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A genetic-Fuzzy Procedure for Solving Fuzzy Multiresponses Problem

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***Abstract*:** This research proposed a procedure that combines genetic algorithm (GA) technique and fuzzy goal programming to optimize process performance in experimental design for fuzzy multiple quality characteristics. Initially, regression models were formulated to relate each replicate of a quality characteristic with the process's controllable factors. The GA technique was then employed to determine the optimal factor settings for each response's replicate. The GA's optimal results were then deployed to develop a fuzzy regression model to relate fuzzy process settings with each quality characteristic. The fuzzy models were adopted to construct the fuzzy desirability and deviation matrices for all quality characteristics. Finally, three optimization models were developed to determine the lower, middle, and upper bounds of optimal factor settings. Three industrial applications, which were widely examined, were employed to illustrate the proposed procedure. Results revealed that the proposed GA-fuzzy procedure efficiently dealt with uncertainty in multiple quality characteristics and process settings by providing fuzzy optimal factor settings rather than crisp values. Such information can support process engineering in understanding the impact of variations/uncertainty on process and product performance and in deciding proper corrective and preventive actions. Compared to the Taguchi method, grey-Taguchi technique, and artificial neural networks approach, the proposed procedure is found efficient in optimizing process performance for multiple quality characteristics under uncertainty.

Keywords: Genetic algorithm, Desirability function, Fuzzy goal programming, Optimization

1. Introduction

To compete effectively in today's marketplace, firms must find ways to manufacture high-quality products at low-cost to meet or exceed customer expectations. Robust design proposed by Taguchi [1] has significantly enhanced product's quality and manufacturing processes' productivity at minimal experimentation costs and efforts. The designed experiments proposed by Taguchi [1] uses orthogonal arrays to study all the process factors with minimum number of experiments. Then, signal-to-noise ratio is used to determine the optimal factor settings of a manufacturing process and the most influential process factors that affect a single quality characteristic of a product or a process. This approach has been widely applied to optimize process performance in many business applications [2-3]. Nowadays, customer interests and product/process functionality require concurrent improvement of multiple quality characteristics of a product. The Taguchi method is a reliable method for optimizing a single quality response of main interest, while it primarily uses engineering judgment to identify the combination of optimal factor settings that enhance multiple quality characteristics [4-6]. This usually increases uncertainty in the decision-making process about the combination of optimal factor settings and does not guarantee concurrent improvement of multiple quality responses. Recently, process engineers should determine the optimal combination of process factor settings of a manufacturing process to enhance multiple quality characteristics of products simultaneously. Therefore, various optimization techniques were proposed in literature to deal with multiresponses problem in the Taguchi method; including the Taguchi methodology and neuro-fuzzy based model [7-9], genetic algorithm [10-12], grey-fuzzy logic Chiang [13], response surface methodology and Taguchi's technique [14], comparisons of efficiency between different systems technique in data envelopment analysis [15], fuzzy goal programming approach [16], Taguchi-based grey relational analysis [17-19], Taguchi methods, neural networks, desirability function, and genetic algorithms [20], particle swarm optimization [21], regression and neural network [22], neural networks and Taguchi method [23], Taguchi technique and upper bound technique [24], fuzzy neural network approach [25], Min-Max model in fuzzy goal programming [26], fuzzy goal programming-regression approach [27], multiple pentagon fuzzy responses [28], non-dominated sorting genetic algorithm II [29]. Nevertheless, most of these approached are deterministic optimization, which were carried out without considering the uncertainty due to measurement and process variations; therefore, the optimal solution will be sensitive to variations of input and process parameters. Hence, an appropriate procedure is required to deal with uncertainty in multi-response problem. Further, customers require conforming product with all observed quality characteristics fall within specified specifications at minimum variability around the process mean and minimal shift of mean from the target. Such customer and process preferences are represented by ranges rather than crisp values. In summary, an effective optimization procedure must be developed to determine fuzzy multiresponses problem in experimental design. The Genetic Algorithm (GA) is an artificial intelligence search metaheuristic that that is particularly well suited to identify the optimal levels of input variables that results in the best/optimal conditions of output variables [30-35]. The algorithm starts by creating an initial population by randomly generating feasible solutions. Then, the sets of chromosomes pass through a self-

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development heuristic. The fitter individuals are then chosen to evolve through crossover and mutation. GA are a popular meta-heuristic that is particularly well suited for this class of problems. The GA requires a fitness function, which represents the objective function and the fitness value that corresponds to the performance of an individual chromosome. To set the fitness function for GA problems, an objective function with a set of variables and constraints can be applied to maximize or minimize a given function. Population is a collection of individuals, which is normally randomly initialized. The two important aspects are the initial population generation and the population size. Furthermore, GA is uniquely distinguished by having a parallel population-based search with stochastic selection of many individual solutions, stochastic crossover and mutation [35-36]. Selection is the process of choosing two parents from the population for crossing. Some of the various selection methods are stochastic uniform, remainder

roulette wheel selection, random selection, rank selection and tournament selection. In crossover, generally two chromosomes (parents) are combined together to generate new chromosomes (offspring). The parents are chosen with preference towards fitness so that offspring inherit good genes. By iteratively applying the crossover operator, genes of good chromosomes appear more frequently in the population, eventually leading to convergence to an overall optimal or near optimal solution. Two typical parameters must be determined, including crossover probability and crossover way. The crossover probabilities of 0.4 through 0.9 are generally proposed. The most common crossover ways, like single-point crossover, multipoint crossover, and uniform crossover, were adopted. In addition, three other crossover forms, including three-parent crossover, ordered crossover, and shuffle crossover. The mutation operator introduces random changes into characteristics of chromosomes, which provides genetic variety and enable the genetic algorithm to search a broader space. The different forms of mutation are constraint dependent, uniform and adaptive feasible. The purpose of mutation is to prevent GA from being trapped into local optimal solutions. A suitable mutation probability is specified according to the mutual of string length. That is, supposing the length of the genetic string consists of eight genes, the mutation probability is 1/8. Flipping mutation is vastly applied to the enlarging solution spaces. Stopping criteria locates what causes the algorithm to terminate-generations, time limit, and fitness limit. The mutation operator consists of altering the genetic information of a member of the population. If the resulted individual has better fitness, it replaces the old individual. In this kind of algorithms, iterations are called "generations," which are processed until (1) a stop condition is meet or (2) the program reaches a predefined limit of generation. This algorithm is applicable to search for the solution of high degree of complexity that often involves attributes that are large, non-linear and discrete in nature. The objective of GA is to find the optimal settings of the input variables to the simulated system that makes the output variables at their best or optimal conditions. Traditional GA are customized to accommodate multi-objective problems by using specialized fitness functions and introducing methods to promote solution diversity [37-42]. However, the GA ignores the uncertainty in the observed measurements between the replicates of each quality characteristic, which may result in distinct optimal factor settings for each replicate of a quality characteristic. To solve this issue, the optimal factor settings for each replicate that are obtained by using GA can be further processed to determine a fuzzy combination of optimal process factor settings for each quality characteristic and/or multiple quality characteristics. An appropriate technique to achieve this objective is the fuzzy goal programming (FGP) technique, which was widely used in optimizing performance for several business applications [26-28]. The FGP utilizes the fuzzy regression models, pay-off matrices, and desirability function to transform multiple objectives into a single equivalent objective function with the consideration of overall desirability. The combined genetic-FGP procedure can identify the fuzzy optimal factor settings to enhance multiple quality responses of a product simultaneously. In this context, this research develops a genetic-fuzzy procedure for optimizing the process performance with multiple quality characteristics under uncertainty. The remaining of this research is outlined as follows. Section two presents the proposed procedure. Section three provides three case studies for procedure illustration. Section four discusses research results. Section five summarizes research conclusions.

2. Proposed Genetic-Fuzzy (G-F) Procedure

Assume there are Q responses to enhances by optimizing a process of J controllable factors. Suppose that the experimental work was repeated, which resulted in having K replicate values of each response. The proposed G-F procedure is depicted in Fig. 1 and is described as follows:

Step 1: Let $y_{rk}(x)$ denotes regression model of replicate k; k=1,…, K, for quality characteristic r; r =1,…, Q, formulated as a function of J process factors, x_f; f=1,…, J. Formulate $y_{rk}(x)$ as follows:

$$y_{rk}(x) = \beta_{0k} + \sum_{f=1}^{v} \beta_{fk} x_f + \sum_{f=1}^{J} \beta_{ffk} x_f^2 + \sum_{g<f} \sum \beta_{fgk} x_f x_g + \varepsilon;\ \forall k, \forall r \quad (1)$$

where β_{0k} is the intercept, and β_{fk}, β_{fgk}, , and β_{ffk} are the crisp coefficients of process factors in the regression model.

Step 2: Formulate an optimization model for each response's replicate utilizing Eq. (1) and a set of constraints on process settings. Use the GA technique to determine the optimal settings of controllable process factors. Repeat this step for all responses' replicates.

Step 3: Formulate the fuzzy multiple regression model, $\tilde{y}_r(\tilde{x})$, for the response r using the regression coefficients of $y_{rk}(x)$ from all response replicates. Let $\tilde{\beta}_0$, $\tilde{\beta}_{ff}$, and $\tilde{\beta}_{fg}$ be by fuzzy number of regression coefficients. Then, the $\tilde{y}_r(\tilde{x})$ can be expressed as stated in Eq. (2).

$$\tilde{y}_r(\tilde{x}) = \tilde{\beta}_0 + \sum_{f=1}^{J} \tilde{\beta}_f \tilde{x}_f + \sum_{f=1}^{J} \tilde{\beta}_{ff} \tilde{x}_f^2 + \sum_{g<f} \sum \tilde{\beta}_{fg} \tilde{x}_f \tilde{x}_g + \varepsilon \qquad \forall r, \forall f \quad (2)$$

where $\tilde{\beta}$ $(\beta^l, \beta^m, \beta^u)$ is obtained using Eq. (3).

$$\tilde{\beta} = (\beta^{\mathrm{m}} = \text{Average}\,\beta, \beta^{\mathrm{l}} = (\beta^{\mathrm{m}} - \delta \mathrm{s}), \beta^{\mathrm{u}} = (\beta^{\mathrm{m}} + \delta \mathrm{s})) \quad (3)$$

The δ is a constant value set by process engineers based on knowledge of allowable variability in process factor levels. The s is the estimated standard of the values of the β coefficient in regression models of all replicates of a quality characteristic. Formulate the $\tilde{y}_r(\tilde{x})$ for all quality characteristics.

Step 4: Let $\tilde{x}_q$ denotes the fuzzy optimal factor settings for the qth response; $q \in Q$, and $y_r(\tilde{x}^q)$ be the value of response r resulted from substituting the values of $\tilde{x}_q$. Construct the $y_r(\tilde{x}^q)$ matrix as displayed in Table 1.

Table 1. The $y_r(\tilde{x}^q)$ matrix.

$\tilde{x}_q$	$\tilde{y}_1(\tilde{x}^q)$	…	$\tilde{y}_r(\tilde{x}^q)$	…	$\tilde{y}_Q(\tilde{x}^q)$
$\tilde{x}^1$	$\tilde{y}_1(\tilde{x}^1)$	…	$\tilde{y}_r(\tilde{x}^1)$	…	$\tilde{y}_Q(\tilde{x}^1)$
⋮	⋮	⋮	⋮	⋮	⋮
$\tilde{x}^Q$	$\tilde{y}_1(\tilde{x}^Q)$	…	$\tilde{y}_r(\tilde{x}^Q)$	…	$\tilde{y}_Q(\tilde{x}^Q)$

Step 5: Formulate the fuzzy desirability function, $\tilde{s}_r(\tilde{y}_r(\tilde{x}))$, of $\tilde{y}_r(\tilde{x})$ depending on the type of the quality characteristic. Mathematically, the $\tilde{s}_r(\tilde{y}_r(\tilde{x}))$ functions for the nominal-the-best (NTB) response is expressed as stated in Eq. (4).

$$\tilde{s}_r(\tilde{y}_r(\tilde{x})) = \begin{cases} \dfrac{\tilde{y}_r(\tilde{x}) - \tilde{y}_{\min}}{\tilde{G} - \tilde{y}_{\min}}, & \tilde{y}_{\min} \le \tilde{y}_r(\tilde{x}^q) \le \tilde{G} \\ \dfrac{\tilde{y}_r(\tilde{x}) - \tilde{y}_{\max}}{\tilde{G} - \tilde{y}_{\max}}, & \tilde{G} \le \tilde{y}_r(\tilde{x}^q) \le y_{\max} \\ 0, & \tilde{y}_r(\tilde{x}^q) \ge \tilde{y}_{\max} \text{ or } \tilde{y}_r(\tilde{x}^q) \le \tilde{y}_{\min} \end{cases} \tag{4}$$

where $\tilde{G}$ is the nominal response value, and $\tilde{y}_{min}$ and $\tilde{y}_{max}$ are the minimal and maximal specified response values, respectively. Further, the $\tilde{s}_r(\tilde{y}_r(\tilde{x}))$ functions for larger-the-better (LTB) and smaller the better (STB) type responses are given in Eqs. (5) and (6), respectively.

$$\tilde{s}_r(\tilde{y}_r(\tilde{x})) = \begin{cases} 0, & \tilde{y}_r(\tilde{x}^q) \le \tilde{y}_{\min} \\ \dfrac{\tilde{y}_r(\tilde{x}) - \tilde{y}_{\min}}{\tilde{y}_{\max} - \tilde{y}_{\min}}, & \tilde{y}_{\min} \le \tilde{y}_r(\tilde{x}^q) \le \tilde{y}_{\max} \\ 1, & \tilde{y}_r(\tilde{x}^q) \ge \tilde{y}_{\max} \end{cases} \tag{5}$$

$$\tilde{s}_r(\tilde{y}_r(\tilde{x})) = \begin{cases} 1, & \tilde{y}_r(\tilde{x}^q) \le \tilde{y}_{\min} \\ \dfrac{\tilde{y}_r(\tilde{x}) - \tilde{y}_{\max}}{\tilde{y}_{\min} - \tilde{y}_{\max}}, & \tilde{y}_{\min} \le \tilde{y}_r(\tilde{x}^q) \le \tilde{y}_{\max} \\ 0, & \tilde{y}_r(\tilde{x}^q) \ge \tilde{y}_{\max} \end{cases} \tag{6}$$

Construct the matrix of $\tilde{s}_r(\tilde{y}_j(\tilde{x}^q))$ values as shown in Table 2.

Table 2. The $\tilde{s}_r(\tilde{y}_j(\tilde{x}^q))$ matrix.

	$\tilde{s}_1(\tilde{y}_1(\tilde{x}^q))$	…	$\tilde{s}_r(\tilde{y}_r(\tilde{x}^q))$	…	$\tilde{s}_Q(\tilde{y}_Q(\tilde{x}^q))$
$\tilde{x}^1$	$\tilde{s}_1(\tilde{y}_1(\tilde{x}^1))$	…	$\tilde{s}_r(\tilde{y}_r(\tilde{x}^1))$	…	$\tilde{s}_Q(\tilde{y}_Q(\tilde{x}^1))$
⋮	⋮	⋮	⋮	⋮	⋮
$\tilde{x}^Q$	$\tilde{s}_1(\tilde{y}_1(\tilde{x}^Q))$	…	$\tilde{s}_r(\tilde{y}_r(\tilde{x}^Q))$	…	$\tilde{s}_Q(\tilde{y}_Q(\tilde{x}^Q))$

Let

$$\tilde{p}_r = \tilde{s}_r(\tilde{y}_r(\tilde{x}^r)), \quad r = 1, \ldots, Q \tag{7}$$

and

$$\tilde{w}_r = Min\left\{\tilde{s}_r(\tilde{y}_r(\tilde{x}^1)), \ldots, \tilde{s}_r(\tilde{y}_r(\tilde{x}^Q))\right\}, \quad r = 1, \ldots, Q \tag{8}$$

Identify the values of $\tilde{p}_r$ and $\tilde{w}_r$.

Step 6: Develop the deviation function, $\tilde{S}_r(\tilde{y}_r(\tilde{x}^q))$, using Eq. (9).

$$\tilde{S}_r(\tilde{y}_r(\tilde{x})) = \frac{\tilde{y}^u_r(\tilde{x}) - \tilde{y}^m_r(\tilde{x})}{1-\lambda}, \quad r = 1, \ldots, Q \tag{9}$$

where λ has a value range between zero and one. Calculate and then list the $\tilde{S}_r(\tilde{y}_r(\tilde{x}^q))$ values as shown in Table 3.

Calculate the values of $\tilde{S}_r(\tilde{y}_r(\tilde{x}^q))$ values then determine the values of $\tilde{P}_r$ and $\tilde{W}_r$ as states in Eqs. (10) and (11), respectively.

$$\tilde{P}_r = \tilde{S}_r(\tilde{y}_r(\tilde{x}^r)), \quad r = 1, \ldots, Q \tag{10}$$

$$\tilde{W}_r = Max\left\{\tilde{S}_r(\tilde{y}_r(\tilde{x}^1)), \ldots, \tilde{S}_r(\tilde{y}_r(\tilde{x}^q))\right\}, \quad r = 1, \ldots, Q \tag{11}$$

Table 3. The $\tilde{S}_r(\tilde{y}_r(\tilde{x}^q))$ matrix.

$\tilde{x}^q$	$\tilde{S}_1(\tilde{y}_1(\tilde{x}^q))$	…	$\tilde{S}_r(\tilde{y}_r(\tilde{x}^q))$	…	$\tilde{S}_Q(\tilde{y}_Q(\tilde{x}^q))$
$\tilde{x}^1$	$\tilde{S}_1(\tilde{y}_1(\tilde{x}^1))$	…	$\tilde{S}_r(\tilde{y}_r(\tilde{x}^1))$	…	$\tilde{S}_Q(\tilde{y}_Q(\tilde{x}^1))$
⋮	⋮	⋮	⋮	⋮	⋮
$\tilde{x}^Q$	$\tilde{S}_1(\tilde{y}_1(\tilde{x}^Q))$	…	$\tilde{S}_r(\tilde{y}_r(\tilde{x}^Q))$	…	$\tilde{S}_Q(\tilde{y}_Q(\tilde{x}^Q))$

Step 7: Formulate a two-objective optimization model as follows:

$$\begin{aligned} & Max\left\{\tilde{s}_1(\tilde{y}_1(\tilde{x})), \ldots, \tilde{s}_r(\tilde{y}_r(\tilde{x}))\right\} \\ & Min\left\{\tilde{S}_1(\tilde{y}_1(\tilde{x})), \ldots, \tilde{S}_Q(\tilde{y}_Q(\tilde{x}))\right\} \\ & s.t \\ & x \in [\text{Factor settings}] \end{aligned}$$

The optimization models with two objective functions can transformed into a single-objective optimization model as follows. Let $\tilde{Z}_r(\tilde{y}_r(\tilde{x}))$ and $\tilde{T}_r(\tilde{y}_r(\tilde{x}))$ are two fuzzy functions indicating the degrees of satisfaction from desirability and robustness, respectively, and are defined as follows:

$$\tilde{Z}_r(\tilde{y}_r(\tilde{x})) = (Z^l_r, Z^m_r, Z^u_r) \tag{12}$$

and

$$\tilde{T}(\tilde{y}_r(\tilde{x})) = (T^l_r, T^m_r, T^u_r) \tag{13}$$

Then, $\tilde{Z}_r(\tilde{y}_r(\tilde{x}))$ and $\tilde{T}_r(\tilde{y}_r(\tilde{x}))$ functions are formulated as shown in Eqs. (14) and (15), respectively.

$$\tilde{Z}_r(\tilde{y}_r(\tilde{x})) = \begin{cases} 0, & \tilde{s}_r(\tilde{y}_r(\tilde{x}^r)) \le \tilde{w}_r \\ \dfrac{\tilde{s}_r(\tilde{y}_r(\tilde{x})) - \tilde{L}_r}{\tilde{p}_r - \tilde{w}_r}, & \tilde{w}_r \le \tilde{s}_r(\tilde{y}_r(\tilde{x}^r)) \le \tilde{p}_r \\ 1, & \tilde{s}_r(\tilde{y}_r(\tilde{x}^r)) \ge \tilde{p}_r \end{cases} \tag{14}$$

$$\tilde{T}_r(\tilde{y}_r(\tilde{x})) = \begin{cases} 1, & \tilde{S}_r(\tilde{y}_r(\tilde{x}^r)) \le \tilde{P}_r \\ \dfrac{\tilde{W}_r - \tilde{S}_r(\tilde{y}_r(\tilde{x}))}{\tilde{W}_r - \tilde{P}_r}, & \tilde{P}_r \le \tilde{S}_r(\tilde{y}_r(\tilde{x}^r)) \le \tilde{W}_r \\ 0, & \tilde{S}_r(\tilde{y}_r(\tilde{x}^r)) \ge \tilde{W}_r \end{cases} \tag{15}$$

To maximize the minimum degree of satisfaction from two objectives, let:

$$Min\ \tilde{Z}_r(\tilde{y}_r(\tilde{x})) = \tilde{Z} \tag{16}$$

and

$$Min\ \tilde{T}_r(\tilde{y}_r(\tilde{x})) = \tilde{T} \tag{17}$$

Finally, let a_1 and a_2 be the weights for desirability and robustness. The final optimization model will be expressed as:

$$
\begin{aligned}
& Max \quad a_1 \tilde{Z} + a_2 \tilde{T} \\
& s.t \\
& \tilde{s}_r(x) - \tilde{Z}(\tilde{p}_r - \tilde{w}_r) \geq \tilde{w}_r \qquad , \ r = 1,\ldots,Q \\
& \tilde{S}_r(x) + \tilde{T}(\tilde{W}_r - \tilde{P}_r) \leq \tilde{P}_r \qquad , \ r = 1,\ldots,Q \\
& a_1 + a_2 = 1 \\
& 0 \leq \tilde{Z} \leq 1 \\
& 0 \leq \tilde{T} \leq 1 \\
& x \in [\text{Factor Levels}].
\end{aligned}
\tag{18}
$$

The fuzzy optimization model shown in model (18) is expressed by three models: lower, middle, and upper denoted by *l*, *m*, and *u*, respectively. Obtain the fuzzy optimal levels, $\tilde{x}^* = (x_1^{l,m,u}, \ldots, x_J^{l,m,u})$, of the controllable process factors. Then, estimate the fuzzy response values, $\tilde{y}^* = (y^l, y^m, y^u)$, at the fuzzy optimal process settings.

Step 8: Apply the proposed procedure on a manufacturing process and then compare the anticipated improvements in the multiple quality characteristics obtained using the proposed genetic-fuzzy procedure and the previously employed approaches.

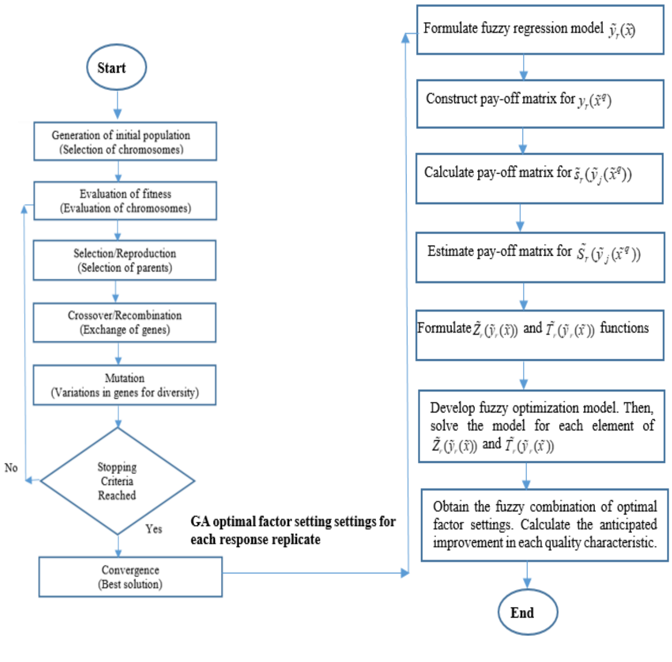

Fig.1. Depiction of the proposed genetic-fuzzy procedure.

3. Illustrations

Three widely studied case studies on the applications of the Taguchi method are provided for illustrating the proposed procedure and presented as follows.

Case I: WEDM process

This case study [5] aimed to optimize the performance of WEDM process for two important responses; y_1: material removal rate (mm^2/min, MRR, LTB) and y_2: surface roughness (mm^2/min, SR, STB). Let the y_{11} and y_{12} represent the first and second replicate of MRR, y_1, respectively. Let the y_{21} and y_{22} represent the first and second replicate of SR, y_2, respectively. Four controllable process factors: the pulse on time (x_1), delay time (x_2), wire feed speed (x_3), and ignition current (x_4), were examined utilizing the L_9 array shown in Table 4.

Table 4: Experimental data for WEDM process.

Ex. *i*	Control factor				MRR (mm^2/min)		SR (mm^2/min)	
	x_1	x_2	x_3	x_4	y_{i11}	y_{i12}	y_{i21}	y_{i22}
1	0.6	4	8	8	46	46	3.2	3.1
2	0.6	6	12	12	48	47	3.3	3.2
3	0.6	8	15	16	42	41	3.3	3.3
4	0.8	4	12	16	56	55	3.8	3.7
5	0.8	6	15	8	50	49	3.4	3.5
6	0.8	8	8	12	52	53	3.2	3.3
7	1.2	4	15	12	70	71	4.2	4
8	1.2	6	8	16	74	73	3.8	3.5
9	1.2	8	12	8	64	64	3.4	3.3

Initially, the multiple regression models were formulated for each of the four response replicates. For illustration, the regression model for y_{11} is expressed as:

$$
\begin{aligned}
y_{11} = & -2.80 + 41.50\,x_1 + 4.30x_2 - 1.25\,x_3 + 3.90x_4 \\
& + 0.13\,x_2x_3 - 0.58\,x_2^2 - 0.14\,x_4^2
\end{aligned}
$$

Further, the controllable factors; x_1, …, x_4, are decided the following acceptable operating ranges of based on experimental knowledge:

$0.6 \leq x_1 \leq 1.2 \quad 4 \leq x_2 \leq 8 \quad 8 \leq x_3 \leq 15 \quad 8 \leq x_4 \leq 16$

The GA technique (selection-stochastic uniform; cross-over fraction of 0.6; Mutation- uniform and ratio of 0.5; cross-over-heuristic and ratio of 1.4; Migration-forward) was then solved to determine optimal factor levels for each replicate as shown in Table 5.

Table 5. The GA optimal factor settings for WEDM process.

Factor	y_1		y_2	
	y_{11}	y_{12}	y_{21}	y_{22}
x_1^*	0.60	0.60	0.83	0.96
x_2^*	4.01	4.01	7.99	7.99
x_3^*	14.99	14.99	8.09	8.13
x_4^*	**8.01**	**8.01**	**8.00**	**8.00**

Utilizing the optimal factor settings in Table 5, the fuzzy optimal factor levels, $\tilde{x}^1$, for y_1 is expressed as

$$
\tilde{x}^1 = \{\tilde{x}_1 = (0.60, 0.60, 0.60);\ \tilde{x}_2 = (4.01, 4.01, 4.01);\ \tilde{x}_3 = (14.99, 14.99, 14.99);\ \tilde{x}_4 = (8.01, 8.01, 8.01)\}
$$

Then, the fuzzy regression, $\tilde{y}_1(\tilde{x})$ for y_1 is then formulated respectively as follows:

$$
\begin{aligned}
\tilde{y}_{1_1}(\tilde{x}) = & (-2.72, -2.69, -2.65) + (41.79, 41.88, 41.97)\tilde{x}_1 + \\
& (2.65, 2.94, 3.29)\tilde{x}_2 + (-1.22, -1.21, -1.20)\tilde{x}_3 + \\
& (4.35, 4.47, 4.59)\tilde{x}_4 + (-0.48, -0.46, -0.43)\tilde{x}_2^2 + \\
& (0.12, 0.12, 0.12)\tilde{x}_2\tilde{x}_3 + (-0.17, -0.17, -0.16)\tilde{x}_2\tilde{x}_4.
\end{aligned}
$$

The fuzzy minimal, $\tilde{y}_{1\min}$, and maximal, $\tilde{y}_{1\max}$, acceptable ranges for y_1 (LTB) were set the fuzzy numbers (37, 37, 37) and (65, 65, 65), respectively. Then, the desirability function $\tilde{s}_1(\tilde{y}_1(\tilde{x}))$ is formulated as:

$$
\tilde{s}_1(\tilde{y}_1(\tilde{x})) = \begin{cases} 0, & \tilde{y}_1(\tilde{x}^q) \leq 37 \\ \dfrac{\tilde{y}_1(\tilde{x}) - 37}{65 - 37}, & 37 \leq \tilde{y}_1(\tilde{x}^q) \leq 65 \\ 1, & \tilde{y}_1(\tilde{x}^q) \geq 65 \end{cases}
$$

Then, the fuzzy regression, $\tilde{y}_2(\tilde{x})$ and $\tilde{s}_2(\tilde{y}_2(\tilde{x}))$ for y_2 are expressed respectively as follows:

$$\tilde{y}_2(\tilde{x}) = (1.18, 1.33, 1.47) + (3.55, 3.92, 4.28)\tilde{x}_1 + (-0.045, -0.044, -0.043)\tilde{x}_2 + (0.025, 0.025, 0.025)\tilde{x}_3 + (0.018, 0.019, 0.02)\tilde{x}_4 + (-1.37, -1.18, -0.98)\tilde{x}_1^2 + (-0.18, -0.17, -0.16)\tilde{x}_1\tilde{x}_2 + (0.007, 0.008, 0.009)\tilde{x}_2^2$$

and

$$\tilde{s}_2(\tilde{y}_2(\tilde{x})) = \begin{cases} 1, & \tilde{y}_2(\tilde{x}^q) \le 2.4 \\ \dfrac{\tilde{y}_2(\tilde{x}) - 4.5}{2.4 - 4.5}, & 2.4 \le \tilde{y}_2(\tilde{x}^q) \le 4.5 \\ 0, & \tilde{y}_2(\tilde{x}^q) \ge 4.5 \end{cases}$$

Table 6 displays the $\tilde{s}_r(\tilde{y}_r(\tilde{x}^q))$ values for both responses, in which both $\tilde{p}_1$ and $\tilde{w}_1$ values are equal to (0.023, 0.141, 0.26), whereas $\tilde{p}_2$ and w_2 are calculated as (0.243, 0.61, 0.967) and (0.243, 0.551, 0.783), respectively. The $\tilde{s}_r(\tilde{y}_r(\tilde{x}^q))$ values are then used to calculate the fuzzy deviation function, $\tilde{S}_r(\tilde{y}_r(\tilde{x}^q))$, for y_1 and y_2 are expressed respectively as:

$$\tilde{S}_1(\tilde{y}_1(\tilde{x}^q)) = 0.038 + 0.108\tilde{x}_1 + 0.342\tilde{x}_2 + 0.009\tilde{x}_3 + 0.148\tilde{x}_4 + 0.031\tilde{x}_2\tilde{x}_2 + 0.002\tilde{x}_2\tilde{x}_3 + 0.006\tilde{x}_4\tilde{x}_4$$

and

$$\tilde{S}_2(\tilde{y}_2(\tilde{x}^q)) = 0.168 + 0.427\tilde{x}_1 + 0.001\tilde{x}_2 + 0.0005\tilde{x}_3 + 0.002\tilde{x}_4 + 0.225\tilde{x}_1^2 + 0.008\tilde{x}_1\tilde{x}_2 + 0.001\tilde{x}_2\tilde{x}_2$$

Table 7 displays the $\tilde{S}_r(\tilde{y}_r(\tilde{x}^q))$ values for both responses, from which the $\tilde{P}_1$ and $\tilde{P}_2$ values are (3.89, 3.89, 3.89) and (0.874. 0.874, 0.874), respectively, while the $\tilde{W}_1$ and $\tilde{W}_2$ values are estimated as (6.717, 6.719, 6.72) and (0.874. 0.874, 0.874), respectively

Table 6. The $\tilde{s}_r(\tilde{y}_r(\tilde{x}^q))$ values of MMR and SR (Case I).

	$\tilde{s}_1(\tilde{y}_1(\tilde{x}^r))$	$\tilde{s}_2(\tilde{y}_2(\tilde{x}^r))$
$\tilde{x}^1$	(0.023, 0.141, 0.26)	(0.32, 0.551, 0.783)
$\tilde{x}^2$	(0.312, 0.536, 0.761)	(0.243, 0.61, 0.967)

Table 7. The $\tilde{S}_r(\tilde{y}_r(\tilde{x}^q))$ values for WEDM process.

	$\tilde{S}_1(\tilde{y}_1(\tilde{x}^q))$	$\tilde{S}_2(\tilde{y}_2(\tilde{x}^q))$
$\tilde{x}^1$	(3.89, 3.89, 3.89)	(0.572, 0.572, 0.572)
$\tilde{x}^2$	(6.717, 6.719, 6.72)	(0.874, 0.887, 0.899)

Finally, the $\tilde{Z}_r(\tilde{y}_r(\tilde{x}))$ function is formulated for the two responses as follows:

$$\tilde{Z}_1(\tilde{y}_1(\tilde{x})) = \begin{cases} 0 & , \tilde{s}_1(\tilde{y}_1(\tilde{x}^1)) \le (0.023, 0.141, 0.26) \\ \dfrac{\tilde{s}_1(\tilde{y}_1(\tilde{x})) - (0.023, 0.141, 0.26)}{(0.023, 0.141, 0.26) - (0.023, 0.141, 0.26)} & , (0.023, 0.141, 0.26) \le \tilde{s}_1(\tilde{y}_1(\tilde{x}^1)) \le (0.023, 0.141, 0.26) \\ 1 & , \tilde{s}_1(\tilde{y}_1(\tilde{x}^1)) \ge (0.023, 0.141, 0.26) \end{cases}$$

and

$$\tilde{Z}_2(\tilde{y}_2(\tilde{x})) = \begin{cases} 0, & \tilde{s}_2(\tilde{y}_2(\tilde{x}^2)) \le (0.243, 0.551, 0.783) \\ \dfrac{\tilde{s}_2(\tilde{y}_2(\tilde{x})) - (0.243, 0.551, 0.783)}{(0.243, 0.61, 0.967) - (0.243, 0.551, 0.783)}, & (0.243, 0.551, 0.783) \le \tilde{s}_2(\tilde{y}_2(\tilde{x}^2)) \le (0.243, 0.61, 0.967) \\ 1, & \tilde{s}_2(\tilde{y}_2(\tilde{x}^2)) \ge (0.243, 0.61, 0.967) \end{cases}$$

In a similar manner, the $\tilde{T}_r(\tilde{y}_r(\tilde{x}))$ functions for y_1 and y_2 are written respectively as:

$$\tilde{T}_1(\tilde{y}_1(\tilde{x})) = \begin{cases} 1, & \tilde{S}_1(\tilde{y}_1(\tilde{x}^1)) \le (3.89, 3.89, 3.89) \\ \dfrac{(6.717, 6.719, 6.72) - \tilde{S}_1(\tilde{y}_1(\tilde{x}))}{(6.717, 6.719, 6.72) - (3.89, 3.89, 3.89)}, & (3.89, 3.89, 3.89) \le \tilde{S}_1(\tilde{y}_1(\tilde{x}^1)) \le (6.717, 6.719, 6.72) \\ 0, & \tilde{S}_1(\tilde{y}_1(\tilde{x}^1)) \ge (6.717, 6.719, 6.72) \end{cases}$$

and

$$\tilde{T}_2(\tilde{y}_2(\tilde{x})) = \begin{cases} 1, & \tilde{S}_2(\tilde{y}_2(\tilde{x}^2)) \le (0.874, 0.887, 0.899) \\ \dfrac{(0.874, 0.887, 0.899) - \tilde{S}_2(\tilde{y}_2(\tilde{x}\ ))}{(0.874, 0.887, 0.899) - (0.874, 0.887, 0.899)}, & (0.874, 0.887, 0.899) \le \tilde{S}_2(\tilde{y}_2(\tilde{x}^2)) \le (0.874, 0.887, 0.899) \\ 0, & \tilde{S}_2(\tilde{y}_2(\tilde{x}^2)) \ge (0.874, 0.887, 0.899) \end{cases}$$

$$\tilde{Z}_r(\tilde{y}_r(\tilde{x})) = \begin{cases} 0, & \tilde{s}_r(\tilde{y}_r(\tilde{x}^r)) \le \tilde{w}_r \\ \dfrac{\tilde{s}_r(\tilde{y}_r(\tilde{x})) - \tilde{L}_r}{\tilde{p}_r - \tilde{w}_r}, & \tilde{w}_r \le \tilde{s}_r(\tilde{y}_r(\tilde{x}^r)) \le \tilde{p}_r \\ 1, & \tilde{s}_r(\tilde{y}_r(\tilde{x}^r)) \ge \tilde{p}_r \end{cases}$$

$$\tilde{T}_r(\tilde{y}_r(\tilde{x})) = \begin{cases} 1, & \tilde{S}_r(\tilde{y}_r(\tilde{x}^r)) \le \tilde{P}_r \\ \dfrac{\tilde{W}_r - \tilde{S}_r(\tilde{y}_r(\tilde{x}))}{\tilde{W}_r - \tilde{P}_r}, & \tilde{P}_r \le \tilde{S}_r(\tilde{y}_r(\tilde{x}^r)) \le \tilde{W}_r \\ 0, & \tilde{S}_r(\tilde{y}_r(\tilde{x}^r)) \ge \tilde{W}_r \end{cases}$$

Finally, the optimization models at the lower, middle and upper; *l, m, and u,* respectively, were constructed. For example, the optimization model at lower bound of the fuzzy number is expressed as:

$$\boldsymbol{Max 0.5 \times S^m + 0.5 \times T^m}$$

$$\boldsymbol{s.t}$$

$$\boldsymbol{-1.418 + 1.492x_1^l + 0.094x_2^l - 0.043x_3^l + 0.155x_4^l - 0.017x_2^l x_2^l + 0.004x_2^l x_3^l - 0.006x_4^l x_4^l - 0S^l \ge 0.023}$$

$$\boldsymbol{1.58 - 1.69x_1^l + 0.021x_2^l - 0.011x_3^l - 0.008x_4^l + 0.653x_1^l x_1^l + 0.084x_1^l x_2^l - 0.003x_2^l x_2^l + 0S^l \ge 0.243}$$

$$\boldsymbol{0.038 + 0.108x_1^l + 0.342x_2^l + 0.009x_3^l + 0.148x_4^l + 0.031x_2^l x_2^l + 0.002x_2^l x_3^l + 0.006x_4^l x_4^l + 2.826T^l \le 6.717}$$

$$\mathbf{0.168 + 0.427}x_1^l + \mathbf{0.001}x_2^l + \mathbf{0.0005}x_3^l + \mathbf{0.002}x_4^l + \mathbf{0.225}x_1^l x_1^l + \mathbf{0.008}x_1^l x_2^l + \mathbf{0.001}x_2^l x_2^l + 0T^l \le \mathbf{0.874}$$

$$0 \le S^l \le 1$$

$$0 \le T^l \le 1$$

$$X = \{x_1^l, x_2^l, x_3^l, x_4^l\} \in [\text{Factor Levels}]$$

In a similar manner, the optimization models were developed at middle and upper bound of the fuzzy number. The three models were then solved using Lingo 11 Software to determine the values of the fuzzy optimal factor settings. The optimization results showed that be the *l*, *m* and *u* optimal factor settings; $x^{*l} = (x_1^*, x_2^*, x_3^*, x_4^*)$, $x^{*m} = (x_1^*, x_2^*, x_3^*, x_4^*)$, and $x^{*u} = (x_1^*, x_2^*, x_3^*, x_4^*)$, respectively, are (0.6, 4.33, 8, 8), (0.6, 4, 8, 8) and (0.60, 4, 8, 8). At these fuzzy optimal factor settings, the calculated $\tilde{y}_1$ and $\tilde{y}_2$ values were (55.72, 57.73, 60.16) and (2.65, 3.16, 3.64), respectively.

Case study II: Gasoline Production Process

Bashiri *et al.* [22] used the regression and artificial neural network (ANN) approaches to enhance the vapor pressure (y_1, RVP, LTB), rate of octane number (y_2, RON, STB) and density (y_3, DEN, LTB) in a gasoline production process. Five controllable process factors were studied utilizing of the L_8 array shown in Table 8.

Table 8. Experimental design for gasoline process.

Ex.	Control factors					Responses (2 replicates)					
i	x_1	x_2	x_3	x_4	x_5	y_{11i}	y_{12i}	y_{21i}	y_{22i}	y_{31i}	y_{32i}
1	3.5	5	3	3	5	64	63	89	88.3	0.751	0.751
2	5	10	3	3	5	63	62.5	93.5	92.4	0.755	0.755
3	3.5	5	5	5	5	62	60.5	88	87.1	0.753	0.754
4	5	10	5	5	5	61.5	60.5	94	93	0.756	0.756
5	5	5	5	3	7.5	63	62	93.2	92.1	0.759	0.759
6	3.5	10	5	3	7.5	62.5	61.5	91.8	91	0.756	0.756
7	5	5	3	5	7.5	62	61	93.5	92.4	0.758	0.759
8	3.5	10	3	5	7.5	61	59.5	91.5	90.5	0.757	0.757

The proposed procedure starts by adopting the GA technique to determine the optimal factor settings for each response replicate. The obtained optimal level values are shown in Table 9. Utilizing these values, the fitted fuzzy regression models $\tilde{y}_1(\tilde{x})$, $\tilde{y}_2(\tilde{x})$, and $\tilde{y}_3(\tilde{x})$ are constructed for responses y_1, and y_2, and y_3, respectively. For example, the $\tilde{y}_1(\tilde{x})$ is represented as:

$$\tilde{y}_1(\tilde{x}) = (70.28583, 70.4165, 70.54717) + (0.106727, 0.29165, 0.476573)x_1 + (-0.43864, -0.3875, -0.33636)x_2 + (-0.78271, -0.625, -0.46729)x_3 + (-0.83157, -0.75, -0.66843)x_4 + (-0.63158, -0.525, -0.41842)x_5 + (0.007387, 0.01, 0.012613)x_2x_3x_5$$

Then, the fuzzy desirability functions; $\tilde{s}_1(\tilde{y}_1(\tilde{x}^q))$, $\tilde{s}_2(\tilde{y}_2(\tilde{x}^q))$, and $\tilde{s}_3(\tilde{y}_3(\tilde{x}^q))$ for $\tilde{y}_1(\tilde{x})$, $\tilde{y}_2(\tilde{x})$, and $\tilde{y}_3(\tilde{x})$ are developed respectively as follows:

$$\tilde{s}_1(\tilde{y}_1(\tilde{x})) = \begin{cases} 0, & \tilde{y}_1(\tilde{x}^r) \le 57 \\ \dfrac{\tilde{y}_1(\tilde{x}) - 57}{66 - 57}, & 57 \le \tilde{y}_1(\tilde{x}^r) \le 66 \\ 1, & \tilde{y}_1(\tilde{x}^r) \ge 66 \end{cases}$$

$$\tilde{s}_2(\tilde{y}_2(\tilde{x})) = \begin{cases} 1, & \tilde{y}_2(\tilde{x}^r) \le 85 \\ \dfrac{\tilde{y}_2(\tilde{x}) - 98}{85 - 98}, & 85 \le \tilde{y}_2(\tilde{x}^r) \le 98 \\ 0, & \tilde{y}_2(\tilde{x}^r) \ge 98 \end{cases}$$

$$\tilde{s}_3(\tilde{y}_3(\tilde{x})) = \begin{cases} 0, & \tilde{y}_3(\tilde{x}^r) \le 0.75 \\ \dfrac{\tilde{y}_3(\tilde{x}) - 0.75}{0.78 - 0.75}, & 0.75 \le \tilde{y}_3(\tilde{x}^r) \le 0.78 \\ 1, & \tilde{y}_3(\tilde{x}^r) \ge 0.78 \end{cases}$$

The values of $\tilde{s}_r(\tilde{y}_r(\tilde{x}^q))$ are then calculated and then listed in Table 10. These values are employed to calculate the $\tilde{S}_r(\tilde{y}_r(\tilde{x}^q))$ values displayed in Table 11. The values of $x^{*l} = (x_1^*, x_2^*, x_3^*, x_4^*, x_5^*)$ are found (5, 5, 3, 3. 7.5). Similarly, the $x^{*m} = (x_1^*, x_2^*, x_3^*, x_4^*, x_5^*)$ and $x^{*u} = (x_1^*, x_2^*, x_3^*, x_4^*, x_5^*)$ values are found to be (5, 5, 3, 3.8, 7.5) and (4.74, 5, 5, 3.8, 7.5), respectively. At these fuzzy optimal factor settings, the corresponding response values of $\tilde{y}_1$, $\tilde{y}_2$ and $\tilde{y}_3$ are calculated as (59.877, 62.379, 65.476), (91.39, 93.17, 94.07) and (0.757, 0.7578, 0.7593), respectively.

Table 9. The GA optimal factor setting for gasoline process.

Factor	y_{11}	y_{12}	y_{21}	y_{22}	y_{31}	y_{32}
x_1^*	4.22	3.50	5.00	5.00	5.00	5.00
x_2^*	10.00	5.04	10.00	10.00	9.94	5.00
x_3^*	3.68	5.00	4.99	4.97	4.99	4.98
x_4^*	5.00	5.00	3.13	3.00	5.00	5.00
x_5^*	7.39	7.48	7.50	7.50	7.50	7.49

Table 10. The $\tilde{s}_r(\tilde{y}_r(\tilde{x}^q))$ values for gasoline process.

	$\tilde{s}_1(\tilde{y}_1(\tilde{x}^q))$	$\tilde{s}_2(\tilde{y}_2(\tilde{x}^q))$	$\tilde{s}_3(\tilde{y}_3(\tilde{x}^q))$
$\tilde{x}^1$	(0.13, 0.41, 0.94)	(0.08,0.54, 0.83)	(0.16, 0.24, 0.31)
$\tilde{x}^2$	(0.12, 0.60, 1.0)	(0.02, 0.24, 0.46)	(0.27, 0.30, 0.32)
$\tilde{x}^3$	(0.04, 0.44, 0.96)	(0, 0.34, 0.63)	(0.30, 0.32, 0.35)

Table 11. The $\tilde{S}_r(\tilde{y}_r(\tilde{x}^q))$ values for gasoline process.

	$\tilde{S}_1(\tilde{y}_1(\tilde{x}^q))$	$\tilde{S}_2(\tilde{y}_2(\tilde{x}^q))$	$\tilde{S}_1(\tilde{y}_1(\tilde{x}^q))$
$\tilde{x}^1$	(13.61, 15.95, 18.34)	(8.53, 11.36, 14.44)	(0.0022, 0.0028, 0.0034)
$\tilde{x}^2$	(17.61, 17.66, 17.71)	(14.16, 14.18, 14.2)	(0.0034, 0.0034, 0.0034)
$\tilde{x}^3$	(16.51, 17.64, 18.77)	(10.466, 12.74, 15.02)	(0.0029, 0.0032, 0.0035)

Case study III: Sputtering Process

Chen *et al.* [43] used the Taguchi-grey relational method optimize parameters of the sputtering process for three quality responses; deposition rate (y_1, LTB, DR), electrical resistively (y_2, STB, ER), and optical transmittance (y_3, LTB, OT). Five controllable process factors; x_1, …, x_5, were investigated via the L_{18} array shown in Table 12.

Table 12. Experimental data for the sputtering process.

Ex. i	Factor					DR		ER		OT	
	x_1	x_2	x_3	x_4	x_5	y_{i11}	y_{i12}	y_{i21}	y_{i22}	y_{i31}	y_{i32}
1	50	0.13	30	25	0	4.5	4.7	14.9	15.3	88.4	88.4
2	50	0.67	60	50	100	5.6	5.6	9.8	9.7	87.7	87.7
3	50	1.33	90	100	200	5.0	4.9	7.9	7.8	88.1	88.1
4	100	0.13	30	50	100	9.6	9.3	5.4	5.6	89.2	89.3
5	100	0.67	60	100	200	11.1	11.3	4.6	4.3	87.1	87.0
⁞	⁞	⁞	⁞	⁞	⁞	⁞	⁞	⁞	⁞	⁞	⁞
15	100	1.33	30	50	200	10.7	10.8	5.5	5.7	88.4	88.3
16	200	0.13	90	50	200	19.5	19.4	1.0	1.1	83.1	83.1
17	200	0.67	30	100	0	22.1	22.0	1.2	1.3	85.7	85.7
18	200	1.33	60	25	100	20.5	20.5	1.4	1.3	83.9	83.7

The regression models were developed for all response replicates. For illustration, the regression models for y_{11} is represented by:

$$y_{11}(x) = -1.94 + 0.12x_1 + 1.24x_2 + 0.02x_3 - 0.01x_4 + 0.01x_5 - 0.003x_1x_2 - 0.0001x_1x_3 + 0.0001x_1x_4 - 0.0001x_1x_5 - 0.014x_2x_3 - 0.000004x_5^2 - 0.000004x_2x_3x_4 \qquad R^2_{\text{adj}} = 98.81\%$$

While, the regression models for y_{12} is written as:

$$y_{12}(x) = -1.86 + 0.12x_1 + 0.98x_2 + 0.04x_3 - 0.02x_4 + 0.01x_5 + 0.0001x_1x_2 - 0.0002x_1x_3 + 0.0001x_1x_4 - 0.0001x_1x_5 - 0.02x_2x_3 + 0.00002x_5^2 + 0.00003x_2x_3x_4 \qquad R^2_{\text{adj}} = 97.93\%$$

The acceptable ranges of the controllable factors are decided as follows:

$$50 \le x_1 \le 200,\ 0.33 \le x_2 \le 1.33,\ 30 \le x_3 \le 90,\ 25 \le x_4 \le 100,\ 0 \le x_5 \le 200$$

The optimal settings of process factors were obtained by GA technique and then displayed in Table 13.

Table 13. GA optimal factor settings for sputtering process.

fact	y_{11}	y_{12}	y_{21}	y_{22}	y_{31}	y_{32}
x_1^*	50.00	50.00	199.68	199.92	99.98	99.99
x_2^*	1.33	1.33	0.34	1.33	1.33	1.33
x_3^*	89.93	30.00	43.62	89.98	89.99	89.88
x_4^*	25.00	25.00	94.75	25.00	75.77	61.83
x_5^*	0.001	0.14	70.28	28.28	4.85	11.88

Utilizing the results in Table 13, the fuzzy regression for $\tilde{y}_1(\tilde{x}), \tilde{y}_2(\tilde{x})$, and $\tilde{y}_3(\tilde{x})$ were developed for the three responses. For illustration, the regression model for $\tilde{y}_1(\tilde{x})$ is written as:

$$\tilde{y}_1(\tilde{x}) = (-1.95,-1.9,-1.85) + (0.12,0.12,0.12)\, x_1 + (0.96,1.11,1.26)\, x_2 + (0.02,0.03,0.04)\, x_3 + (-0.02,-0.017,-0.012)\, x_4 + (0.01,0.01,0.01)\, x_5 + (-0.003,-0.001,0.0003)x_1x_2 + (-0.0002,-0.0001,-0.0001)x_1x_3 + (0.0001,0.0001,0.0001)x_1x_4 + (-0.0001,-0.0001,-0.0001)x_1x_5 + (-0.02,-0.02,-0.01)x_2x_3 + (-.00001,0.00001,0.00003)x_2x_3x_4 + (-0.00001,0.00001,0.00002)x_5^2$$

Then, the fuzzy desirability functions, $\tilde{s}_r(\tilde{y}_r(\tilde{x}^q))$, are constructed for all quality responses. The $\tilde{s}_1(\tilde{y}_1(\tilde{x}^q))$, for example, is developed as:

$$\tilde{s}_1(\tilde{y}_1(\tilde{x}^q)) = \begin{cases} 0, & \tilde{y}_1(\tilde{x}^q) \le 4 \\ \dfrac{\tilde{y}_1(\tilde{x}) - 4}{24 - 4}, & 4 \le \tilde{y}_1(\tilde{x}^q) \le 24 \\ 1, & \tilde{y}_1(\tilde{x}^q) \ge 24 \end{cases}$$

The corresponding $\tilde{Z}_1(\tilde{y}_1(\tilde{x}))$ and $\tilde{T}_j(\tilde{y}_1(\tilde{x}))$ are then formulated as follows:

$$\tilde{Z}_1(\tilde{y}_1(\tilde{x})) = \begin{cases} 0, & \tilde{s}_1(\tilde{y}_1(\tilde{x}^q)) \le (0.013,0.054,0.15) \\ \dfrac{\tilde{s}_1(\tilde{y}_1(\tilde{x})) - (0.013,0.054,0.15)}{(0.013,0.054,0.15) - (0.013,0.054,0.15)}, & (0.013,0.054,0.15) \le \tilde{s}_1(\tilde{y}_1(\tilde{x}^q)) \le (0.013,0.054,0.15) \\ 1, & \tilde{s}_1(\tilde{y}_1(\tilde{x}^q)) \ge (0.013,0.054,0.15) \end{cases}$$

$$\tilde{T}_1(\tilde{y}_1(\tilde{x}) = \begin{cases} 1, & \tilde{S}_1(\tilde{y}_1(\tilde{x}^q)) \le (3.68,6.59,9.49) \\ \dfrac{(11.71,12.22,21.15) - \tilde{S}_1(\tilde{y}_1(\tilde{x}))}{(11.71,12.22,21.15) - (3.68,6.58,9.49)}, & (3.68,6.58,9.49) \le \tilde{S}_1(\tilde{y}_1(\tilde{x}^q)) \le (11.71,12.22,21.15) \\ 0, & \tilde{S}_1(\tilde{y}_1(\tilde{x}^q)) \ge (11.71,12.22,21.15) \end{cases}$$

The $\tilde{Z}_j(x)$ and $\tilde{T}_j(x)$ functions of the other responses are formulated in a similar manner.

Finally, the optimization models constructed at the l, m, and u bounds and then solved to obtain the fuzzy optimal settings of process factors shown in Table 14.

Table 13. The fuzzy optimal factor settings for sputtering process.

Factor	Model l	Model m	Model u
x_1^*	198.29	198.906	59.743
x_2^*	0.13	0.618	1.33
x_3^*	30.00	30.00	30.00
x_4^*	25.00	43.35	25.00
x_5^*	0.00	29.336	0.00

Substituting the fuzzy optimal settings shown in Table 13, the estimated values of $\tilde{y}_1$, $\tilde{y}_2$ and $\tilde{y}_3$ are (7.074, 20.54, 21.34), (1.27, 1.51, 9.62) and (87.42, 88.28, 88.53), respectively.

4. Results and Discussion

A comparison between the optimization results obtained using the GA-fuzzy procedure and those adopted on the studied case studies is conducted as follows:

- For the WEDM process [5] as shown in Fig. 2, the initial MRR and SR values were 52.15 and 3.43, respectively. Using the Taguchi method (GA-Fuzzy), the values of MRR and SR values at the optimal factor settings are 68.56 (55.72, 57.73, 60.16) and 3.45 (2.65, 3.16, 3.64), respectively. It is noticed that the Taguchi method provides the largest improvement in MRR, whereas the GA-fuzzy procedure resulted in the largest improvement in SR. However, the Taguchi method does not rely on mathematical relation between process factors and each response. Moreover, it ignores preferences on product and process settings, and lacks the ability to deal with uncertainty due to measurement and process variations.

- For the gasoline production process [22] as shown in Fig. 3, using the ANN technique (GA-fuzzy procedure) the obtained RVP (LTB), RON (STB) and DEN (LTB) values are calculated as 58.99 (59.87, 62.37, 65.477), 93.96 (91.39, 93.17, 94.07), and 0.76 (0.757, 0.7578, 0.7593), respectively. It is clear that the GA-fuzzy provide the largest improvements in RVP and RON, respectively. While, both approaches provide almost the same improvement in DEN. Although the ANN is widely used, it has a number of limitations, such as, its "black box" nature and limited ability to explicitly determine possible causal relationships, large computational burden, and proneness to overfitting. Moreover, the ANN requires sufficient data set for training and validation. Finally, the used ANN approach ignored the fuzziness nature in quality characteristics and process settings.

- For the sputtering process [43] as shown in Fig. 4, it is noticed that the DR (LTB), ER (STB) and OT (LTB) values at initial factor settings are 21.81, 1.61 and 85.73, respectively. The corresponding values at the optimal settings by using the Grey-Taguchi method are 22.19, 1.25 and 86.85, respectively. Using the GA-Fuzzy procedure, the DR, ER, and OT values range from 7.074 to 21.34, 1.27 to 9.62, and 87.42 to 88.3, respectively. It is seen that the grey-Taguchi method provides slightly larger improvements in the three quality characteristics. Using the GA-fuzzy approach the u, l, and u values of DR (= 21.34), ER (=1.27), and OT (= 88.3), respectively, are slightly differ from those obtained by the Taguchi method and the grey relational approach. However, the grey relational analysis is based on ranking rather than mathematical modeling and thus it may not provide optimal process settings. Besides, Taguchi method and grey relational approach failed in handling fuzziness of the quality characteristics, which is due to measurement and process variations. For illustration, using the GA-fuzzy procedure the DR values at the u (=21.34) and l (= 7.074) levels differ significantly because of the existence of variations, which reveals the effectiveness of this approach in handling uncertainty.

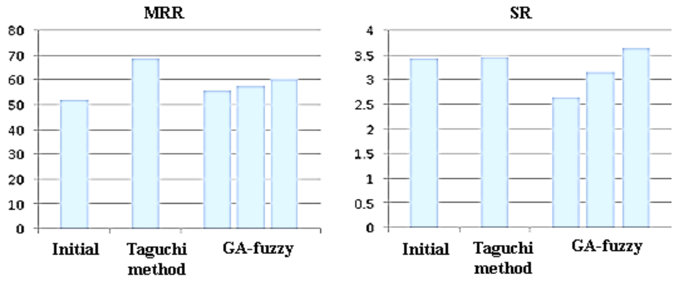

Fig. 2. Results comparison for WEDM process.

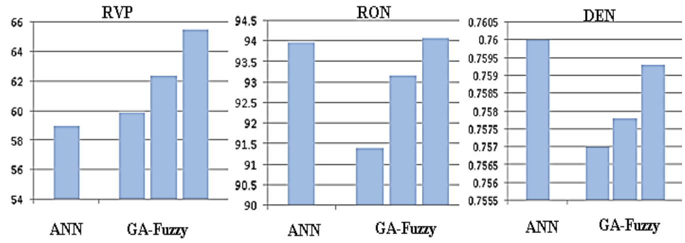

Fig. 3. Results comparison for gasoline process.

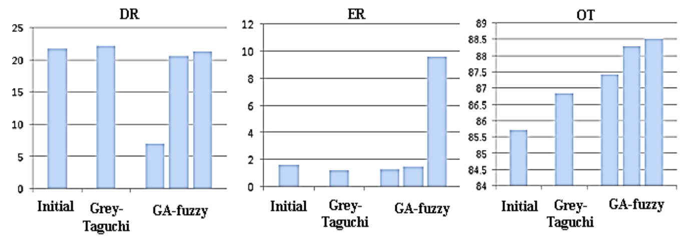

Fig. 4. Results comparison for sputtering process.

In summary, the proposed GA-fuzzy procedure has several benefits, including:

i. It effectively deals with uncertainty due to fuzzy quality characteristics and process settings by providing fuzzy rather than crisp optimal factor settings.
ii. It is relies on mathematical models to depict the relationships between the quality characteristic and process factors. This enables process engineers evaluate and predict accurately values of quality characteristic and process performance under fuzziness.
iii. It handles uncertainty/variations in the observations between the replicates of each response, which improves accuracy in determining the optimal factor settings.
iv. It conducts two-stage optimization. In the first stage, it uses the genetic algorithm technique, which is found effective providing optimal/near optimal solutions, to determine the crisp optimal factor settings for each response. Then, it adopts fuzzy goal programming to handle fuzziness in responses and process settings and then optimizing performance for multiple responses.

Nevertheless, the GA-fuzzy approach requires a moderate knowledge and proficiency in statistics and computer skills. In addition, process engineers should possess extensive understanding of experimental design and analysis, regression modelling and optimization. This may increase the complexity of this approach.

5. Conclusions

This research developed a GA-fuzzy procedure for optimizing a manufacturing process for multiple characteristics under uncertainty. The GA technique was initially adopted to optimize factor levels for each response's replicate separately, which were then employed to formulate a fuzzy regression for each response. The fuzzy desirability function and deviation functions were developed and finally utilized in constructing the fuzzy optimization model. Three case studies in manufacturing were used for illustration. In contrast to previously used approaches, results showed that the proposed procedure has efficiently optimized process settings under measurement and process variations. In conclusions, the proposed procedure provides valuable information to process engineers about the relationships between quality characteristics and process factors and the impact of uncertainty on product and process performance. Such feedback can support them in taking proper corrective and preventive actions. Future research considers combining neural networks and fuzzy goal programming to optimize process performance under uncertainty.

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