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Effect of Uncertainty in Optimal Inventory Policy for Manufacturing Products

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Abstract: To provide the best storage facilities for an items is one of the main part for any inventory management system. There are many categories for perishable items which deteriorates at various rates due to temperature and some other environment conditions. This research study developed a fuzzy inventory model by considering the time varying demand. The model incorporates the linear decreasing demand with signed distance method. In inventory system the reliability of any procedure is the significant property in research work in which some parameters are very difficult to assign the values or nearly unreal. Fuzzy inventory models are quite useful in practice in such cases. The effectiveness of this system is shown through the consequence of fuzzy parameters on total inventory cost were considered, and a new improved model was modified and also for demonstrated the relationship between crisp and fuzzy environment.

Keywords: *Inventory system, Fuzzy, Signed distance method, variable demand.*

1. Introduction

Perishable products decay in various situations and manners; according to types of items and available storage facilities in a way of rate of deterioration and starting point. Many products of food, vegetable, cosmetic, pharmaceutical, blood bank etc., are fail to maintain their freshness of a long time due to environment, humidity, temperature and some other places problems. Therefore the smart and best storage facilities for items are the most important part for inventory management. Deterioration is one of the most important factors which is considerable when we analysing the inventory models. Holding cost also play an important roles for inventory system because of product storing time increase the total inventory cost. The first inventory model investigated was Economic Order Quantity (EOQ) inventory model by Harris (1915). Hadley and Whitin (1963) studied the theory of inventory systems with deterministic and probabilistic parameters, types of inventory models and inventory costs etc.

One of the key factors in attains the optimum profit is managing the inventory management is minimize the total inventory cost for any business organization. Some recent contributions in the field of inventory modelling are (Singh and Malik (2008, 2009, 2010, 2011); Sharma et al. (2013); Kumar et al. (2016, 2017 & 2019); Malik et al. (2016, 2017, 2019); Vashisth et al. (2015, 2016); Malik and Sharma (2011); Tyagi et al. (2022a); Tyagi et al. (2022b); Yadav and Malik (2014); Singh et al. (2011, 2014)). The research article written by Malik et al. (2018) suggests an inventory model with variable cost coefficients. The article of Gupta et al. (2013) provides a

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closed form solution for optimal ordering policies. Some other related research work can be found in Malik et al. (2008) proposes that the inventory model with time dependent demand. Malik et al. (2010) claims that most attempts in the literature try to clarify the importance of supply chain management in decision making.

In realistic conditions, for new products the probability of quality, price, demand, costs are not known or say fixed due to suitable information or deficiency of previous data. So these parameters and variables are considered as fuzzy parameters due to their uncertain behavior. In last three decades, the fuzzy theory especially has been magnificently applied and generated important methods and applications in the various engineering, science, medical and industrial problems. In inventory control and management, the fuzzy theory is a ground-breaking solution method to transform the optimal outcomes. Many inventory models were developed based on the crisp environment because of its general practice phenomena. If some inventory constraints are in fuzzy form then the resultant objective cost function are also in fuzzy form. Zadeh (1965) introduce the concept of fuzzy theory, which is a road map for the topic of uncertainty. Due to handling the uncertainty in inventory costs and demand rate in the inventory system applied the fuzzy set theory. Guiffrida (2010) examined the fuzzy inventory model in which EOQ models and Economic Production Quantity (EPQ) models, joint Economic Lot Sizing (ELS) models, Single period models, multi-items and multi-period inventory models discussed in detail.

Bellman and Zadeh (1970) investigate the relevance of fuzzy goals constraints which discuss the concepts decision making with fuzzy environment in which maximization decision is compact for functional equation is obtained. Kao and Hsu (2002) work on the study of inventory system considered fuzzy based demand which is a trapezoidal fuzzy number and applied Yager's ranking method to obtain the optimal fuzzy based objective cost function. Chang et al. (2006) investigate the fuzzy optimal objective cost function for order quantity, lead time in fuzzy demand using centroid method of defuzzification. Among these techniques/methods based on fuzzy environment such as costs, demand, objective function with Chang et al. (2004); Zimmermann (1985); Vujosevic and Petrovic (1996); Yao and Lee (1999); Halim et al. (2010); Yung et al. (2007); Yong et al. (2010), De (2021) etc., have drawn more and more attention in fuzzy theory.

Yao and Chiang (2003) work to find out using signed distance and centroid method to determine the optimal order quantity by defuzzifying the objective cost function and also compare the final results with these two methods. Chou (2009) performed a fuzzy EOQ model using Graded mean integration, function principal and Kuhn Tucker conditions in which parameters are in trapezoidal fuzzy number form. Liu and Iwamura (1998) reviewed most recently work on a new approach, through fuzzy objective function is optimized. Dutta et al. (2007) reviewed most recently research work on improved fuzzy inventory model where fuzziness and randomness environment using probabilistic mean values of parameters. Over the years, some research work has examined in the literature that the delicacy fuzzy based inventory model with different conditions such as Malik and Singh (2011 & 2013), Malik et al. (2012). Singh et al. (2014) studied theoretically the literature survey about soft computing methods with inventory control techniques.

Daniel et al. (2016) analysis the decision making policies in dynamic models using fuzzy theory. Shekarian et al. (2017) studied theoretically the systematic review on fuzzy inventory model with a sample of large data is considered. Priyan and Manivannan (2017) based their research work on a modelling of supply chain model for optimal inventory policies in fuzzy environment. Sarkar and Mahapatra (2017) focused on fuzzy inventory model in which fuzzy demand and variable lead time is proposed. Malik et al. (2018) proposed a new solution for inventory control model with time varying demand for useful in the manufacturing system. Hollah and Fergany (2019) focused on the inventory system in which stochastic deterioration rate with Pareto distribution function of demand. Recently, Malik and Garg (2021) investigate the relevant inventory model with fuzzy constraints for getting the optimum objective cost function.

In this research work a fuzzy inventory system for a deteriorating product (especially seasonal products) with linear decreasing demand is examined in rough environment (including fuzzy and crisp constraints). The resultant inventory ordering cost, holding cost and deteriorating costs are considered as fuzzy forms. A signed distance method is applied to obtain the optimal inventory decision and total costs. Finally, a crisp as well as fuzzy based sensitivity analysis is proposed and used for minimize the total inventory cost to obtain the optimal result.

2. Model

2.1 Assumptions and Notations

Formulating an appropriate inventory system is one of the most important anxieties for any business organization. This research work examined a fuzzy inventory model for decaying products in which coefficients of inventory costs and objective cost function are in fuzzy in nature which is solved by signed distance technique. The demand rate in this model follows time varying (decreasing function of time).

Shortages are not considered in this research work. This work study the fuzzy inventory model with succeeding assumptions and notations {~ sign used for the fuzzy constraints}:

(i) The demand of the product is $d(t) = (d_1 - d_2 t)$.

(iii) Shortage is not allowed to occur in this research work.

(iii) The replenishment rate is instantaneous i.e., lead time is assumed to be zero.

(iv) Let $\tilde{P}$ and $\tilde{Q}$ be any two fuzzy numbers: $\tilde{P} \oplus \tilde{Q} = (p_1 q_1,\ p_2 q_2,\ p_3 q_3,\ p_4 q_4)$, where $\tilde{P} = (p_1,\ p_2,\ p_3,\ p_4)$ and $\tilde{Q} = (q_1, q_2, q_3, q_4)$.

(vi) Triangular fuzzy number $\phi = (\phi_1, \phi_2, \phi_3)$, where $\phi_1 = \phi - \Delta_1, \phi_2 = \phi, \phi_3 = \phi + \Delta_2$.

The membership function of ϕ is

$$\mu_\phi(\tilde{\phi}) = \begin{cases} \dfrac{\phi - \phi_1}{\phi_2 - \phi_1} & \phi_1 \le \phi \le \phi_2 \\ \dfrac{\phi_3 - \phi}{\phi_3 - \phi_2} & \phi_2 \le \phi \le \phi_3. \\ 0 & \text{otherwise} \end{cases}$$

C_O	The order cost per unit product
C_h	The holding cost per unit product
$N_1(t)$	The inventory level in the time interval $(0, t_0)$
$N_2(t)$	The inventory level in the time interval (t_0, T)
N_0	The initial inventory level
χ	Deterioration rate
C_d	The Deterioration cost per unit product
F_{TC}	The inventory cost function
$\tilde{F}_{TC}$	The inventory cost function in fuzzy model

2.2 Mathematical Analysis

In this research, we worked on a new approached fuzzy based inventory system with variable demand. This research work on a demand and supply base system. Here we discuss two models: Crisp and Fuzzy model. In the time interval (0, T), inventory level slowly-slowly decreases to meet demands of the products. The proposed system under consideration the stock level in which decreases by a unit rate, over the cycle as shown in the below figure.1.

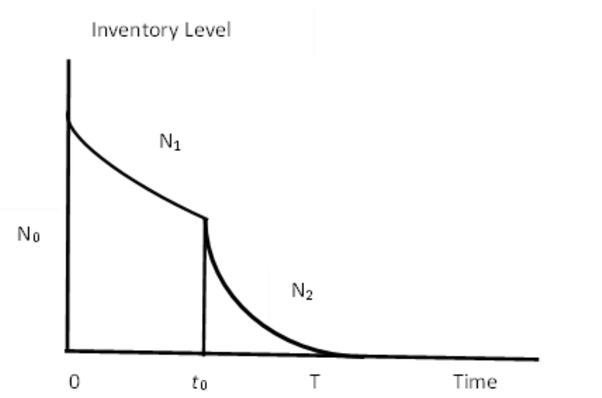

Figure 1- Graphical representation of Inventory system

2.2.1 Crisp Model

For manufacturing system, one of the most important issues is controlling and managing inventory. In this article, an improved inventory system is designed for manufacturing system. In the proposed system, we consider that there is no deterioration in the time interval $(0, t_0)$ and inventory level is depleted due to only demand rate. Therefore, the inventory level $N_1(t)$ describing in the following differential equation:

$$\frac{dN_1(t)}{dt} = -(d_1 - d_2 t), \qquad 0 \le t \le t_0 \qquad \ldots(1)$$

with the boundary condition

$$N_1(0) = N_0 \qquad \ldots(2)$$

The result of the above differential equation (1) is

$$N_1(t) = N_0 - \left(d_1 t - d_2 \frac{t^2}{2}\right), \qquad 0 \le t \le t_0 \qquad \ldots(3)$$

During the time interval (t_0, T) the inventory system depleted due to demand and deterioration rate. Therefore the inventory level $N_2(t)$ describing in the following differential equation:

$$\frac{dN_2(t)}{dt} + \chi N_2(t) = -(d_1 - d_2 t), \qquad t_0 \le t \le T \qquad \ldots(4)$$

with the boundary condition

$$N_2(T) = 0 \qquad \ldots(5)$$

The result of the above differential equation (4) is

$$N_2(t) = -\left[\frac{d_1 - d_2 t}{\chi} + \frac{d_2}{\chi^2}\right] + \left[\frac{d_1 - d_2 T}{\chi} + \frac{d_2}{\chi^2}\right] . e^{\chi (T-t)} \qquad \ldots(6)$$

In the inventory system, the inventory levels $N_1(t)$ and $N_2(t)$ at the time t_0 are equal due to continuity. From equation (3) and (6), we have

$$N_1(t_0) = N_2(t_0)$$

or

$$N_0 = \begin{pmatrix} \left(d_1 t_0 - d_2 \frac{t_0^2}{2}\right) - \left[\frac{d_1 - d_2 t_0}{\chi} + \frac{d_2}{\chi^2}\right] \\ + \left[\frac{d_1 - d_2 T}{\chi} + \frac{d_2}{\chi^2}\right] . e^{\chi (T - t_0)} \end{pmatrix} \quad \ldots(7)$$

Present value of Inventory Ordering Cost

$OC = C_O$ … (8)

Present value of Inventory Holding Cost

$$IH_C = C_h \left(\int_0^{t_0} N_1(t) dt + \int_{t_0}^{t_0 + t_1} N_2(t) dt \right)$$

$$= C_h \left[\left\{ N_0 . t_0 - \left(d_1 \frac{t_0^2}{2} - d_2 \frac{t_0^3}{6} \right) \right\} - \frac{d_1}{\chi^2} (1 + \chi t_1) \right.$$

$$\left. + \frac{d_2}{\chi} \left(\frac{t_1^2}{2} + t_1 t_0 + \frac{t_0}{\chi} - \frac{1}{\chi^2} \right) + \frac{1}{\chi} \left(\frac{d_1 - d_2 T}{\chi} + \frac{d_2}{\chi^2} \right) . e^{\chi t_1} \right] \quad \ldots(9)$$

Present value of Inventory Deteriorating Cost

$$ID_C = C_d \left(\int_{t_0}^{t_0 + t_1} \chi \, N_2(t) dt \right)$$

$$= \chi . C_d \left[-\frac{d_1}{\chi^2} (1 + \chi t_1) + \frac{d_2}{\chi} \left(\frac{t_1^2}{2} + t_1 t_0 + \frac{t_0}{\chi} - \frac{1}{\chi^2} \right) \right.$$

$$\left. + \frac{1}{\chi} \left(\frac{d_1 - d_2 T}{\chi} + \frac{d_2}{\chi^2} \right) . e^{\chi t_1} \right] \quad \ldots(10)$$

Total Inventory cost function is

$$F_{TC}(t_0, t_1) = \frac{1}{T} \left(OC + IH_C + ID_C \right)$$

$$= \frac{1}{T} \left[C_O + C_h \left\{ N_0 . t_0 - \left(d_1 \frac{t_0^2}{2} - d_2 \frac{t_0^3}{6} \right) \right\} + (C_h + \chi . C_d) \left\{ -\frac{d_1}{\chi^2} (1 + \chi t_1) \right. \right.$$

$$\left. \left. + \frac{d_2}{\chi} \left(\frac{t_1^2}{2} + t_1 t_0 + \frac{t_0}{\chi} - \frac{1}{\chi^2} \right) + \frac{1}{\chi} \left(\frac{d_1 - d_2 T}{\chi} + \frac{d_2}{\chi^2} \right) . e^{\chi t_1} \right\} \right] \quad \ldots(11)$$

2.2.2 Optimum Solution Method

For obtain the optimum solution, we obtain the values of t_0, T, N_0, F_{TC} for this inventory system. For minimize the total inventory cost F_{TC}, we have

$$\frac{\partial F_{TC}}{\partial t_0} = 0, \ \frac{\partial F_{TC}}{\partial t_1} = 0,$$

$$\left(\frac{\partial^2 F_{TC}}{\partial t_0^2} \right) . \left(\frac{\partial^2 F_{TC}}{\partial t_1^2} \right) - \left(\frac{\partial^2 F_{TC}}{\partial t_0 \partial t_1} \right)^2 > 0$$

and $\left(\frac{\partial^2 F_{TC}}{\partial t_0^2} \right) > 0.$

2.2.3 Fuzzy Model

Most of the researchers proposed their models for forecasting the optimal solution and optimal ordering quantity assumed by constant demand and deterioration rate. Generally, the inventory system with deteriorating products, such as Vegetable, fruits, packing food products contains the rough constraints, like inaccurate inventory costs, fuzzy holding costs, ordering costs etc. In this research work we consider the inventory ordering cost ($\tilde{C}_O$), inventory holding cost ($\tilde{C}_h$) and inventory deteriorating costs ($\tilde{C}_d$) are in fuzzy form. Therefore, due to our consideration, the objective function $\left(F_{TC}\right)$ becomes $\left(\tilde{F}_{TC}\right)$.

2.2.4 Signed Distance Method

Signed distance method is a well-known method and applied in many uncertainty domain of the study. The fuzzy constraints of the proposed inventory model: inventory ordering cost ($\tilde{C}_O$), inventory holding cost ($\tilde{C}_h$) and inventory deteriorating costs ($\tilde{C}_d$) are defined as

(i)
$C_h \in [C_h - \Delta_1, \ C_h - \Delta_2]$, where $0 < \Delta_1 < C_h$ and $0 < \Delta_1 \Delta_2$.

(ii)
$C_O \in [C_O - \Delta_3, \ C_O - \Delta_4]$, where $0 < \Delta_3 < C_O$ and $0 < \Delta_3 \Delta_4$.

(iii)
$C_d \in [C_d - \Delta_5, \ C_d - \Delta_6]$, where $0 < \Delta_5 < C_d$ and $0 < \Delta_5 \Delta_6$.

Using the signed distance method, the defuzzification of inventory ordering cost ($\tilde{C}_O$), inventory holding cost ($\tilde{C}_h$) and inventory deteriorating costs ($\tilde{C}_d$) are defined as

(i) $d(\tilde{C}_h, 0) = C_h + \frac{1}{4}(\Delta_2 - \Delta_1)$.

(ii) $d(\tilde{C}_O, 0) = C_O + \frac{1}{4}(\Delta_4 - \Delta_3)$.

(iii) $d(\tilde{C}_d, 0) = C_d + \frac{1}{4}(\Delta_6 - \Delta_5)$.

Fuzziness can be related to objective cost functions with constrained involved. Zimmermann (1976) proposed the fuzzy based both objective function and constraints. Here the fuzzy objective cost function $\left(\tilde{F}_{TC}\right)$ can be written as in form

$$\tilde{F}_{TC}(t_0, t_1) = \left(F_{TC1} + F_{TC2} + F_{TC3} \right)$$

$$F_{TC1}(t_0, t_1) = \frac{1}{T} \left[(C_O - \Delta_3) + (C_h - \Delta_1) \left\{ N_0 . t_0 - \left(d_1 \frac{t_0^2}{2} - d_2 \frac{t_0^3}{6} \right) \right\} \right.$$

$$+\left(\left(C_h-\Delta_1\right)+\chi.\left(C_d-\Delta_5\right)\right)\left\{-\frac{d_1}{\chi^2}\left(1+\chi t_1\right)\right.$$

$$\left.\left.+\frac{d_2}{\chi}\left(\frac{t_1^2}{2}+t_1t_0+\frac{t_0}{\chi}-\frac{1}{\chi^2}\right)+\frac{1}{\chi}\left(\frac{d_1-d_2T}{\chi}+\frac{d_2}{\chi^2}\right).e^{\chi t_1}\right\}\right] \qquad \ldots(12)$$

$$F_{TC2}\left(t_0,t_1\right)=F_{TC}\left(t_0,t_1\right)$$

$$F_{TC3}\left(t_0,t_1\right)=\frac{1}{T}\left[\left(C_O-\Delta_4\right)+\left(C_h-\Delta_2\right)\left\{N_0.t_0-\left(d_1\frac{t_0^2}{2}-d_2\frac{t_0^3}{6}\right)\right\}\right.$$

$$+\left(\left(C_h-\Delta_2\right)+\chi.\left(C_d-\Delta_6\right)\right)\left\{-\frac{d_1}{\chi^2}\left(1+\chi t_1\right)\right.$$

$$\left.\left.+\frac{d_2}{\chi}\left(\frac{t_1^2}{2}+t_1t_0+\frac{t_0}{\chi}-\frac{1}{\chi^2}\right)+\frac{1}{\chi}\left(\frac{d_1-d_2T}{\chi}+\frac{d_2}{\chi^2}\right).e^{\chi t_1}\right\}\right] \qquad \ldots(13)$$

Speciously, the fuzzy method examined in this paper can be applied to inventory system to get the optimum inventory policies. The defuzzification of the objective cost function is

$$\tilde{d}\left(F_{TC}\right)=F_{TC}\left(t_0,t_1\right)+\frac{1}{T}\left[\frac{1}{4}\left(\Delta_4-\Delta_3\right)+\frac{1}{4}\left(\Delta_2-\Delta_1\right)\left\{N_0.t_0-\left(d_1\frac{t_0^2}{2}-d_2\frac{t_0^3}{6}\right)\right\}\right.$$

$$+\left(\frac{1}{4}\left(\Delta_2-\Delta_1\right)+\chi.\frac{1}{4}\left(\Delta_6-\Delta_5\right)\right)\left\{-\frac{d_1}{\chi^2}\left(1+\chi t_1\right)\right.$$

$$\left.\left.+\frac{d_2}{\chi}\left(\frac{t_1^2}{2}+t_1t_0+\frac{t_0}{\chi}-\frac{1}{\chi^2}\right)+\frac{1}{\chi}\left(\frac{d_1-d_2T}{\chi}+\frac{d_2}{\chi^2}\right).e^{\chi t_1}\right\}\right] \qquad \ldots(14)$$

3. Numerical Analysis

For the importance of demand and supply based inventory system, this research work investigates the fuzzy based inventory model for non-instantaneous decaying products with time varying demand under fuzzy constraints. To demonstrate the validity and relevancy of this work, first we consider the numerical examples and discuss the sensitivity analysis of constraints used in this system with objective optimal total cost function. The solution procedure of the inventory system for the objective optimum cost function and order quantity illustrated through the following examples:

Example.1. Here we assumed the following constraints values for the developed model: Demand rate d_1=600 and d_2=4 units per annum; Inventory ordering cost (O_C) =1200; Deterioration rate (χ) =0.8; Inventory holding cost (C_h) =0.4; Inventory deteriorating cost (C_d) =0.05. Using these constraints values, the proposed inventory system have been solved by non-linear optimization techniques with various parameters values, and the sensitivity analysis with resultant are presented in table 1. The optimum solution is obtained with the MATLAB software after some iterations, t_0^*=112.411, $T^*(=t_0+t_1)$ =116.118, N_0^*=42174.129 and F_{TC}^*=6760.671. It can be decided that the output of this analyses (Table 1) and discusses the obtained result is acceptable and can be used in business industries for decision making. The below sensitivity analysis and graphical results demonstrate the efficiency of our developed system.

Table 1 Results for crisp model

Changes in		t_0	t_1	N_0	F_{TC}
d_1	-10%	101.140	4.133	34157.004	5479.358
	600	**112.411**	**3.707**	**42174.129**	**6760.671**
	+10%	123.676	9.559	51034.729	8177.200
d_2	-10%	124.920	2.905	46862.973	7509.603
	4	**112.411**	**3.707**	**42174.129**	**6760.671**
	+10%	102.175	1.496	38337.557	6148.103
χ	-10%	112.411	1.742	42174.130	6760.661
	0.8	**112.411**	**3.707**	**42174.129**	**6760.671**
	+10%	112.411	1.381	42.174.130	6760.677
C_d	-10%	112.411	1.925	42174.128	6760.670
	0.05	**112.411**	**3.707**	**42174.129**	**6760.671**
	+10%	112.411	6.677	42174.130	6760.672
O_C	-10%	112.402	3.507	42172.789	6751.738
	1200	**112.411**	**3.707**	**42174.129**	**6760.671**
	+10%	112.420	3.916	42175.470	6769.603
C_h	-10%	112.401	6.713	42172.640	6085.671
	0.4	**112.411**	**3.707**	**42174.129**	**6760.671**
	+10%	112.419	2.162	42175.348	7435.671

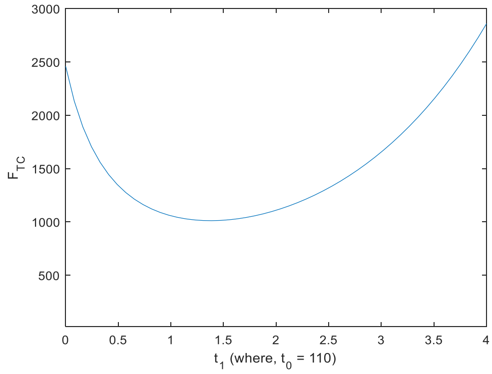

Figure 2- Optimum inventory cost function (Crisp model) vs. t_1

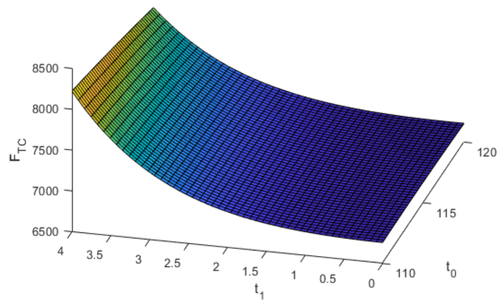

Figure 3- Optimum inventory cost function (Crisp model) vs. t_0 and t_1

Example.2. This example analyses the fuzzy model and discusses the obtained result and its implication to achieved the optimum objective cost function and ordering policies. Using the above record and interchange the objective cost functions and explain again with the same set of explanations determined in the first example. The sensitivity analysis, use of numerical results and concept of fuzzy model discussed in table 2.

The optimum solution is obtained with the MATLAB software after some iterations: t_0^*=112.410, $T^*(=t_0+t_1)$ =115.202, N_0^*=42174.045 and F_{TC}^*=6718.483. From a real world study, data are collected from the business industries and corresponding fuzzy numbers are designed. From these fuzzy numbers and signed distance method, the optimum cost function is calculated.

Table 2 Results for fuzzy model

Changes in		t_0^*	t_1^*	N_0^*	dF_{Tc}
d_1	-10%	101.139	3.095	34156.910	5445.186
	600	**112.410**	**2.792**	**42174.045**	**6718.483**
	+10%	123.676	3.955	51034.652	8126.153
d_2	-10%	124.919	9.056	46862.897	7462.728
	4	**112.410**	**2.792**	**42174.045**	**6718.483**
	+10%	102.174	9.424	38337.464	6109.750
χ	-10%	112.410	1.512	42174.045	6718.480
	0.8	**112.410**	**2.792**	**42174.045**	**6718.483**
	+10%	112.410	0.000	42174.045	6718.484
C_d	-10%	112.410	1.397	42174.045	6718.481
	0.05	**112.410**	**2.792**	**42174.045**	**6718.483**
	+10%	112.410	5.195	42174.045	6718.485
O_C	-10%	112.401	2.635	42172.696	6719.551
	1200	**112.410**	**2.792**	**42174.045**	**6718.483**
	+10%	112.419	2.958	42175.394	6719.416
C_h	-10%	112.400	5.081	42172.535	6043.484
	0.4	**112.410**	**2.792**	**42174.045**	**6718.483**
	+10%	**112.419**	**1.627**	**42175.279**	**7393.483**

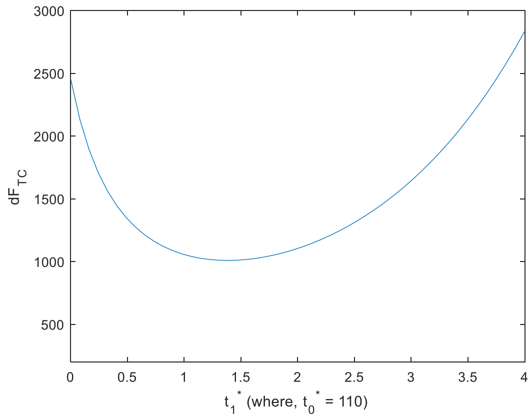

Figure 4- Optimum inventory cost function (Fuzzy model) vs. t_1

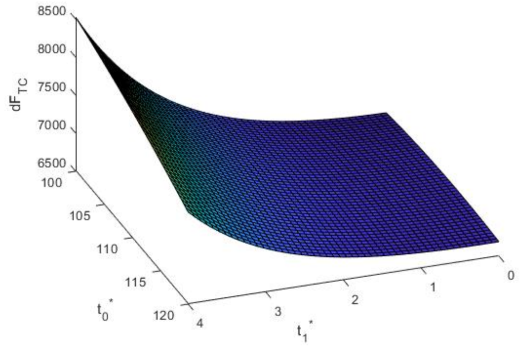

Figure 5- Optimum inventory cost function (Fuzzy model) vs. t_0 and t_1

4. Sensitivity Analysis

A numerical solution for finding the optimum values of initial inventory level and total inventory cost which provided to demonstrate adequately that the proposed system is feasible and efficient. However, the new improved model which we examined does not guarantee to obtain the optimal solution but after the sensitivity analysis of the objective function with constraints demonstrate the relevancy and validity of this model. Here the sensitivity analysis is implemented with the changing the values (+10%, 0%, -10%) of the constraints taking one constraints unchanged. The following observations are noted on the basis of the sensitivity analysis:

(i) From the above table 1 and 2, we see that the cost objective function obtained in fuzzy model is minimum compare to crisp model.

(ii) From the above table 1 and 2, we see that if the cycle time increases than the objective cost function F_{TC} as well as $\tilde{F}_{TC}$ are also increases.

(iii) From the above table 1 and 2, we see that if the demand parameter d_1 is increases than the objective cost function F_{TC} as well as $\tilde{F}_{TC}$ are also increases but if demand parameter d_1 is increases than the objective cost function F_{TC} as well as $\tilde{F}_{TC}$ are also decreases due to linear decreasing demand.

(iv) From the above table 1 and 2, we see that the increases the cost of C_h, C_O and C_d also increases the objective cost function F_{TC} as well as $\tilde{F}_{TC}$.

Conclusion

The effect of constraints involved in this inventory system with total inventory cost and sensitivity analysis is illustrated to demonstrate the optimal output. In this study a fuzzy inventory system with time varying (linearly decreasing demand) is formulated. This research work has been solved in both cases crisp and fuzzy environment. For crisp system, the optimal inventory cost is minimize with optimization techniques and in fuzzy system, the optimal inventory cost is minimize with signed distance method. This study will help the experts of seasonal items such as fruits, vegetable, packed food, medicine, cosmetic items etc. Finally numerical analysis and sensitivity analysis of total inventory cost with various constraints are illustrated to obtain the optimum result and, finally the fuzzy environment is better in comparison of crisp environment for optimizing (minimize) the total inventory cost of the system. This analysis and formulation examined are in general framework and can be protracted to contain some other fuzzy based method and fuzzy logic relations with various constraints of demand, deterioration, advertisement, price, backlogging, profit, production, trade credit and inflation etc.

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