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Effectiveness of Logarithmic Entropy Measures for Pythagorean Fuzzy Sets in diseases related to Post COVID Implications under TOPSIS Approach

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Abstract: Following the second wave of Covid-19 infections in India, individuals are now arriving to hospitals with a variety of symptoms, not simply for mucormycosis, a fungal infection. The most common symptoms are extreme tiredness, drowsiness, body and joint pain, mental fog, and fever, but pneumonia, collapsed lungs, heart attacks, and strokes have all been reported. Pythagorean fuzzy sets (PFSs) proposed by Yager [42] offers a novel technique to characterize uncertainty and ambiguity with greater precision and accuracy. The idea was developed specifically to describe uncertainty and ambiguity mathematically and to provide a codified tool for dealing with imprecision in real-world circumstances. This article addresses novel logarithmic entropy measures under PFSs. Additionally, numerical illustration is utilized to ascertain the strength and validity of the proposed entropy measures. Application of the measures is used in detecting diseases related to Post COVID 19 implications through TOPSIS method. Comparison of the suggested measures with the existing ones is also demonstrated.

Keywords: *Decision making, Entropy measure, Intuitionistic fuzzy set, Logarithmic function, Pythagorean fuzzy sets, Weighted entropy measures,*

1. Introduction

Although most patients with COVID-19 recover within weeks after becoming unwell, some people develop post-COVID symptoms. After being infected with the virus that causes COVID-19, people might develop a wide range of new, returning, or chronic health problems known as post-COVID disorders. Even those who were asymptomatic when infected can develop post-COVID symptoms. For varying durations of time, these disorders can cause various uncertain kinds and combinations of health problems. To deal with such uncertainty in data, Zadeh [46] proposed FSs theory, in which each element is characterised by association grade, non-association grade lying between 0 and 1. Extension to this, Atanassov [6] fostered the idea of intuitionistic fuzzy sets (IFSs) for the better portrayal on vulnerability where the membership grade (δ) and non-membership grade (ζ) both are real numbers, and their summation is under 1. The difference between 1 and the summation precedes to hesitancy grade. The idea of IFSs appears to be practical in modelling many real-life circumstances like medicinal findings [11-13, 16, 35-37], career endurance [15], selection procedure [14], and multi-criteria decision-making [18-20] amongst others.

There are circumstances where $\delta + \zeta \geq 1$ unlike the cases obtained in IFSs. This inadequacy in IFSs surely preceded to a configuration, called PFSs. PFSs proposed by Yager [42-44] is a novel tool to contract with imprecision considering association degree δ and non-association ζ satisfying the conditions $\delta + \zeta \leq 1$ or $\delta + \zeta \geq 1$, and it follows that $\delta^2 + \zeta^2 + \eta^2 = 1$, where η is the PFS index. Different investigators hypothetically exploited the notion of Yager's [44] Pythagorean fuzzy sets and employed it in the field of dynamic, clinical finding, design acknowledgment and a lot more reasonable issue. To negotiate the dynamic issue with PFSs, Zhang and Xu [49] anticipated a similarity technique to arrange fuzzy PIS and fuzzy NIS. They extended the technique of order preference by similarity to ideal solution to ascertain the divergence between every option, separately. Some essential tasks for PFSs aggregation operators along with their significant properties was also discussed [32]. Another strategy for PFSs dynamic issues with the assistance of accumulation administrators and divergence measures has been created [47]. Aggregation operation using TOPSIS was also deliberated. Further, Yager [45] presented a portion of the fundamental set tasks for PFSs and set up the connection between Pythagorean membership grade and complex grade. Likewise, the arrangements of multicriteria dynamic with fulfillments through Pythagorean membership grade have been done. Many researchers analysed MADM approach using TOPSIS method [26]. Many Researchers [2-5, 8-9, 17, 24, 29, 40] and many more have applied TOPSIS method in various problems of decision making like supplier selection, selection of land, robotics, medical diagnosis, ranking of water quality, human resource selection personnel problem, and many other real-life situations flavoured with fuzzy sets and generalized fuzzy sets.

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In the information measure theory, the idea of entropy measure is used to determine the level of uncertainty of a set. For IFS, Burillo and Bustince [10] pioneered the concept of entropy. For IFSs, Hung and Yang [26] provide an axiomatic definition of entropy. Vlachos and Sergiadis [39] developed a mathematical model to compare FS and IFS similarity. The entropy for the ambiguous set was created [50]. The generalised entropy of order α and degree β for IFS was reported [21]. Garg [22] described an entropy weight and aggregation operators-based approach for solving DM problems. The distinct and generalized form of the entropy metrics for IFSs was introduced [23]. The ambiguous entropy measure for a complicated soft set was proposed by [34]. Peng and Selvachandran [31]. Interval type-2 fuzzy logic system was demonstrated for power quality improvement [1]. Implementation Based on Hybrid Structure for power System Using an Interval Type-2 TSK Fuzzy Logic Controller was discussed [28].

In this article, we are exploring the resourcefulness of entropy measures of PFSs in the application to identify disease of the patients suffering from Post COVID implications. This paper is organized as follows: Section 2 introduces preliminaries of FSs, IFSs, PFSs and other recent entropies developed. Section 3 comprises of the concept of proposed logarithmic entropy measures of PFSs. We introduce logarithmic entropy and weighted entropy measures of the PFSs and its numerical computations to validate our measures. Application is also provided in Section 4 using TOPSIS approach. Section 5 compares the new entropy measures with the existing similarity measure by an example. Finally, Section 6 summarizes the document and delivers directions for future experiments.

2. Preliminaries

In this segment, we bring in some fundamental theories associated to FSs, IFSs and PFSs used in the outcome.

Definition 2.1. [46]. A fuzzy set $\mathcal{M}$ in $\mho$ is characterized by a membership function:

$$\mathcal{M} = \{\langle u, \delta_{\mathcal{M}}(u)\rangle | u \in \mho\} \tag{1}$$

where $\delta_{\mathcal{M}}(u): \mho \to [0,1]$ is a measure of belongingness of degree of participation of an element $u \in \mho$ in $\mathcal{M}$.

Definition 2.2. [6]. An IFS $\mathcal{M}$ in $\mho$ is given by

$$\mathcal{M} = \{\langle u, \delta_{\mathcal{M}}(u), \zeta_{\mathcal{M}}(u) \rangle | u \in \mho\} \tag{2}$$

where $\delta_{\mathcal{M}}(u), \zeta_{\mathcal{M}}(u): \mathcal{M} \to [0,1]$, and $0 \le \delta_{\mathcal{M}}(u) + \zeta_{\mathcal{M}}(u) \le 1$, $\forall\ u \in \mho$. The number $\delta_{\mathcal{M}}(u)$ and $\zeta_{\mathcal{M}}(u)$ represents, respectively, the participation and non-participation grade of the element u to the set P. For each IFS $\mathcal{M}$ in $\mho$, if

$$\eta_{\mathcal{M}}(u) = 1 - \delta_{\mathcal{M}}(u) - \zeta_{\mathcal{M}}(u), \forall\, u \in \mho. \tag{3}$$

Then $\eta_P(x)$ is the degree of indeterminacy of u to $\mho$.

Definition 2.3. [42]. An IFS $\mathcal{M}$ in $\mho$ is given by
$\mathcal{M} = \{\langle u, \delta_{\mathcal{M}}(u), \zeta_{\mathcal{M}}(u) \rangle | u \in \mho\}$
where $\delta_{\mathcal{M}}(u), \zeta_{\mathcal{M}}(u): \mathcal{M} \to [0,1]$, and with the condition

$$0 \le \delta^2_{\mathcal{M}}(u) + \zeta^2_{\mathcal{M}}(u) \le 1, \forall\, u \in \mho \tag{4}$$

and the degree of indeterminacy for any PFS $\mathcal{M}$ and $u \in \mho$ is given by

$$\eta^2_{\mathcal{M}}(u) = \sqrt{1 - \delta^2_{\mathcal{M}}(u) - \zeta^2_{\mathcal{M}}(u)} \tag{5}$$

Definition 2.4. [49]. Let $\mathcal{M}$ be a PFS of E and $\lambda > 0$, then following are the operators:

$$\lambda\mathcal{M} = \left\{< u, \left(\sqrt{1 - \left(1 - \delta^2_{\mathcal{M}}(u)\right)^{\lambda}}, \left(\zeta_{\mathcal{M}}(u_i)\right)^{\lambda}\right) > | u \in \mho\right\} \tag{6}$$

$$\mathcal{M}^{\lambda} = \left\{< u, \left(\left(\delta_{\mathcal{M}}(u_i)\right)^{\lambda}, \sqrt{1 - \left(1 - \zeta^2_{\mathcal{M}}(u_i)\right)^{\lambda}}\right) > | u \in \mho\right\} \tag{7}$$

The score function is defined as

$$S(\mathcal{M}) = \delta^2_{\mathcal{M}}(u_i) - \zeta^2_{\mathcal{M}}(u_i) \tag{8}$$

Definition 2.5. [30]. For PFSs $\mathcal{M} = [\delta_{\mathcal{M}}(u),\ \zeta_{\mathcal{M}}(u)]$, entropy measures $\mho_1, \mho_2$: $PFSs(\mho) \to [0,1]$ as

$$E_{LIN1}(\mathcal{M}) = \frac{1}{n(\sqrt{2}-1)}\sum_{i=1}^{n}\left[\sin\left(\frac{\pi\left(\tau^2+\delta^2_{\mathcal{M}}(u_i)-\zeta^2_{\mathcal{M}}(u_i)\right)}{4\tau^2}\right) + \sin\left(\frac{\pi\left(\tau^2-\delta^2_{\mathcal{M}}(u_i)+\zeta^2_{\mathcal{M}}(u_i)\right)}{4\tau^2}\right) - 1\right] \tag{9}$$

$$E_{LIN2}(\mathcal{M}) = \frac{1}{n(\sqrt{2}-1)}\sum_{i=1}^{n}\left[\sin\left(\frac{\pi\left(\tau^2+\delta^2_{\mathcal{M}}(u_i)-\zeta^2_{\mathcal{M}}(u_i)\right)}{4\tau^2}\right) + \sin\left(\frac{\pi\left(\tau^2-\delta^2_{\mathcal{M}}(u_i)+\zeta^2_{\mathcal{M}}(u_i)\right)}{4\tau^2}\right) - 1\right] \tag{10}$$

Definition 2.6. [25]. For PFSs $\mathcal{M} = [\delta_{\mathcal{M}}(u),\ \zeta_{\mathcal{M}}(u)]$, entropy measure $\mho_3 : PFSs(\mho) \to [0,1]$ as

$$E_{HAN}(\mathcal{M}) = \frac{1}{2n}\sum_{i=1}^{n}\left[2 - \frac{\delta_{\mathcal{M}}(u_i)}{2\tau} - \frac{\zeta_{\mathcal{M}}(u_i)}{2\tau} - \frac{|\delta_{\mathcal{M}}(u_i)-\delta_{\mathcal{M}}(u_i)|}{2\tau}\right] \tag{11}$$

Definition 2.7. [38]. For PFSs $\mathcal{M} = [\delta_{\mathcal{M}}(u),\ \zeta_{\mathcal{M}}(u)]$, entropy measure $\mho_4 : PFSs(\mho) \to [0,1]$ as

$$E_{TS}(\mathcal{M}) = \frac{1}{n}\sum_{i=1}^{n}\left[1 - \left|\delta^2_{\mathcal{M}}(u_i) - \frac{1}{3}\right| - \left|\zeta^2_{\mathcal{M}}(u_i) - \frac{1}{3}\right|\right] \tag{12}$$

Definition 2.8. [7]. For PFSs $\mathcal{M} = [\delta_{\mathcal{M}}(u),\ \zeta_{\mathcal{M}}(u)]$, entropy measure $\mho_5, \mho_6 : PFSs(\mho) \to R^+ \cup \{0\}$ as

$$E_{ATHIRA1}(\mathcal{M}) = \frac{1}{\sqrt{2}-1}\sum_{j=1}^{m}\sum_{i=1}^{n}\left[\sin\left(\frac{\pi\left(1+\delta^2_{\mathcal{M}}(u_i)-\zeta^2_{\mathcal{M}}(u_i)\right)}{4}\right) + \sin\left(\frac{\pi\left(1-\delta^2_{\mathcal{M}}(u_i)+\zeta^2_{\mathcal{M}}(u_i)\right)}{4}\right) - 1\right] \tag{13}$$

$$E_{ATHIRA1}(\mathcal{M}) = \frac{1}{\sqrt{2}-1}\sum_{j=1}^{m}\sum_{i=1}^{n}\left[\cos\left(\frac{\pi\left(1+\delta^2_{\mathcal{M}}(u_i)-\zeta^2_{\mathcal{M}}(u_i)\right)}{4}\right) + \cos\left(\frac{\pi\left(1-\delta^2_{\mathcal{M}}(u_i)+\zeta^2_{\mathcal{M}}(u_i)\right)}{4}\right) - 1\right] \tag{14}$$

3. Entropy Measures for PFSs

Firstly, we reminiscence the self-evident definition of similarity for Pythagorean fuzzy sets.

Preposition 1. [31]. Let $A = \{x_i, \delta_A(x_i), \zeta_A(x_i) | x_i \in X\}$ and $B = \{x_i, \delta_B(x_i), \zeta_B(x_i) | x_i \in X\}$ be two PFS(X), the entropy for $E: PFSs \to [0,1]$ which is a crisp function should meet the requirements of the following properties:

(P1) Boundedness: $0 \le E(A), E(B) \le 1$

(P2) Crispness: If $\delta_A(x_i) = 0, \zeta_A(x_i) = 1$ or $\delta_A(x_i) = 1$, $\zeta_A(x_i) = 0$, then $E(A) = 0$.

(P3) Separability: $\delta_A(x_i) = \zeta_A(x_i), then\ E(A) = 1$.

(P4) *Duality:* $E(A^c) = E(A)$.

(P5) *Inequality:* $E(A) \le E(B)$, if $\delta_A^2(x_i) \le \delta_B^2(x_i)$, $\zeta_A^2(x_i) \le \zeta_B^2(x_i)$ or $\delta_A^2(x_i) \ge \delta_B^2(x_i)$, $\zeta_A^2(x_i) \ge \zeta_B^2(x_i)$ for all $x_i \in X$.

In several circumstances, the weight of the elements $x_i \in X$ must be considered. For instance, in decision making, the attributes usually have distinct significance, and thus ought to be designated unique weights. In extension to the results obtained, we propose fuzzy entropy for PFSs as follows:

Let $P \in$ PFS (X) such that $X = \{x_1, x_2 \dots, x_n\}$ then

$$E_{PFSL}(P) = -log_2\left[\frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2}\right] \tag{15}$$

$$E_{WPFSL}(P) = -log_2\left[\frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\omega_i\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2}\right] \tag{16}$$

where, $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of $x_i (i = 1,2,\dots,n)$, with $\omega_k \in [0,1]$, $k = 1,2,\dots,n$, $\sum_{k=1}^{n}\omega_k = 1$. If $\omega = \left(\frac{1}{n}, \frac{1}{n}, \dots \frac{1}{n}\right)^T$, then the weighted entropy measure reduces to proposed entropy measure i.e., if we take $\omega_k = 1$, $k = 1,2,\dots,n$, then $E_{WPFSL}(P) = E_{PFSL}(P)$.

Theorem 3.1. The Pythagorean fuzzy entropy measures $E_{PFSL}(P)$ and $E_{WPFSL}(P)$ defined in equations (15) - (16) are valid measures of Pythagorean fuzzy entropy.

Proof. All the necessary four conditions to be an entropy measure are satisfied by the new measures as follows:

(P1) Boundedness: $0 \le E_{PFSL}(P), E_{WPFSL}(P) \le 1$

Proof. For $\mathbf{E_{PFST}(P)}$: By the definition of PFSs, we have $\boldsymbol{0 \le \delta_P(x_i) \le 1}$ and $\boldsymbol{0 \le \zeta_P(x_i) \le 1}$. This implies that $\boldsymbol{0 \le \delta_P^2(x_i) \le 1}$ and $\boldsymbol{0 \le \zeta_P^2(x_i) \le 1}$. We have, $\Rightarrow \boldsymbol{0 \le |\delta_P^2(x_i) - \zeta_P^2(x_i)| \le 1}$.

$\Rightarrow 0 \le \frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\} \le 1$

$\Rightarrow 0 \le \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2} \le \frac{1}{2}$

$\Rightarrow \frac{1}{2} \le \frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2} \le 1$

$\Rightarrow 0 \le -log_2\left[\frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2}\right]$

$\Rightarrow 0 \le \mathrm{E_{PFSL}(P)} \le 1$.

Measure $E_{WPFSL}(P)$ can be proved similarly.

(P2) Crispness: $\mathrm{E_{PFSL}(P)}$, $E_{WPFSL}(P) = 0$, if P is a crisp set.

Proof. For $E_{PFST}(P)$: If P is a crisp set i.e., if $\delta_P(x_i) = 0, \zeta_P(x_i) = 1$ or $\delta_P(x_i) = 1, \zeta_P(x_i) = 0$, then $|\delta_P^2(x_i) - \zeta_P^2(x_i)| = 1$.

$\Rightarrow \frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2} = 1$

$\Rightarrow -log_2\left[\frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2}\right] = 0$

$\Rightarrow E_{PFSL}(P) = 0$.

Measure $E_{WPFSL}(P)$ can be proved similarly.

(P3) Separability: $E_{PFSL}(P), E_{WPFSL}(P) = 1 \Leftrightarrow \delta_P(x_i) = \zeta_P(x_i)$.

Proof. For $E_{PFS}(P)$: For all $x_i \in X$, if $\delta_P(x_i) = \zeta_P(x_i)$ or $\delta_P^2(x_i) = \zeta_P^2(x_i)$, then

$|\delta_P^2(x_i) - \zeta_P^2(x_i)| = 0$. Hence, $\frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2} = \frac{1}{2}$.

$\Rightarrow -log_2\left[\frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2}\right] = 1$

Therefore, $E_{PFSL}(P) = 1$. If $E_{PFSL}(P) = 1$, this implies,

$\frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2} = \frac{1}{2}$

$\Rightarrow |\delta_P^2(x_i) - \zeta_P^2(x_i)| = 0$.

Therefore $\delta_P^2(x_i) = \zeta_P^2(x_i)$. Hence $\delta_P(x_i) = \zeta_P(x_i)$.

Measure $E_{WPFSL}(P)$ can be proved similarly.

(P4) Complement: $E_{PFST}^c(P) = E_{PFST}(P)$ and $E_{WPFSL}^c(P) = E_{PFSL}(P)$

Proofs are self-explanatory and straight forward.

(P5) Inequality: $E(P) \le E(Q)$**,** if P is crisper than Q, i.e., if $\delta_P^2(x_i) \le \delta_Q^2(x_i)$, $\zeta_P^2(x_i) \le \zeta_Q^2(x_i)$ or $\delta_P^2(x_i) \ge \delta_Q^2(x_i)$, $\zeta_P^2(x_i) \ge \zeta_Q^2(x_i)$ for all $x_i \in X$.

Proof. For $E_{PFS}(P)$: For $x_i \in$ X, we have

$0 \le \delta_P(x_i) \le \delta_Q(x_i) \le 1$ and $1 \ge \zeta_P(x_i) \ge \zeta_Q(x_i) \ge 0$. This implies that

$0 \le \delta_P^2(x_i) \le \delta_Q^2(x_i) \le 1$ and $1 \ge \zeta_P^2(x_i) \ge \zeta_Q^2(x_i) \ge 0$.

This we have, $|\delta_P^2(x_i) - \zeta_P^2(x_i)| \ge |\delta_Q^2(x_i) - \zeta_Q^2(x_i)|$

From the above we can write,

$\frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2} \ge \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_Q^2(x_i) - \zeta_Q^2(x_i)|\}}{2}$

$\Rightarrow \frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2} \ge \frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_Q^2(x_i) - \zeta_Q^2(x_i)|\}}{2}$

$\Rightarrow -log_2\left[\frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_P^2(x_i) - \zeta_P^2(x_i)|\}}{2}\right] \le -log_2\left[\frac{1}{2} + \frac{\frac{1}{n}\sum_{i=1}^{n}\{|\delta_Q^2(x_i) - \zeta_Q^2(x_i)|\}}{2}\right]$

$\Rightarrow E_{PFSL}(P) \le E_{PFSL}(Q)$.

Similar proof can be made $E_{WPFSL}(P) \le E_{WPFSL}(Q)$.

3.1 Numerical Verification of the Entropy Measures

Now, we will verify whether proposed entropy measures satisfy above four properties:

Example 1. Let $P, Q \in PFS(X)$ for $X = \{x_1, x_2, x_3\}$. Suppose $P = \{\langle x_1, 0.2, 0.7\rangle, \langle x_2, 0.4, 0.9\rangle, \langle x_3, 0.3, 0.8\rangle\}$ and $Q = \{\langle x_1, 0.3, 0.6\rangle, \langle x_2, 0.7, 0.8\rangle, \langle x_3, 0.4, 0.6\rangle\}$

Calculating the entropy using proposed measures as follows:

$E_{PFSL}(P) = -log_2\left[\frac{1}{2} + \frac{\frac{1}{3}\sum_{i=1}^{n}\{|0.2^2 - 0.7^2| + |0.4^2 - 0.9^2| + |0.3^2 - 0.8^2|\}}{2}\right]$

$$= -log_2\left[\frac{1}{2} + \frac{1.65}{6}\right] = -log_2(0.775) = 0.367731785 \tag{17}$$

$E_{PFSL}(Q) = -log_2\left[\frac{1}{2} + \frac{\frac{1}{3}\sum_{i=1}^{n}\{|0.3^2 - 0.6^2| + |0.7^2 - 0.8^2| + |0.4^2 - 0.6^2|\}}{2}\right]$

$$= -log_2\left[\frac{1}{2} + \frac{0.62}{6}\right] = -log_2(0.6033) = 0.728972803 \tag{18}$$

From equation (17) and (18), it has been concluded that $E(P) \le E(Q)$**,** if $\delta_P^2(x_i) \le \delta_Q^2(x_i)$, $\zeta_P^2(x_i) \le \zeta_Q^2(x_i)$ or $\delta_P^2(x_i) \ge \delta_Q^2(x_i)$, $\zeta_P^2(x_i) \ge \zeta_Q^2(x_i)$ for all $x_i \in X$.

Moreover, if we cconsider weights assigned be $\omega = \{0.5, 0.3, 0.2\}$ and using proposed entropy measures stated in equation (13), numerical values are 0.765294317 and 0.897901812. Hence, in this case also, $E(P) \le E(Q)$. Extended Euclidean distance measure, proposed by Szmidt, and J. Kacprzyk [35], is used to find the weighted divergence as

$$D_E(P,Q) = \sqrt{\frac{1}{2}\omega_i\left[\{\delta_P^2(x_i) - \delta_Q^2(x_i)\}^2 + \{\zeta_P^2(x_i) - \zeta_Q^2(x_i)\}^2 + \{\eta_P^2(x_i) - \eta_Q^2(x_i)\}^2\right]} \tag{19}$$

4. TOPSIS Approach to Logarithmic Entropy Measures for PFSs

This segment presents MADM issue under PFSs environment. A viable dynamic methodology is proposed to manage such MADM issues. Each decision matrix in MADM techniques has four main components as (a) criteria, (b) alternatives, (c) weight or relative importance of each attribute and (d) assessment value of alternatives with respect to the criteria. An algorithm of the proposed technique is too introduced which will be applied in selection procedure of a marketing expert in any manufacturing organization in this session.

The procedure of the TOPSIS method can be depicted in the subsequent flowchart in figure 1 as follows:

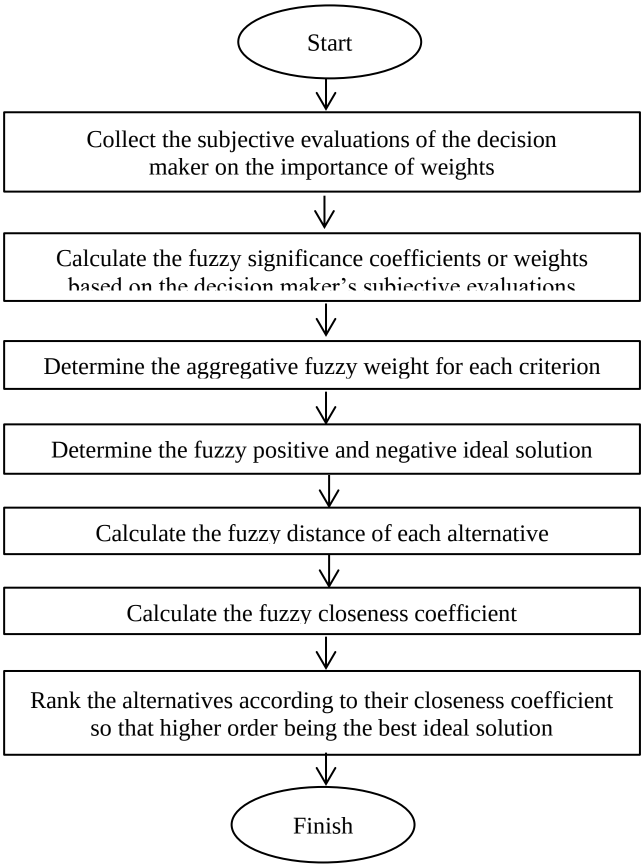

Figure 1. Flowchart for TOPSIS Approach

4.1 A Case Study

In India, it is observed that most people suffering from corona virus disease will return completely within a few weeks. However, many people with mild illness still have signs after recovery. These indications are thought to be a side effect of treatment that lasts more than four weeks after the patient's diagnosis of the virus. The following are some of the most common long-term indications and symptoms: Post COVID cough ($\boldsymbol{\mathcal{S}_1}$), joint aches and muscle pain ($\boldsymbol{\mathcal{S}_2}$), fatigue and dyspnea ($\boldsymbol{\mathcal{S}_3}$), weight loss and poor appetite ($\boldsymbol{\mathcal{S}_4}$), loss of taste and smell ($\boldsymbol{\mathcal{S}_5}$) and sleep disorder ($\boldsymbol{\mathcal{S}_6}$). Six different diseases: cardiac arrest ($\boldsymbol{D^1}$), diabetic ($\boldsymbol{D^2}$), lung fibrosis ($\boldsymbol{D^3}$), pneumonia ($\boldsymbol{D^4}$), kidney failure ($\boldsymbol{D^5}$) and brain stroke ($\boldsymbol{D^6}$) are to be evaluated as Pythagorean fuzzy number by the doctors under the six symptoms criteria, whose weights are completely unknown, is presented in the table 1 as follows:

Table 1. Patient's Disease Pythagorean relation

Relation	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_3$	$\mathcal{S}_4$	$\mathcal{S}_5$	$\mathcal{S}_6$
D^1	<0.8, 0.4>	<0.4, 0.9>	<0.2, 0.3>	<0.5, 0.8>	<0.3, 0.6>	<0.4, 0.7>
D^2	<0.8, 0.1>	<0.4, 0.8>	<0.5, 0.4>	<0.6, 0.4>	<0.6, 0.5>	<0.7, 0.5>
D^3	<0.5, 0.8>	<0.4, 0.7>	<0.3, 0.5>	<0.4, 0.7>	<0.8, 0.1>	<0.4, 0.9>
D^4	<0.9, 0.4>	<0.1, 0.8>	<0.4, 0.2>	<0.5, 0.7>	<0.7, 0.2>	<0.7, 0.4>
D^5	<1.0, 0.0>	<0.6, 0.1>	<0.5, 0.4>	<0.4, 0.6>	<0.2, 0.6>	<0.4, 0.5>
D^6	<0.4, 0.5>	<0.7, 0.2>	<0.6, 0.3>	<0.2, 0.5>	<0.8, 0.4>	<0.2, 0.6>

Now, we compute the overall entropy of each criteria using equation (15) as

$\boldsymbol{E_{PFSL}(\mathcal{S}_1)} = 0.3770696$ $\quad \boldsymbol{E_{PFSL}(\mathcal{S}_2)} = \mathbf{0.432779}$
$\boldsymbol{E_{PFSL}(\mathcal{S}_3)} = 0.8236772$ $\quad \boldsymbol{E_{PFSL}(\mathcal{S}_4)} = 0.664669201$
$\boldsymbol{E_{PFSL}(\mathcal{S}_5)} = \mathbf{0.53882}$ $\quad \boldsymbol{E_{PFSL}(\mathcal{S}_6)} = 0.59219407$

Next, we calculate the weight of each criteria using equation

$$\boldsymbol{w_j} = \frac{\boldsymbol{1 - E_{PFSL}(D^j)}}{\boldsymbol{n - \sum_{j=1}^{n} E_{PFSL}(D^j)}} \tag{20}$$

as $w_1 = 0.242311$ $\quad w_2 = 0.22064$ $\quad w_3 = 0.068587$
$w_4 = 0.130439$ $\quad w_5 = 0.179392$ $\quad w_6 = 0.158631$

Computing positive ideal solution (PIS) and negative ideal solution (NIS) for each alternative as
$\psi^+ = \left\{\begin{matrix}(0.9, 0.0), (0.7, 0.1), (0.6,0.2), (0.6,0.4), (0.8,0.1), \\ (0.7,0.4)\end{matrix}\right\}$ and
$\boldsymbol{\psi^-} = \left\{\begin{matrix}(\mathbf{0.4}, \mathbf{0.8}), (\mathbf{0.1}, \mathbf{0.9}), (\mathbf{0.2}, \mathbf{0.5}), (\mathbf{0.2}, \mathbf{0.8}), (\mathbf{0.2}, \mathbf{0.6}), \\ (\mathbf{0.2}, \mathbf{0.9})\end{matrix}\right\}$

We compute distance measure values by using equation (19) as

$$\boldsymbol{D^+_{WPFSL} = D(D^j, \psi^+)} = \sqrt{\frac{1}{2}\boldsymbol{\omega_i}\left[\begin{matrix}\{\boldsymbol{\delta}^2_{\mathbf{P}}(\mathbf{x_i}) - \boldsymbol{\delta}^{2+}_{\mathbf{Q}}(\mathbf{x_i})\}^2 + \{\boldsymbol{\zeta}^2_{\mathbf{P}}(\mathbf{x_i}) - \boldsymbol{\zeta}^{2+}_{\mathbf{Q}}(\mathbf{x_i})\}^2 + \\ \{\boldsymbol{\eta}^2_{\mathbf{P}}(\mathbf{x_i}) - \boldsymbol{\eta}^{2+}_{\mathbf{Q}}(\mathbf{x_i})\}^2\end{matrix}\right]} \tag{21}$$

$$\boldsymbol{D^-_{WPFSL} = D(D^j, \psi^-)} = \sqrt{\frac{1}{2}\boldsymbol{\omega_i}\left[\begin{matrix}\{\boldsymbol{\delta}^2_{\mathbf{P}}(\mathbf{x_i}) - \boldsymbol{\delta}^{2+}_{\mathbf{Q}}(\mathbf{x_i})\}^2 + \{\boldsymbol{\zeta}^2_{\mathbf{P}}(\mathbf{x_i}) - \boldsymbol{\zeta}^{2+}_{\mathbf{Q}}(\mathbf{x_i})\}^2 + \\ \{\boldsymbol{\eta}^2_{\mathbf{P}}(\mathbf{x_i}) - \boldsymbol{\eta}^{2+}_{\mathbf{Q}}(\mathbf{x_i})\}^2\end{matrix}\right]} \tag{22}$$

The obtained results are summarized in table 2.

Table 2. Separation measures for ideal solutions for 6 diseases

	D^1	D^2	D^3	D^4	D^5	D^6
$D(D^j, \psi^+)$	0.8480	0.4790	0.6653	0.5622	0.4781	0.4734
$D(D^j, \psi^-)$	0.4456	0.6761	0.4584	0.6526	0.8360	0.9459

Relative closeness coefficient with respect to each decision maker can be found as

$$\boldsymbol{R_j} = \frac{\boldsymbol{D(D^j, \psi^-)}}{\boldsymbol{D(D^j, \psi^+) + D(D^j, \psi^-)}} \tag{23}$$

where $\mathbf{0} \leq \boldsymbol{R_j} \leq \mathbf{1}, \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{n}$

Ranking for 6 diseases is depicted in table 3 as follows:

Table 3. Ranking for 6 diseases

Alternatives	*R_j value*	*Ranking*
Cardiac arrest (D^1)	0.3444863	6
Diabetes (D^2)	0.5853234	3
Lung fibrosis (D^3)	0.4079063	5
Pneumonia (D^4)	0.5372033	4
Kidney failure (D^5)	0.6361803	2
Brain stroke (D^6)	0.6664158	1

Based on the relative closeness of each disease, we rank these diseases. It has been found from the above table that the brain stroke (D^6) is the optimal disease and are classified as $D^6 \succ D^5 \succ D^2 \succ D^4 \succ D^3 \succ D^1$. The ranking of the alternatives can be shown in figure 2 as follows:

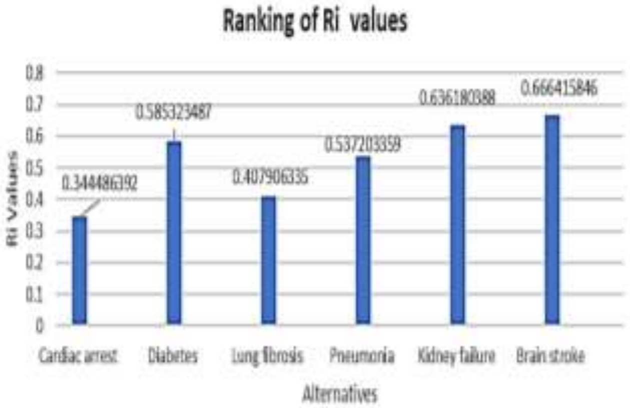

Figure 2. Ranking of Alternatives

5. Conclusion

We introduce some novel entropy measures between PFSs based on the logarithmic function in this study. The desirable combinations and their characteristics are thoroughly investigated. To demonstrate the efficacy of the suggested entropy measures, we provide several paradoxical instances that demonstrate how existing measures fail in specific situations, whereas the new one categorizes the items. The proposed entropy measures and distance measure are then used to solve MADM problem. To demonstrate consistency, the numerical findings are compared to previous ones. The proposed method reveals that the produced solution is a good compromise over the existing ones and is conservative in nature. In the future, we will broaden the recommended measures to include more uncertain and fuzzy scenarios.

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