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A Study on the Estimation of Ultrafine Dust (PM2.5) Prediction Model by Vector Error Correction (VECM)

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Abstract- This study presents an ultrafine dust (PM2.5) prediction model using a vector error correction model, and daily time series data of ultrafine dust (UFD), nitrogen dioxide (NO2), and carbon monoxide (CO) observed in Jung-gu, Seoul from January 1, 2017 to October 31, 2021. From the Granger causality test for prediction model estimation, it was found that the vector time series model can be applied, and the model was turned out as the VAR(2) model according to minimum information criterion. Using this, the prediction model was concluded as VECM(2), a model having intercept and no linear trend, as a result of performing three cointegration coefficient tests to select VECM(p). Therefore, the prediction model was presented by calculating the long-term parameter estimate, the error correction coefficient estimate, and the parameter estimate estimated by the model. And as a result of performing model diagnosis on the residual time series vector obtained after fitting the VECM(2) model, it was found that there was no cross-correlation until the lag 12, meaning that the VECM(2) prediction model in this study was a reliable model.

Keywords: *Vector time series analysis, VECM, Ultrafine dust (PM2.5), Cointegration coefficient tests, Multivariate Portmanteau test Portmanteau test*

1. Introduction

Recently interest in fine dust at home and abroad is rapidly increasing. As the public interest and demand for resolving the fine dust problem increase, the Korean government is actively promoting related policies to respond to fine dust in Korea. The main sources of fine dust are business sites, construction machinery, power plants, automobiles, air conditioning, air-conditioning, fugitive dust, biological combustion, and the use of organic solvents. Ultrafine dust generated by the reaction of hazardous substances from factory chimneys or automobile exhausts with substances in the air is mainly generated when chemical fuels such as coal or petroleum are burned or when gases are emitted from factories and automobiles [1]. Ultrafine dust contains substances harmful to the body, such as nitrate, sulfate, and ammonium, which are known to be more harmful than fine dust. (Figure 1) shows the change in the annual average concentration of fine dust and ultrafine dust (atmospheric environment information in 2020) at 52 air monitoring stations across the country (period: 2000 to 2020). The fine dust concentration peaked at 64μg/m^3 in 2002 and has been steadily decreasing until recently, and in 2020, it showed the lowest concentration at 35μg/m^3. Ultrafine dust started to be measured in 2015 and has been steadily decreasing, showing the lowest concentration of 19μg/m^3 in 2020.

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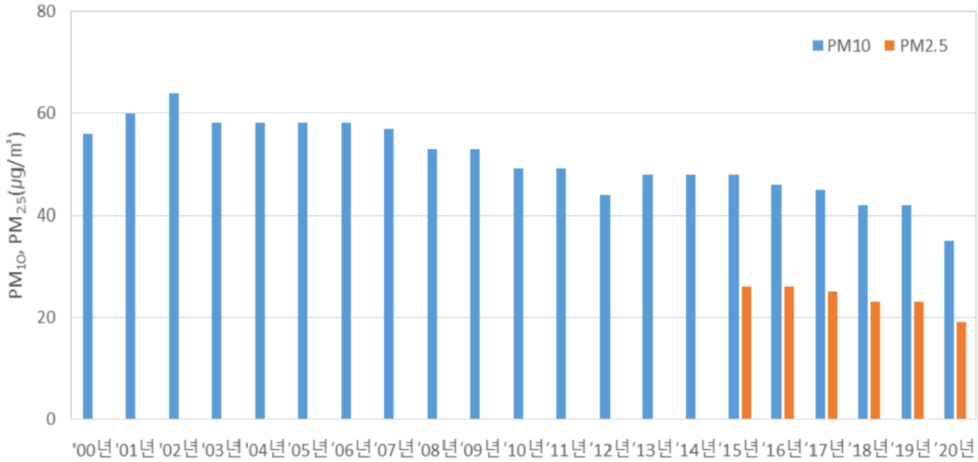

Figure 1- Annual change of fine dust (PM10, PM2.5)

Although the annual average concentration of fine and ultrafine dust in Korea is showing a decreasing trend overall, it is far below the recommended standards (PM10:20μg/m^3, PM2.5:10μg/m^3) suggested by the World Health Organization. Thus, people are dissatisfied with the government's measures against fine dust and are anxious about fine dust.

2. Related research

As the social and economic problems caused by fine dust and the seriousness of its harmful effects on the human body have been raised, research on fine dust is being actively in progress. It has been studied that fine dust is known to increase excess mortality and cardiovascular and respiratory diseases [2]. Ultrafine dust is particularly dangerous for respiratory and cardiovascular diseases in women and children [3], and it has been studied that the hospitalization rate for children with asthma increases by 3.45% for every 10 ㎍/㎥ increase in ultrafine dust concentration [4]. Fine dust has been shown to have a large impact not only on the human body but also on society and economy. High concentration of fine dust causes people to avoid going out, which can lead to a decrease in interaction and consumption among people, which can eventually weaken the vitality of the national economy [5]. It was also analyzed that the concentration of fine dust had a negative impact on sales of retail outlets [6]. As statistical prediction model studies, it was revealed that the peripheral distribution of fine dust has a thick tail and strong dependence [7], and Shon et al. (2016) studied a prediction model that applied CMAQ, a numerical model for air quality prediction [8]. It was found that factors highly correlated with fine dust include nitrogen dioxide and carbon monoxide, and they also generate secondary fine dust and have a strong correlation with fine dust [9]. Recently, research on prediction models using artificial intelligence (machine learning, deep learning) has been actively conducted. However, the accuracy of prediction models differs from statistical probability models to artificial intelligence models depending on the characteristics of each region and industry. In addition, there are not many studies on prediction using the vector time series model. Therefore, in this study, vector time series data (ultrafine dust (UFD), nitrogen dioxide (NO2), carbon monoxide (CO)) are used to estimate and predict the prediction model using a vector error correction model.

3. Research model

3.1 Vector error correction model (VECM)

When there are several related time series data and the prediction value can be obtained from the related time series data, it will be more efficient than forecasting using only one time series data if. The method of analyzing multiple time series at the same time is called vector time series analysis. Vector time series analysis is intended to analyze the dynamic characteristics and interactions of multiple time series variables while simultaneously considering them, and does not prescribe specific variables as dependent variables in advance. It is not always a good idea to fit the model after differentiating the nonstationary time series data. The vector error correction model, which is one of vector time series analysis, is a useful model to analyze the relationship between data when nonstationary time series data are cointegrated. Nonstationary time series in cointegration relationship can be modeled by vector error correction model. The p-lag vector error correction model, VECM(p), is defined as (Equation 1) [10].

$$\nabla Z_t = \delta(\mathrm{t}) + \alpha\beta' Z_{t-1} + \sum_{i=1}^{p-1} \Phi_i \nabla Z_{t-i} + \varepsilon_t \qquad (1)$$

Where $\nabla Z_t = Z_t - Z_{t-1}$, α, and β are $l \times m$ matrix respectively, Φ_i is an $l \times l$ matrix, $\delta(t) = \delta_0 + \delta_1 t$ is a deterministic trend term, and δ_0 and δ_1 are $l \times 1$ constant vectors. The vector error correction model is a model that corrects the loss of information with the correction term

$\alpha\beta' Z_{t-1}$ in the process of fitting the vector model after all time series variable are differentiated.

3.2 Granger causality test

It is to view whether vector time series data can be analyzed by vector time series model [11]. The hypotheses used in this study are shown in (Table 1).

Table 1. Hypotheses

TEST 1	$H_{10}: \{UFD\} \nleftarrow \{\{NO2\}, \{CO\}\}$ $H_{11}: \{UFD\} \leftarrow \{\{NO2\}, \{CO\}\}$
TEST 2	$H_{20}: \{NO2\} \nleftarrow \{\{UFD\}, \{CO\}\}$ $H_{21}: \{NO2\} \leftarrow \{\{UFD\}, \{CO\}\}$
TEST 3	$H_{30}: \{CO\} \nleftarrow \{\{UFD\}, \{NO2\}\}$ $H_{31}: \{CO\} \leftarrow \{\{UFD\}, \{NO2\}\}$

3.3 Cointegration coefficient test

If $l \times l$ matrix Π is a perfect coefficient matrix, namely $rank(\Pi) = l$, then all time series of Z_t are $I(0)$ stationary time series. If $rank(\Pi) = 0$, it means $\Pi = 0$, and since the cointegration vector does not exist, the vector error correction model cannot be applied. In this case, all time series of Z_t are $I(1)$ nonstationary time series and are predicted by applying VAR(p) model to the differential time series. However, if $0 < rank(\Pi) = m < l$, there are m independent linear combination equations that becomes stationary, and it is predicted by the vector error correction model. Then, the cointegration coefficient test means a test that determines the number of columns of $\Pi = \alpha\beta'$ that are linearly independent, that is, the value of the cointegration coefficient m . Hypothesis for this test is as (Equation 2).

$$H_0: m = m_0 \;\; vs. \;\; H_1: m > m_0 \tag{2}$$

Where $m = rank(\Pi)$ is the number of cointegration relations. When constructing the VECM(p) model from the VAR(p) model, various models can be considered depending on the decisive factors included in the model. For the cointegration test used in this study, the vector error correction model was determined by considering three models. Trace statistics were used to determine the presence or absence of the cointegration vector, and the models considered are as follows [12], [13]. The first model considered is the case where there is no intercept in the VECM(p) term, as shown in (Equation 3),

$$\nabla Z_t = \alpha\beta' Z_{t-1} + \sum_{i=1}^{p-1} \Phi_i \nabla Z_{t-i} + \varepsilon_t \tag{3}$$

The second model considered is the case where the error correction term has intercept, as shown in (Equation 4),

$$\nabla Z_t = \alpha(\beta_0 + \beta' Z_{t-1}) + \sum_{i=1}^{p-1} \Phi_i \nabla Z_{t-i} + \varepsilon_t \tag{4}$$

The third model considered is the case where the VECM(p) term has intercept and there is no linear trend, as shown in (Equation 5).

$$\nabla Z_t = \delta_0 + \alpha\beta' Z_{t-1} + \sum_{i=1}^{p-1} \Phi_i \nabla Z_{t-i} + \varepsilon_t \tag{5}$$

And the trace statistics for testing the null hypothesis that there are   cointegration vectors are as shown in (Equation 6).

$$\lambda_{trace} = -n \sum_{i=r+1}^{k} \log(1 - \lambda_i) \tag{6}$$

where n is the number of observations, and λ_i are the eigenvalues.

3.4 Model diagnostic test

After fitting the vector error correction model, the multivariate Portmanteau test statistic to test whether the correlation remains in the residual time series vector is as shown in (Equation 7) [14].

$$Q(k) = n^2 \sum_{k=1}^{K} (n-k)^{-1}\, tr\{\hat{\rho}_k(e) \widehat{\Sigma_\varepsilon^{-1}}\, [\widehat{\rho_k}(e)]'\} \tag{7}$$

where $\widehat{\rho_k}(e)$ is the k lag sample cross autocorrelation matrix of the residual time series vector e_t , $\widehat{\Sigma_\varepsilon}$ is the estimator of Σ_ε , which is the covariance matrix of the multivariate white noise process, n is the size of the time series data, k is the appropriate time lag, and l is the number of univariate time series constituting the multivariate time series. Hypothesis for this is as (Equation 8).

$$H_0: \rho_1(e) = \rho_2(e) = \cdots = \rho_k(e) = 0 \qquad H_1: Not\ H_0 \tag{8}$$

where $\rho_k(e)$, $k = 1,2,\cdots,K$

4. Results

4.1 Stationary time series transformation and unit root test

In order to convert the multivariate time series data (ultrafine dust, nitrogen dioxide, carbon monoxide) used in this study to stationary time series data, a power transformation was performed to stabilize the variance. As shown in Figure 2, the time series data showed a trend.

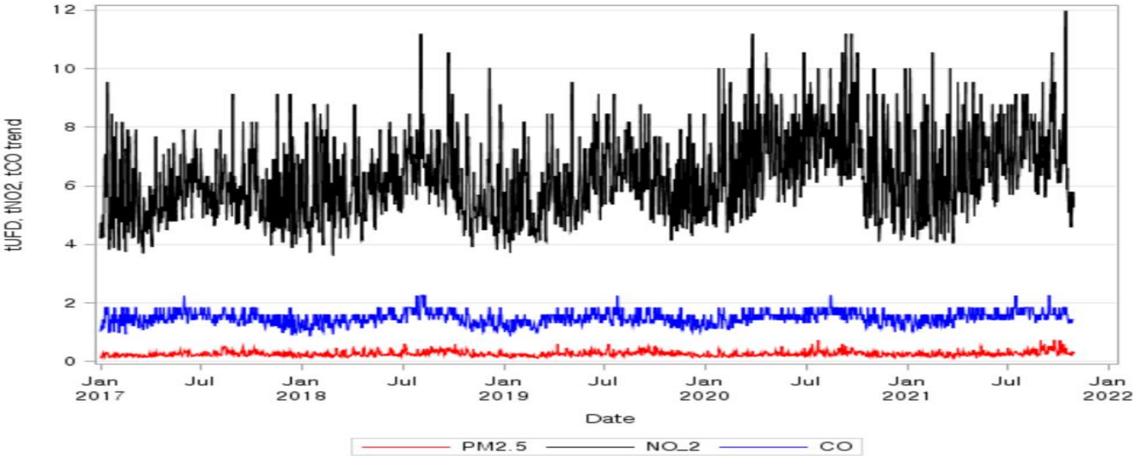

Figure 2- Time series plot after power transformation

As a result of removing the trend by first differentiating the multivariate time series data (ultrafine dust, nitrogen dioxide, carbon monoxide) subjected to power transformation, and checking the autocorrelation analysis diagram, it was shown truncated to 0 after p = 2 lag in the sample partial autocorrelation function (SPACF). After checking the sample partial autoregression matrix (SPAM) and the sample cross-correlation matrix (SCCM) between time series data to determine p in the initial model VAR(p) model, which is the previous stage of the cointegration test, p = 4 and q = 2 have been identified. As a result of confirming the AICC statistic value, which is the minimum information criterion (MINIC) method, by competing models including these two models, the minimum AICC value was identified as the VAR(2) model with p=2 as shown in Table 2.

Table 2. Model identification by MINIC

Lag	MA0	MA1	MA2
AR0	-11.3225	-11.5434	-11.8132
AR1	-12.3812	-12.6355	-12.6683
AR2	-12.8735	-12.6893	-12.6549
AR3	-12.4947	-12.6377	-12.6102
AR4	-12.5339	-12.7292	-12.5486

Therefore, as a result of performing the unit root test with p = 2, the p-value of the Tau test was less than the significance level (0.05) as shown in Table 3, and all of them were confirmed as stationary time series data.

Table 3. ADF unit root test

Variable	Type	Tau	Pr < Tau
∇tUFD	Zero Mean	-35.32	<.0001
	Single Mean	-35.31	<.0001
	Trend	-35.30	<.0001
∇tNO2	Zero Mean	-38.00	<.0001
	Single Mean	-37.99	<.0001
	Trend	-37.98	<.0001
∇tCO	Zero Mean	-36.16	<.0001
	Single Mean	-36.15	<.0001
	Trend	-36.14	<.0001

4.2 Granger causality test

To test if stationary time series data can be analyzed by the vector time series model, the Granger causality test was done with VAR(2) model with $\alpha = 0.05$. As a result, (Table 4) show that all of p-values from chi-square test appear smaller than the significance level, so each time series data can be explained by its own and the past values of the other two time series, respectively.

Table 4. Granger causality test

Test	DF	Chi-Square	Pr > ChiSq
Test 1	4	34.70	0.0001
Test 2	4	9.88	0.0225
Test 3	4	57.22	0.0001

4.3 Cointegration coefficient test and model identification

The trace statistic of the cointegration coefficient test proposed by Johansen was used. The model was identified by testing three cases in order to select an appropriate VECM(p) out of VAR(p) model; VECM(p) term has no intercept (Case 1), the error correction term has intercept (Case 2), and the VECM(p) term with intercept and no linear trend (Case 3). The result of the cointegration test with no intercept in VECM(p) term is as shown in (Table 5), and in this case, the cointegration relationship does not exist. That is, it can be seen that rank=0 because H_0 is adopted as trace=35.5668 < 5% and Critical Value=42.35, where $H_0\!: m = 0\ \ vs.\ H_1\!: m > 0$.

Table 5. Cointegration coefficient test (Case 1)

H0 : Rank= r	H1 : Rank > r	Trace	5% Critical Value	Drift in ECM	Drift in Process

0	0	35.5668	42.35	NOINT	Constant
1	1	12.4342	13.64		
2	2	2.5904	3.84		

When the error correction term has intercept, the result of the cointegration test is as shown in (Table 6), and in this case, it can be seen that there is one cointegration relationship. That is, it can be seen that rank=1 because H_0 is adopted as trace=10.6371 < 5% and Critical Value=13.36, where $H_0: m = 1\ vs.\ H_1: m > 1$.

Table 6. Cointegration coefficient test (Case 2)

H0 : Rank=r	H1 : Rank > r	Trace	5% Critical Value	Drift in ECM	Drift in Process
0	0	56.5339	28.83	Constant	Constant
1	1	10.6371	13.64		
2	2	3.2656	4.72		

When the VECM(p) term has intercept and there is no linear trend, the result of the cointegration test is as shown in (Table 7), and it can be seen that there is one cointegration relationship in this case as well. That is, it can be seen that rank=1 because H_0 is adopted as trace=8.1736 < 5% and Critical Value=10.46, where $H_0: m = 1\ vs.\ H_1: m > 1$.

Table 7. Cointegration coefficient test (Case 3)

H0 : Rank=r	H1 : Rank > r	Trace	5% Critical Value	Drift in ECM	Drift in Process
0	0	52.3915	21.57	Constant	Linear
1	1	8.1736	10.46		
2	2	2.1206	3.48		

Since there is one cointegration relationship in the models of Case 2 and Case 3, chi-square test for hypothesis $H_0: Case\ 2\ vs.\ H_1: Case\ 3$ for proper model identification was performed. And the result is shown in (Table 8). That is, in the χ^2 test of Rank=1, the p-value is 0.0172, which is less than 0.05, so H_0 is rejected and H_1 is adopted. Therefore, the vector error correction model was identified as VECM(2), as shown in (Equation 9).

$$\nabla Z_t = \delta_0 + \alpha\beta' Z_{t-1} + \Phi_1 \nabla Z_{t-1} + \varepsilon_t \tag{9}$$

Table 8. Model identification

Rank	DF	Chi-Square	Pr > ChiSq
0	3	9.28	0.0924
1	2	8.14	0.0172
2	1	5.12	0.0153

4.4 Model estimation

The VECM(2) model estimated above is obtained as (Equation 10) from the long-term parameter estimate and error correction coefficient estimate (Table 9), and parameter estimate value (Table 10).

$$\nabla Z_t = \begin{pmatrix} 0.01528 \\ 1.77266 \\ 0.11603 \end{pmatrix} + \begin{pmatrix} 0.02185 & -0.00566 & 0.01010 \\ 2.54796 & -0.66000 & 1.17784 \\ 0.16652 & -0.04613 & 0.07694 \end{pmatrix} \begin{pmatrix} UFD_{1,t-1} \\ NO2_{2,t-1} \\ CO_{3,t-1} \end{pmatrix} + \begin{pmatrix} -0.22985 & 0.00881 & 0.02402 \\ -1.38103 & 0.17534 & -0.95302 \\ -0.01421 & 0.05531 & -0.39752 \end{pmatrix} \begin{pmatrix} \nabla UFD_{t-1} \\ \nabla NO2_{t-1} \\ \nabla CO_{t-1} \end{pmatrix} \tag{10}$$

Table 9. Estimation of long-term parameters and error correction coefficients

Beta Estimates			
Variable	1	2	3
UFD	1.00000	1.00000	1.00000
NO2	-0.25903	-0.01123	0.06666
CO	0.46227	-0.02650	-1.02715
Alpha Estimates			
Variable	1	2	3
UFD	0.02185	-0.43958	0.00338
NO2	2.54795	-3.06491	0.77527
CO	0.16652	-0.62181	0.39373

Table 10. Parameter estimation

Equation	Parameter	Estimate	Variable
D_UDF	CONST1	0.01528	1
	AR1_1_1	0.02185	UFD(t-1)
	AR1_1_2	-0.00566	NO2(t-1)
	AR1_1_3	0.01010	CO(t-1)
	AR2_1_1	-0.22985	D_UFD(t-1)
	AR2_1_2	0.00881	D_NO2(t-1)
	AR2_1_3	0.02402	D_CO(t-1)
D_NO2	CONST2	1.77216	1
	AR1_2_1	2.54795	UFD(t-1)
	AR1_2_2	-0.66001	NO2(t-1)
	AR1_2_3	1.17784	CO(t-1)
	AR2_2_1	-1.38103	D_UFD(t-1)
	AR2_2_2	0.17534	D_NO2(t-1)
	AR2_2_3	-0.95302	D_CO(t-1)
D_CO	CONST3	0.11603	1

AR1_2_1	0.16652	UFD(t-1)
AR1_3_2	-0.04313	NO2(t-1)
AR1_3_3	0.07698	CO(t-1)
AR2_3_1	-0.01421	D_UFD(t-1)
AR2_3_2	0.05531	D_NO2(t-1)
AR2_3_3	-0.39752	D_CO(t-1)

4.5 Model diagnosis and prediction

As a result of the significance test of the cross-correlation matrix for the residual time series vector obtained after fitting the VECM(2) model (Equation 10), it was found to follow the white noise process, and as shown in the multivariate Portmanteau test (Table 11), cross-correlation did not exist until the maximum lag of 12. That is, the p-values of the chi-square statistics are greater than the significance level of 0.05 at all lags, indicating that there is no cross-correlation.

Table 11. Model diagnostic test

Up To Lag	Chi-Square	Pr > ChiSq
3	13.46	0.0762
4	20.03	0.2230
5	27.72	0.2964
6	33.75	0.1853
7	48.76	0.1512
8	61.21	0.3204
9	67.54	0.2527
10	76.64	0.2459
11	82.59	0.1992
12	94.71	0.3937

Using the VECM(2) model (Equation 10), the predicted values after the 1-lag of the fitting period, the predicted values after the multi-lag of the forecast period (6 months), and the 95% confidence interval are presented graphically. (Figure 3) is a prediction for ultrafine dust (PM2.5), (Figure 4) is a prediction for nitrogen dioxide (NO2), and (Figure 5) is a prediction for carbon monoxide (CO). The prediction results of ultrafine dust, nitrogen dioxide, and carbon monoxide were found to decrease overall.

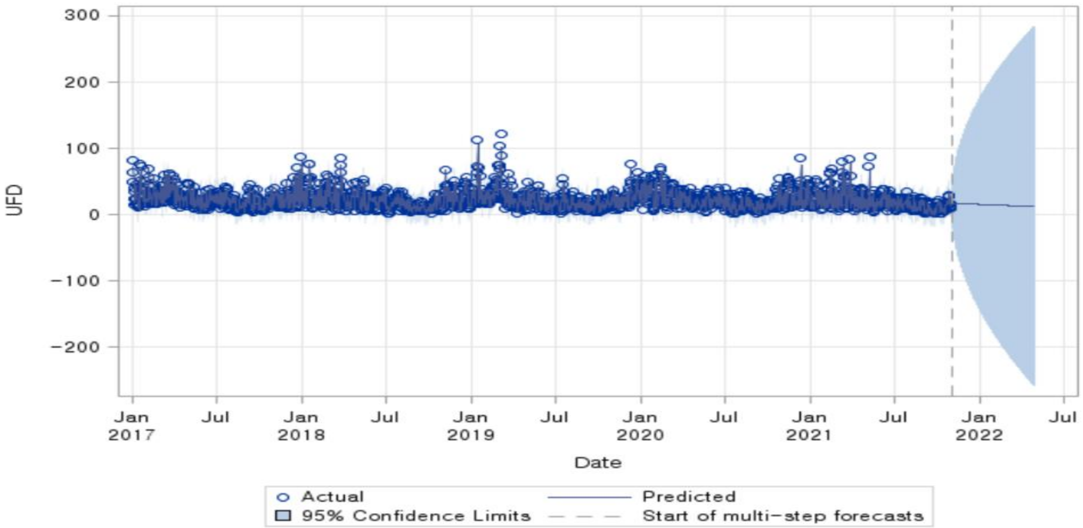

Figure 3- PM2.5 prediction by VECM(2)

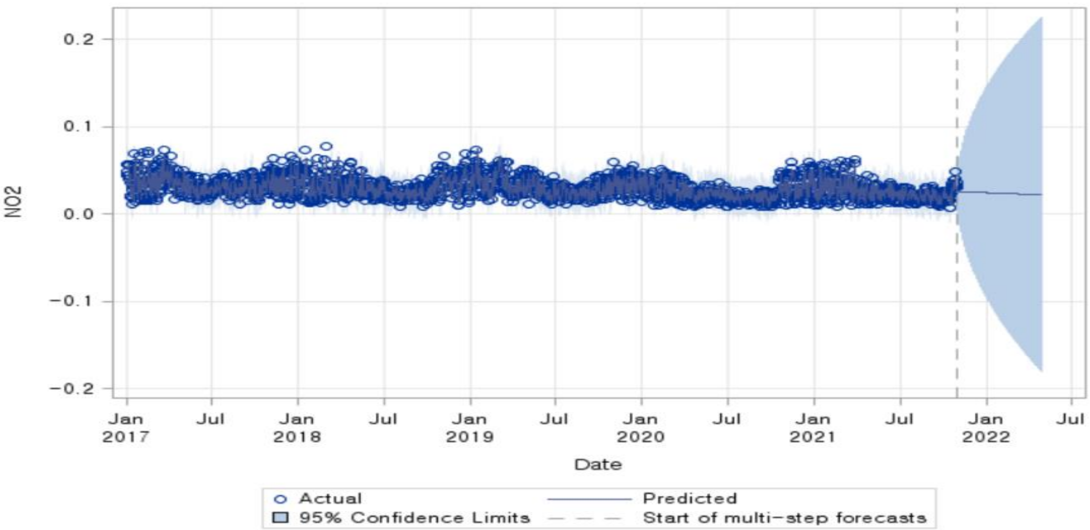

Figure 4- NO2 prediction by VECM(2)

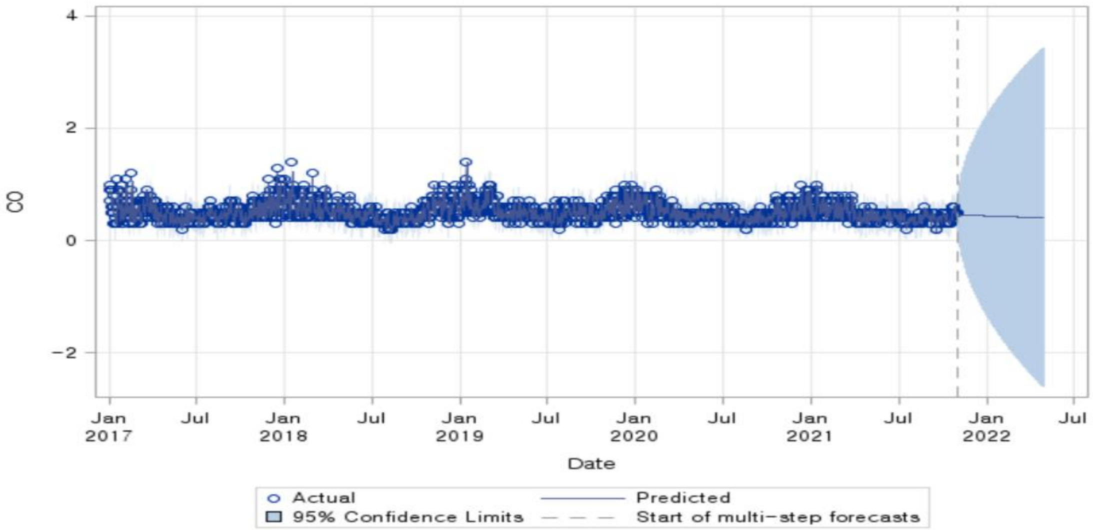

Figure 5- CO prediction by VECM(2)

5. Conclusion

The public's interest in air pollution has increased with the advancement of industry, rapid changes in weather conditions such as climate change, and the rise of public awareness and sensibility levels. In particular, fine dust can be seen to have a wider range of effects than other pollution because it is easy to be exposed to an unspecified number of people. Exposure to fine dust can cause or aggravate heart and lung-related diseases, which in turn can affect the increase in death. In addition, the risk of cardiovascular disease, respiratory disease, and lung cancer increases when exposed for a long time to a place with a high concentration of fine dust. In Korea, the severity of air pollution is rising to the extent that there is a report confirming that the concentration of ultrafine dust is the second highest among OECD member countries. In fact, as a result of the 2018 National Environmental Awareness Survey published by the Korea Environmental Policy Evaluation Institute, it was confirmed that more than 3 out of 10 people think that air quality improvement such as fine dust is the most urgent among environmental problems. Various research groups around the world, including Korea, have established and implemented many countermeasures to solve the problem of fine dust through various studies. In this study, using data of ultrafine dust, nitrogen dioxide, and carbon monoxide in Jung-gu, Seoul, the causal relationship between variables was reviewed, and a prediction model was presented. As a result of prediction using the model presented, ultrafine dust, nitrogen dioxide, and carbon monoxide are predicted to decrease. Fine dust has been studied to have many negative effects on the human body, ecosystem, industry, and society. Fine dust generation factors are greatly affected by weather factors (temperature, humidity, wind direction, wind speed, precipitation, etc.) and external inflows. In order to solve the fine dust problem, research on the generation factors and prediction modeling should be continuously conducted.

6. Acknowledgments

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※ MSIT: Ministry of Science and ICT, MOE: Ministry of Education research was supported

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