\theta_1 + l_2 c(\theta_1 + \theta_2) + l_4 s(\theta_1 + \theta_2) \\
0 \\
-l_2 s(\theta_1 + \theta_2) + l_4 c(\theta_1 + \theta_2) \\
l_1 c(\theta_1 + \theta_2) + l_4 s(\theta_1 + \theta_2) \\
0 \\
0
\end{bmatrix}
\]

(5)

3. Stiffness Modelling

The following relationship can be stated between actuated torques and the corresponding external forces and moments exerted on end-effector of the prosthesis can be expressed as:

\[
\mathbf{f} = \mathbf{J}^T \mathbf{\omega}
\]

(6)

where \(\mathbf{f}\) and \(\mathbf{\omega}\) are the 3x1 vectors and \(\mathbf{f} = [f_1, f_2, f_3] \mathbf{\omega} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]\) represents the corresponding external forces & moments exerted on end-effector of the prosthesis. There are two major changes happens in the manipulator because of its motion. First change happens angular position of the joints due to the torques/forces applied to the joints.

\[
\mathbf{\Gamma} = \mathbf{K}_d \Delta \theta
\]

(7)

where \(\mathbf{K}_d = \text{diag}[K_{d1}, K_{d2}, K_{d3}]\) denotes joint stiffness matrix and \(\Delta \theta = [\delta \theta_1, \delta \theta_2, \delta \theta_3]^T\) represents change in the positions of the joints. Second change happens on the end-effector of the manipulator due to the external force and moment applied to the end-effector of the manipulator.
\[ \omega = K_s \Delta \]  

where \( K_s \) illustrates the Cartesian stiffness matrix of the manipulator and \( \Delta \) is the change in the end-effector of the manipulator.

The following important identity is obtained by applying partial differentiation to the (Equation.6) with respect to \( \theta \).

\[ \frac{\partial \Gamma}{\partial \theta} = \left( \frac{\partial \Gamma}{\partial \theta} \right)^T \partial \omega + \frac{\partial \Gamma}{\partial \theta} \partial X \partial \theta \]  

(Equation.9) can be written as follows:

\[ K_\theta = K_c + \frac{1}{k} K_s J^T \]  

where \( K_c \) is the complementary stiffness matrix can be written for a 3DOF robotic manipulator as follows

\[ K_c = \left[ \frac{\partial J^T}{\partial \theta_1} \frac{\partial J^T}{\partial \theta_2} \frac{\partial J^T}{\partial \theta_3} \right] \]  

The stiffness matrix seen at the end-effector of the manipulator can be simplified as:

\[ K_s = \frac{f^T (K_\theta K_c)}{J^T} \]  

In order to find joint stiffness matrix, the Cartesian stiffness matrix can be simplified by ignoring \( K_c \) as follows:

\[ K_s = \frac{f^T K_s J^T}{J} \]  

(Equation.8) can be rewritten by substituting (Equation.12). (Equation.14) as follows.

\[ \omega = f^T K_s J^T \Delta \]  

(Equation.14) can be rearranged as

\[ \Delta = K_\theta J^T \omega \]  

(Equation.15) can be rewritten as follows

\[ \Delta = A \omega \]  

where \( x \) and \( A \) include 6x1 vector of joint compliances and 6x6 matrix having external forces/moments and elements of Jacobian matrices.

\[ x = [1/k_{\theta_1} 1/k_{\theta_2} 1/k_{\theta_3} \bar{f}] \]  

4. Stiffness Analysis

Cartesian stiffness matrix can be obtained by using (Equation.11) as follows

\[ K_c = \begin{bmatrix}
(-l_1c\theta_1 - d_3s\theta_2)f_x + (-l_1s\theta_1 + d_3c\theta_1)f_y & 0 & 0 & 0 & 0 & 0 \\
0 & c\theta_2f_x + s\theta_1f_y & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]  

A prosthesis end effector is forced to track a trajectory from its zero position \( (\theta_1 = \theta_2 = \theta_3 = 0) \) to final position \( (\theta_1 = 150, \theta_2 = 15, \theta_3 = 40) \) to identify joint stiffness values. The travel time of robot trajectory is planned as 3 seconds at 100 Hz frequency. The joint stiffness values \( (K_{\theta_1}, K_{\theta_2} \text{ and } K_{\theta_3}) \) along the trajectory are identified. Since trajectory frequency 100 Hz, 300 sample of joint stiffness values are obtained. The arithmetic averages of these sample values gives joint stiffness values. In this manipulator (Equation.16) can be written for prosthesis to find joint compliance values

\[ \Delta = \begin{bmatrix}
j_{11} & j_{12} & 0 & 0 & 0 & 0 \\
j_{21} & j_{22} & 0 & 0 & 0 & 0 \\
j_{31} & j_{32} & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]  

where Jacobean parameters;

\[ j_{11} = -l_1s\theta_1 - l_2s(\theta_1 + \theta_2) - l_4c(\theta_1 + \theta_2) \]

\[ j_{12} = -l_2s(\theta_1 + \theta_2) + l_4c(\theta_1 + \theta_2) \]

\[ j_{21} = l_1s\theta_1 + l_2c(\theta_1 + \theta_2) + l_3s(\theta_1 + \theta_2) \]

\[ j_{22} = l_2c(\theta_1 + \theta_2) - l_3s(\theta_1 + \theta_2) \]

This equation is reorganized to obtain (Equation.16).

\[ \begin{bmatrix}
\delta_{\theta_1} & \delta_{\theta_2} & \delta_{\theta_3} \\
j_{21} & j_{22} & 0 \\
j_{31} & j_{32} & 0 \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{k_{\theta_1}} & 0 & 0 \\
0 & \frac{1}{k_{\theta_2}} & 0 \\
0 & 0 & \frac{1}{k_{\theta_3}} \\
\end{bmatrix} \]  

5. Stiffness Verification

In order to compute the end effector displacements of proposed above knee prosthesis, the external forces are acted on anatomical position of human. Anatomical position is the erect position of the body with the face directed forward, the arms at the side, and the palms of the hands facing forward. It was used as a reference position for describing the relation of body parts to one to another [10]. Cartesian stiffness matrix is calculated by using (Equivalent.12).

Typical stiffness values of \( K_\theta = diag. \{10^5, 10^5, 10^5\} \text{ N/mm/rad} \) are chosen for initial values. \( K_\theta \) is constant, because it lets to the same joint stiffness values for it’s different initial values [4].

In this study, in order to have displacement values of the end-effector, an experimental study is performed. The magnitudes of force vector are implemented from 0 Newton to 200 Newton with a step size of 10 Newton as shown in (Table.2).
Table 2. Stiffness verification results for prosthesis

<table>
<thead>
<tr>
<th>Force Vector $(F_x, F_y, F_z)$ (Newton)</th>
<th>Deflection Calculated</th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$\delta_x$ (mm)</td>
<td>$\delta_y$ (mm)</td>
<td>$\delta_z$ (mm)</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>0</td>
<td>0</td>
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<td>0.0005472</td>
<td>0.0001</td>
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<td>0.001641</td>
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<tr>
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<td>0.0007</td>
<td>0.0011</td>
<td>0.002</td>
<td></td>
</tr>
</tbody>
</table>

(Figure 3) represents the deflection values along the x, y and z-axes of prosthesis end-effector under the forces exerted on the prosthesis as shown in (Table.2). (Figure 3) represents the deflection values along the x, y and z-axes of prosthesis end-effector under the forces exerted on the prosthesis as shown in (Table.2).

Figure 3. Force-deflection curve for $F_x, F_y, F_z$

In (Figure.4), (Figure.5) and (Figure.6), $F_x, F_y$ and $F_z$ forces are presented in separately.

6. Conclusion

In this study, the stiffness analysis of the proposed above-knee prosthesis is presented. The translational displacement of above knee prosthesis is obtained when the prosthesis is subjected to the external forces from 0 Newton to 200 Newton with a step size 10 Newton. The computed displacements along x, y and z axes of the prosthesis’s end-effector are illustrated by a table and figures. These results can be used to design and manufacture an above knee prosthesis. Results can also help researchers to choose the material which will be used.

References


