

## Particle Swarm Optimization Design of Optical Directional Coupler Based on Power Loss Analysis

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**Abstract:** In this work, possible design is presented as an optimization problem for an optical directional coupler by using particle swarm optimization (PSO). In this PSO design, identical, parallel, lossless and slab optical waveguides are supposed to be coupled to each other weakly. The power loss and the propagation constant change of TE and TM modes in these coupled optical waveguides are analysed by the modal analysis and PSO. PSO design of an optical directional coupler is an optimization problem consisting of input variables and design parameters within a fitness function (FF). FF is the power loss of TE and TM modes. PSO should minimize the FF and obtain design criteria. The analysis shows that the PSO results are compatible with modal analysis results. The availability of the optical coupler design by PSO has been presented and tested successfully.

**Keywords:** Optical fiber, Optical communication, Optical directional coupler, Particle swarm optimization

### 1. Introduction

Optical fiber is the basic element of optical networks. The elements such as optical directional couplers, optical fiber sensors, optical amplifiers, optical filters, optical reflectors and optical detectors are commonly used in optical communication networks. In this work, an optical directional coupler is analysed and optimized considering the optical communication principles.

An optical directional coupler consists of two parallel optical waveguides or two bent optical waveguides or one straight and one bent optical waveguides. Evanescent fields affect each other because the distance between the axis of the optical waveguides is much smaller than the working wavelength. Coupling can occur between different modes propagating in distinct guides and also between different modes propagating in the same guide. In this study, coupling between two distinct parallel optical guides is investigated and optimized with PSO.

The PSO algorithm was introduced in 1995 as an efficient intelligent evolutionary optimization, which can be applied to solve most optimization problems [1]. In engineering, PSO has been successfully applied to various areas [2]. Optic studies with PSO are also common in engineering [3, 4].

The remainder of this paper is organized as follows: In Section 2, the coupling mechanism of an optical directional coupler is analysed by using the Coupled Mode Theory and Perturbation Theory. The interaction within an optical directional coupler, which consists of identical, slab, parallel, weakly guiding and lossless optical waveguides, is analysed for a time dependent term of  $\exp(j\omega t)$  [5, 6]. TE and TM modes are determined by using the Maxwell Equations, Helmholtz Equations and dielectric-dielectric boundary conditions [7-10]. The power loss is analysed by

Poynting Theorem in order to use in PSO as FF. In Section 3, applicability of PSO in optical directional coupler is investigated by taking relevant criteria, parameters and constraints. The propagation constant change for each mode is calculated and drawn with the result values of PSO. Satisfactory numerical and graphical results are presented. The study is concluded in section 4 and determined that the PSO results are compatible with the analytical results.

### 2. Coupling Analysis in Optical Directional Coupler

An optical directional coupler consists of two parallel optical waveguides or two bent optical waveguides or one straight and one bent optical waveguides. Evanescent fields affect each other because the distance between the axis of the optical waveguides is much smaller than the working wavelength. Mutual coupling between optical waveguides is expressed with the help of Coupled Mode Theory and Perturbation Theory.

The propagation constant change  $\beta$  is analysed for a time dependent term of  $\exp(j\omega t)$  with coupling coefficients which are independent of the propagation direction as follows:

$$\Delta\beta = j(c_{12}c_{21})^{1/2} \quad (1)$$

where  $c_{12}$  and  $c_{21}$  are the coupling coefficients of the optical guides affecting each other.

The change occurs in the form of an increase or decrease between the propagation constants of coupled modes. It takes complex or real values depending on whether the coupled waveguides are lossy or lossless. The value is a real value if lossless, a complex value if lossy [5, 6].

#### 2.1. Propagation Constant Change in the Coupled Uncladded Optical Waveguides

In Figure 1, coupled, parallel, identical, slab and uncladded optical waveguides are present. The radius of the core is  $d$ , and the

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distance between the core axis is  $U$ . The refractive index of the core is  $n_1$  and the refractive index of the region surrounding the core is  $n_2$ . The propagation is available with the z-direction in the waveguides extending to infinity with the y-direction.

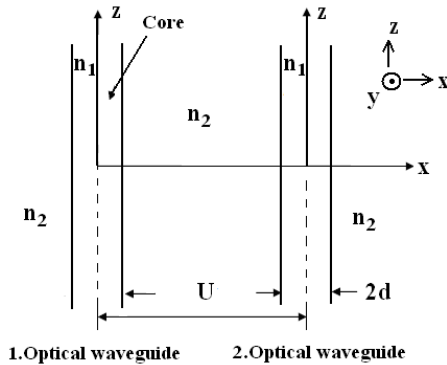


Figure 1. The coupling between two parallel, identical, slab and uncladded optical waveguides

TE and TM modes are examined as a result of solving the Maxwell Equations, Helmholtz Equations and boundary conditions through optical waveguides [5-14]. TE even and odd guided field definitions in the core and the surrounding region respectively are given by the following equation:

$$E_y = \begin{cases} A \begin{cases} \cos(\kappa x) \\ \sin(\kappa x) \end{cases} & , 0 \leq x \leq d \\ B \exp(-\gamma(|x| - d)) & , d \leq x \leq \infty \end{cases} \quad (2)$$

Where " $\kappa$ " is the eigenvalue of core region and  $\gamma$  is the eigenvalue of the region surrounding the cores. The power loss by using Poynting theorem is given by:

$$2\alpha = \frac{2 \operatorname{Im}(\gamma)}{\beta(1 + \gamma d)} \begin{cases} \cos^2(\kappa d) \\ \sin^2(\kappa d) \end{cases} \exp(-2\gamma(|x| - d)) \quad (3)$$

The propagation constant change within two couple identical, parallel, slab and uncladded optical waveguides having the same  $\beta$  propagation constant is expressed as the following:

$$|\Delta\beta| = \begin{cases} \frac{k_0^2 (n_1^2 - n_2^2) \left[ \frac{\gamma^2}{\beta^2 (1 + \gamma d)^2} \right]^{1/2}}{4\gamma} \\ \begin{cases} \cos^2(\kappa d) \\ \sin^2(\kappa d) \end{cases} \exp^2(\gamma d) \exp(-\gamma U) \end{cases} \quad (4)$$

TM even and odd guided field definitions in uncladded optical waveguides in the core and the surrounding region respectively are as follows:

$$H_y = \begin{cases} A \begin{cases} \cos(\kappa x) \\ \sin(\kappa x) \end{cases} & , 0 \leq x \leq d \\ B \exp(-\gamma(|x| - d)) & , d \leq x \leq \infty \end{cases} \quad (5)$$

The power loss by using Poynting theorem is given by:

$$2\alpha = \begin{cases} 2 \operatorname{Im}(\gamma) \frac{1}{\beta \left[ d + \frac{n_1^2 n_2^2}{\gamma} \frac{\kappa^2 + \gamma^2}{n_2^4 \kappa^2 + n_1^4 \gamma^2} \right]} \\ \begin{cases} \cos^2(\kappa d) \\ \sin^2(\kappa d) \end{cases} \exp(-2\gamma(|x| - d)) \end{cases} \quad (6)$$

The propagation constant change within two couple identical, parallel, slab and uncladded optical waveguides having the same  $\beta$  propagation constant is expressed as follows:

$$|\Delta\beta| = \begin{cases} \frac{\omega^2 \varepsilon_0 n_1^2 (n_1^2 - n_2^2)}{4\gamma} \frac{1}{\beta \left( d + \frac{n_1^2 n_2^2}{\gamma} \frac{\kappa^2 + \gamma^2}{n_2^4 \kappa^2 + n_1^4 \gamma^2} \right)} \\ \begin{cases} \cos^2(\kappa d) \\ \sin^2(\kappa d) \end{cases} \exp^2(\gamma d) \exp(-\gamma U) \end{cases} \quad (7)$$

In this PSO design, the modes corresponding to azimuthal mode number  $v=l$  are investigated.

$V_c$  is the normalized frequency and the relation is given by:

$$(\kappa d)_c \cong V_c = \nu \frac{\pi}{2} \quad (8)$$

$\nu = 0, 1, 2, 3 \dots$

## 2.2. Propagation Constant Change in the Couple Cladded Optical Waveguides

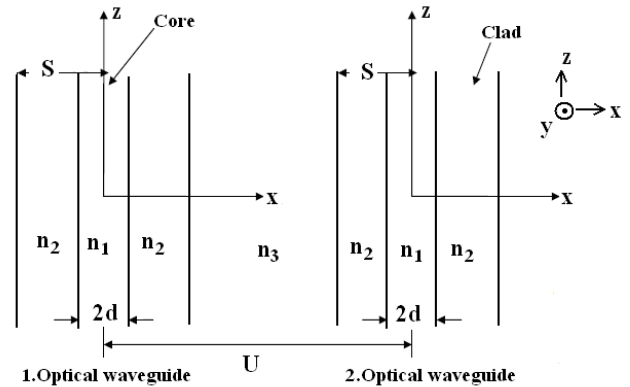


Figure 2. The coupling between two parallel, identical, slab and cladded optical waveguides

In Figure 2, coupled parallel, identical, slab and cladded optical waveguides are present. The radius of the core is  $d$ , and the radius with cladding is  $S$ . The distance between the core axis is  $U$ . The refractive index of the core is  $n_1$  and the refractive index of the cladding is  $n_2$ . The refractive index of the region surrounding the cladding is  $n_3$ . The propagation is available with the z-direction in the waveguides extending to infinity with the y-direction [1-10]. The electric field in the core, cladding and surrounding region is expressed as the following:

$$E_y = \begin{cases} A \begin{cases} \cos(\kappa x) \\ \sin(\kappa x) \end{cases} & , 0 \leq x \leq d \\ B \exp(\gamma x) + C \exp(-\gamma x), & d \leq x \leq S \\ F \exp(-\rho x) & , S \leq x \leq \infty \end{cases} \quad (9)$$

where

$$\rho = (\beta^2 - n_3^2 k_0^2)^{1/2} \quad (10)$$

is the eigenvalue of the region surrounding the cladding.

The propagation constant change within two couple identical cladded optical waveguides having the same  $\beta$  propagation constant is expressed as:

$$|\Delta\beta| = \left\{ \frac{4\kappa^2 \gamma^3 \rho}{\beta(1 + \gamma d)(\gamma + \rho)^2 (\kappa^2 + \gamma^2)^{1/2}} \right\} \exp[-2\gamma(S - d)] \exp[-\rho(U - 2S)] \quad (11)$$

The power loss by using Poynting theorem is defined as the following equation:

$$2\alpha = \frac{8\kappa^2 \gamma^3 \text{Im}(\rho)}{\beta(1 + \gamma d)(\kappa^2 + \gamma^2) |\gamma + \rho|^2} \exp(-2\gamma(S - d)) \quad (12)$$

The magnetic field in the core, cladding and surrounding region as follows:

$$H_y = \begin{cases} A \begin{cases} \cos(\kappa x) \\ \sin(\kappa x) \end{cases}, & 0 \leq x \leq d \\ B \exp(\gamma x) + C \exp(-\gamma x), & d \leq x \leq S \\ F \exp(-\rho x) & , S \leq x \leq \infty \end{cases} \quad (13)$$

The propagation constant change within two couple identical cladded optical waveguides having the same  $\beta$  propagation constant is expressed by:

$$|\Delta\beta| = \frac{\left\{ \frac{n_1^2 n_2^2 \kappa^2 \gamma^2 [2 \exp((\gamma - \rho)(S - d)) - 1]}{\exp(-2\gamma(S - d)) \exp(-\rho(U - 2S))} \right\}}{\beta [(n_1^4 \kappa^2 + n_1^4 \gamma^2) \gamma d + n_1^2 n_2^2 (\kappa^2 + \gamma^2)]} \quad (14)$$

The power loss by using Poynting theorem is defined as the following:

$$2\alpha = \left\{ \frac{8n_1^2 n_2^4 |n_3|^4 \kappa^2 \gamma^3 \text{Im}(\frac{\rho}{n_3})}{\beta [(n_1^4 \kappa^2 + n_1^4 \gamma^2) \gamma d + n_1^2 n_2^2 (\kappa^2 + \gamma^2)] |n_2^2 \rho + n_2^2 \gamma|^2} \right\} \exp(-2\gamma(S - d)) \quad (15)$$

### 3. PSO Design of Optical Directional Coupler

#### 3.1. PSO

In Particle Swarm Optimization (PSO), solutions are called particles in the search space. All particles have fitness values evaluated by the fitness function and velocity information that directs their movements. Particles follow the existent optimum particles in the problem space.

PSO is initiated with a random particle swarm and is updated to search for the best value. Each particle is updated according to the best value obtained in each of the iterations. One of the updates is related to the best fit value of the particle titled pbest (personal best). This value is kept in the memory to be used later. The second best value is the global best value (gbest) obtained by any particle in the swarm. It is the best global value in the swarm.

The swarm matrix where D is the swarm dimension and n is the particle number is given below:

$$x = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1D} \\ x_{21} & x_{22} & \dots & x_{2D} \\ x_{31} & x_{32} & \dots & x_{3D} \\ \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & \dots & x_{nD} \end{bmatrix}_{n \times D}$$

According to swarm matrix;  $i^{\text{th}}$  particle;

$$x_i = [x_{i1}, x_{i2}, x_{i3}, \dots, x_{iD}]$$

And the personal best fit value obtained until that point pbest:

$$pbest_i = [p_{i1}, p_{i2}, p_{i3}, \dots, p_{iD}]$$

And the global best in the population is:

$$gbest = [p_1, p_2, p_3, \dots, p_D]$$

Velocity vector that presents the amount of transformation for  $i^{\text{th}}$  particle in each position is displayed as follows:

$$v_i = [v_{i1}, v_{i2}, v_{i3}, \dots, v_{iD}]$$

The velocity and position of the particle are updated according to the equations below respectively:

$$v_i^{k+1} = v_i^k + c_1 \cdot rand_1^k \cdot (pbest_i^k - x_i^k) + c_2 \cdot rand_2^k \cdot (gbest^k - x_i^k)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1}$$

Where  $k$  is the number of iterations and  $i$  is the number for the particle. When the particle swarm matrix is composed of  $n$  lines, there is an  $i^{\text{th}}$  line as well. The learning factors  $c_1$  and  $c_2$  values pull the particle towards pbest and gbest values.  $c_1$  and  $c_2$  are generally selected as equals and in  $[0,4]$  range.  $c_1$  allows the particle to move according to its own experiences whereas  $c_2$  allows it to move according to the experiences of the other particles in the swarm.

#### 3.2. Optical Directional Coupler Design

In this study, the optical directional coupler is chosen to investigate the usage of PSO in optical communication systems.

Figure 3 shows a basic illustration for the PSO design of optical directional coupler with uncladded optical waveguides. The input variables are mode type, frequency of the mode,  $n_1$  and the value range of  $n_1$  and  $d$ . The output variable is the exact value of  $n_1$  and  $d$ .

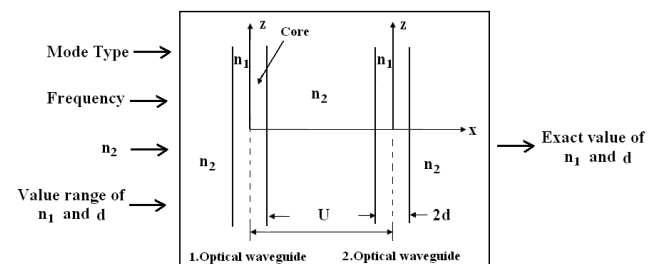


Figure 3. Optical Directional Coupler with uncladded fibers

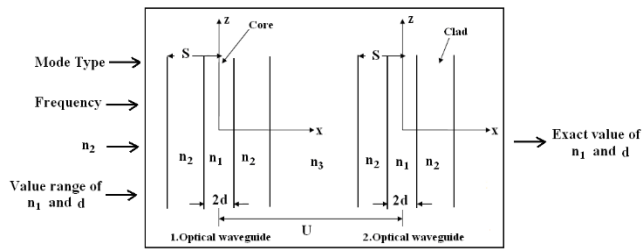


Figure 4. Optical Directional Coupler with cladded fibers

Figure 4 shows a basic illustration for the PSO design of optical directional coupler with cladded optical waveguides. The refractive index of the region surrounding the cladding is air and  $n_3=1$ . The input variables are mode type, frequency of the mode,  $n_2$  and the value range of  $n_1$  and  $d$ . The output variable is the exact value of  $n_1$  and  $d$ .

The optimal design is aimed at taking relevant criteria, parameters and constraints. PSO should minimize the FF and obtain design criteria and constraints. In this study, FF is the powerloss of TE and TM modes obtained by using the Poynting theorem.

In the beginning of the algorithm a certain constraint is adjusted because of the azimuthal mode number for both design criteria and design parameters by the designer. Initial population matrix size is  $10 \times 2$ . The row number indicates the number of particles in the population and the column denotes  $n_1$  and  $d$  parameters. The learning factors  $c_1, c_2$  and  $w$  are 1.4, 1.4 and 0.9 respectively while the algorithm runs for 100 iterations. Propagation constant change is calculated and drawn with the result values of PSO.

### 3.3 Numerical and Graphical Results

A directional coupler is designed consisting of uncladded and cladded fibers guided with TE, TM even and odd modes. Table 1 shows that design criteria of directional coupler worked with uncladded waveguides.

Table 1. PSO design criteria of an optical directional coupler with uncladded waveguides.

Mode type	Frequency (THz)	$n_1$	$n_2$	The value range of d for single-mode ( $\times 10^{-7}$ )
TE even	200	1.5	1.49	1.77 - 3.54
TE odd	352	1.5	1.46	1.90 - 2.25
TM even	190	1.77	1.45	1.56 - 2.33
TM odd	230	1.48	1.33	1.57 - 1.96

As mentioned before, PSO starts with a random particle swarm. PSO algorithm runs for TE, TM even and odd modes considering design constraints given in Table 1. Optimal results which satisfy min loss power equality are given in Table 2.

Table 2. PSO results of  $n_1$  and  $d$

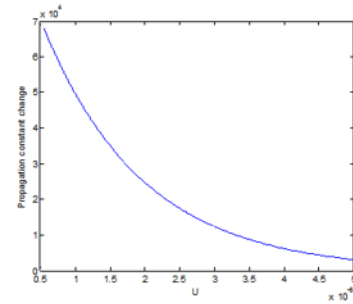
Mode type	$n_1 (>n_2)$	$d (\times 10^{-7})$
TE even	1.53 ( $>1.49$ )	2.57
TE odd	1.74 ( $>1.46$ )	2.18
TM even	1.69 ( $>1.45$ )	1.90
TM odd	1.66 ( $>1.33$ )	1.70

For comparison, using PSO results of output  $n_1$  and  $d$  values, optical directional couplers with uncladded fibers are designed and drawn (Figure 5, Figure 6).

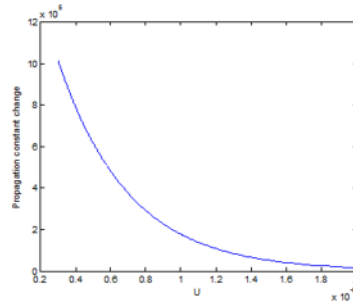
In conclusion, the coupling between TE modes is more efficient than the coupling between TM modes. In addition, propagation constant change decreases with increasing distance between two axis of the fiber core.

After a directional coupler consisting of uncladded fibers, a directional coupler with cladded fibers is taken. Table 3 shows the

PSO design criteria of a directional coupler with TE, TM modes.

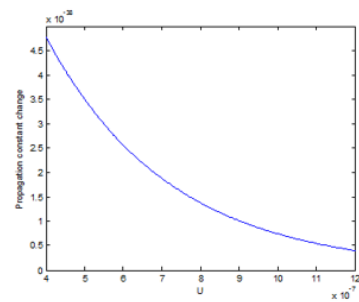


(a)

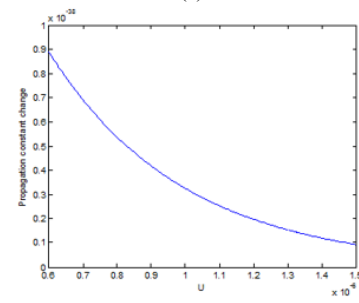


(b)

Figure 5. TE even and odd modes guided in uncladded coupled optical waveguides



(a)



(b)

Figure 6. TM even and odd modes guided in uncladded coupled optical waveguides

Table 3. PSO design criteria of an optical directional coupler with cladded waveguides.

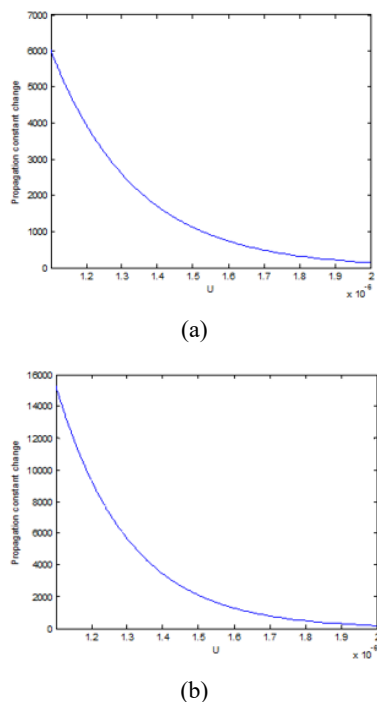
Mode type	Frequency (THz)	$n_1$	$n_2$	The value range of d for single-mode ( $\times 10^{-7}$ )
TE even	220	1.49	1.33	1.58 - 2.71
TE odd	180	1.77	1.46	1.55 - 2.43
TM even	200	1.5	1.49	1.77 - 3.54
TM odd	195	2.42	1.48	1.22 - 2.78

Optimum results which satisfy min loss power equality are shown in Table 4.

For comparison, using PSO results of output  $n_1$  and  $d$  values, optical directional couplers with cladded fibers are designed and drawn (Figure 7, Figure 8).

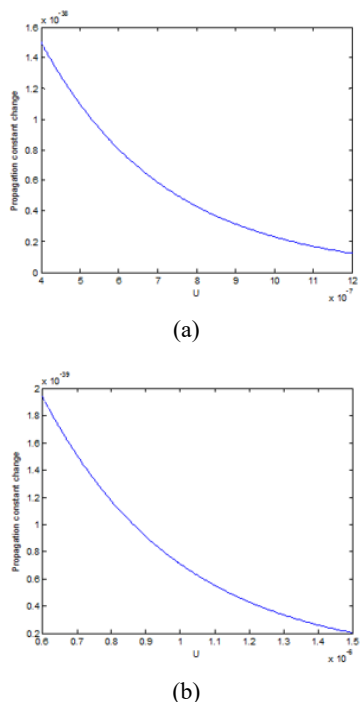
**Table 4.** PSO results of  $n_1$  and  $d$

Mode type	$n_1 (> n_2)$	$d (x10^{-7})$
TE even	1.44 (>1.33)	2.11
TE odd	1.90 (>1.46)	2.05
TM even	1.52 (>1.49)	2.66
TM odd	1.55 (>1.48)	1.58



**Figure 7.** TE even and odd modes guided in cladded coupled optical waveguides

As seen in the graphics, the coupling between the uncladded optical fibers is more effective than the coupling between the cladded optical fibers. Also, the coupling between TE modes is more efficient than the coupling between TM modes. Moreover, propagation constant change decreases with increasing distance between two axis of the fiber core.



**Figure 8.** TM even and odd modes guided in cladded coupled optical waveguides

#### 4. Conclusions

As a result of the modal analysis and PSO, the coupling between TE modes is more efficient than the coupling between TM modes. In addition, propagation constant change decreases with increasing distance between two axis of the fiber core. Moreover, the coupling between the uncladded optical fibers is more effective than the coupling between the cladded optical fibers. Analytical results are in agreement with the results of the PSO. PSO is a fast and good alternative method for designing a complex optical directional coupler.

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