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Fixed Point of Mappings in Ĝ–Dualistic Partial Metric Space Endowed with a Graph

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Abstract
This paper aims is to find out the fixed points for two mappings in **Ĝ** - dualistic partial metric space with the help of graph by using **Ĝ** - Convergence Comparison Property (**Ĝ** - CCP). An application is also given to find fixed point with the help of boundary value problem.
Keywords: **Ĝ** - Dualistic partial metric space, **Ĝ** -CCP, Fixed point, Graph.

1 .Introduction

In 1922, a vital research in fixed point theory is due to Banach, see [7] who gave Banach's contraction principal.After this many extensions had appeared in the literature see [4, 20].

In 1992, the concept of partial metric space was given by Matthews[14]. Partial metric space just replaces one condition of usual metric space i.e self distance not necessarily zero.The concepts like how to find convergence of sequences , cauchy sequences , completeness of spaces , etc can be seen in [1, 10, 11, 14, 18, 19]. In 1996, Neil [15] developed the idea of Dualistic partial metric space(DPMS) and established the relationship between dualistic and partial metric space. In partial metric space(PMS), range is restricted to non negative real numbers , but in dualistic partial metric space range is extended from non - negative real numbers to set of real numbers i.e ℝ. He found many properties of topological space and axioms of DPMS . Oltra and Valero [16] established the concept of fixed point in DPMS.Nazam et al.[8] gave the concept of rational type contractive conditions of fixed point.Recently Nazam et al.[9]initiate the view of fixed point in dualistic metric space and found an application with the help of boundary value problem in DPMS.Jachymski [6],introduced the concept of graphs in the area of fixed point and proved many results of fixed point theory by using concept of graphs. Motivated by[3, 6, 13, 17] ,we obtain fixed points for two mappings in **Ĝ** - dualistic partial metric space with the help of graph by using **Ĝ** - Convergence Comparison Property(**Ĝ** - CCP). An application is also given to find fixed point with the help of boundary value problem.Firstly we present some definitions related to graphs see in [5, 6, 7, 12]. Throughout this paper, the product of **Å**×**Å** (which is diagonal) is represented by Δ where **Å** $\neq \phi$.Choose **Ĝ** as a graph,where set of vertices are indicated by V(**Ĝ**) coincides with **Å**, and E(**Ĝ**) contains loops as well as edges. Thus a pair **Ĝ** = (V(**Ĝ**), E(**Ĝ**)) represents a graph. Let **Ĝ** $^{-1}$ represent a change in **Ĝ** i.e E(**Ĝ** $^{-1}$)= {(w,v)|(v,w) ∈E(**Ĝ**)} and **G̈** indicates an undirected graph from **Ĝ** , when the direction of the set of edges is not considered .

Thus E(**Ĝ**) ∪E(**Ĝ** $^{-1}$) = E(**G̈**)

2 .Definitions and Preliminaries

Definition 2.1. Consider a graph **Ĝ** = (V, E), V(**Ĝ**) = **Å**, E(**Ĝ**) = { (u,v)∈ **Å**×**Å** } a pair (**Å**, d) defined on **Å** ≠ ϕ is termed as **Ĝ** - Metric space if d: **Å**×**Å** → $[0,\infty)$ met the following postulates:

(d1) $d(u,v) \geq 0$ and $d(u,v) = 0$ iff $u = v$,

(d2) $d(u,v) = d(v,r)$,

(d3) $d(u,w) \leq d(u,v) + d(v,w)$

for all u , v, w ∈E(**Ĝ**)

Then the pair (**Å**, d) termed as a **Ĝ** - metric space.

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Definition 2.2 [14]**.** A pair ($\mathbf{Å}$, P) on $\mathbf{Å} \neq \phi$ is said to be a partial metric space if
P: $\mathbf{Å}\times\mathbf{Å} \rightarrow [0, \infty)$ satisfies the following axioms:
(P1) $P(u,u) = P(u,v) = P(v,v)$, iff u = v,
(P2) $P(u,u) \leq P(u,v)$,
(P3) $P(u,v) = P(v,u)$,
(P4) $P(u,w) + P(w,w) \leq P(u,w) + P(w,v)$
$\forall$ u , v, w in $\mathbf{Å}$
Then a pair ($\mathbf{Å}$, P) termed as a partial metric space(PMS).

Neil[15] did a remarkable change in definition of PMS.He extended the range from[o, ∞) to $\mathbb{R}$. This leads us to metric space which is DPMS .

Definition 2.3 [9] A pair ($\mathbf{Å}$, D) on $\mathbf{Å} \neq \phi$ is termed as dualistic -partial metric space if
D:$\mathbf{Å}\times\mathbf{Å} \rightarrow \mathbb{R}$ satisfies the following axioms:
(D1) $D(u,u) = D(u,v) = D(v,v) \Leftrightarrow$ v = u,
(D2) $D(u,u) \leq D(u,v)$,
(D3) $D(u,v) = D(v,u)$,
(D4) $D(u,v) + D(w,w) \leq D(u,w) + D(w,v)$
for all u , v, w $\in \mathbf{Å}$
Then a pair ($\mathbf{Å}$, D) termed as a DPMS.

Remark. Any PMS ⇒DPMS but DPMS⇏ PMS. We give an illustration in support of our remark.

Example 2.4. Consider a map D: $[-4,4] \times [-4,4] \rightarrow \mathbb{R}$ defined as max(v,w)= D(v,w)
$\forall$ v,w ∈[-4,4]. We observe that D satisfies all the axioms of DPMS i.e (D1) to (D4).Thus D is a DPMS. On the other hand, we see D is not PMS due to D(-1,-2)= -1 i.e not in the set of non - negative real numbers.

Neil[15] showed that any dualistic patrial metric space ($\mathbf{Å}$, D) generates a topology (which is T_0 topology) Ʊ(D) on $\mathbf{Å}$ and possesses a base along with a family of D - balls
$\{B_D(v,u): v \in Å, u > 0\}$ and $B_D(v,u) = \{w \in \mathbf{Å}: D(v,w) < u + D(v,v)\}$.

To obtain more information related to concepts like convergence , cauchy sequences , completeness of DPMS in [9].

Definition 2.5. Consider a DPMS ($\mathbf{Å}$, D) where $\mathbf{Å} \neq \phi$. If we establish DPMS on
E($\hat{\mathbf{G}}$) = {(v,w))∈ $\mathbf{Å}\times\mathbf{Å}$ } structure of graph $\hat{\mathbf{G}}$ =(V,E), V($\hat{\mathbf{G}}$) = $\mathbf{Å}$, then the new structure is called a $\hat{\mathbf{G}}$ -DPMS i.e D:$\mathbf{Å}\times\mathbf{Å} \rightarrow \mathbb{R}$ met the following axioms:
(D1) $D(u,u) = D(u,v) = D(v,v) \Leftrightarrow$ u = v,
(D2) $D(u,u) \leq D(u,v)$,
(D3) $D(u,v) = D(v,u)$,
(D4) $D(u,v) + D(w,w) \leq D(u,w) + D(w,v)$
$\forall$ u , v, w ∈E($\hat{\mathbf{G}}$) Then a pair ($\mathbf{Å}$, D) termed as a $\hat{\mathbf{G}}$ - DPMS.
If ($\mathbf{Å}$, D) is a $\hat{\mathbf{G}}$ - DPMS and a mapping ℓ_D: $\mathbf{Å}\times\mathbf{Å} \rightarrow [0, \infty)$ given by
D(v,w) - D(v,v) = ℓ_D(v,w), v, w∈E($\hat{\mathbf{G}}$) is $\hat{\mathbf{G}}$ - quasi metric space(define on graph (V, E) = $\hat{\mathbf{G}}$)with Ʊ(D) = Ʊ(ℓ_D).Furhter if $\ell_D$is $\hat{\mathbf{G}}$ - quasi metric space implies ℓ^*_D(s,t) = max{ ℓ_D(v,w) , ℓ_D(w,v)} is $\hat{\mathbf{G}}$ -metric space(define on graph (V, E) = $\hat{\mathbf{G}}$.

Example 2.6 . Define a function D:$\mathbf{Å}\times\mathbf{Å} \rightarrow \mathbb{R}$ as D(v,w) = P(v,w) - P(v,v) - P(w,w), where $\mathbf{Å}$ = [-5,5], P is PMS on non empty set $\mathbf{Å}$. Thus ($\mathbf{Å}$, D) is $\hat{\mathbf{G}}$ -DPMS with a graph (V, E) = $\hat{\mathbf{G}}$, E($\hat{\mathbf{G}}$) = { (v,w)∈ $[-5,5] \times [-5,5]$ } contains loops and V($\hat{\mathbf{G}}$) = [-5,5].Thus a couple ($\mathbf{Å}$, D) is a
$\hat{\mathbf{G}}$ - DPMS.

Definition 2.7. Consider a $\hat{\mathbf{G}}$ - DPMS, (V,E) = $\hat{\mathbf{G}}$ represents the graph and E($\hat{\mathbf{G}}$) represents a set of edges and $\mathbf{Å}$ = V($\hat{\mathbf{G}}$), where $\mathbf{Å} \neq \phi$, then
(1) A sequence { v_n } in $\hat{\mathbf{G}}$ -Dualistic partial metric space $\hat{\mathbf{G}}$ - DPMS converges to point v such that (v_n , v) in E($\hat{\mathbf{G}}$) iff D(v,v) = $\lim_{n\rightarrow\infty} D(v, v_n)$.
(2) A sequence { v_n } in $\hat{\mathbf{G}}$ -Dualistic partial metric space($\hat{\mathbf{G}}$ -DPMS) called cauchy with (v_n , v) in E($\hat{\mathbf{G}}$) and $\lim_{n\rightarrow\infty} D(v_m, v_n)$ exists as well as finite.
(3) A $\hat{\mathbf{G}}$ -DPMS ($\mathbf{Å}$, D) is termed as a complete space if we take any sequence { v_n } which is cauchy in $\hat{\mathbf{G}}$ -DPMS converges w.r.t Ʊ(D) and D(v_n , v_m) = D(v,v) as m, n → ∞ for any (v_n , v_m) in E($\hat{\mathbf{G}}$) **.**If we take [0,∞) in place of $\mathbb{R}$ then the Definition 2.5 reduces to $\hat{\mathbf{G}}$ **-** partial metric space and consequently Definition 2.7 holds for $\hat{\mathbf{G}}$ **-** partial metric space.

Definition 2.8 [2]**.** Suppose that ($\mathbf{Å}$, D) is a $\hat{\mathbf{G}}$ - DPMS with a graph $\hat{\mathbf{G}}$ = (V , E), $\mathbf{Å}$ = V($\hat{\mathbf{G}}$) ,
$\phi \neq \mathbf{Å}$, E($\hat{\mathbf{G}}$).contains loops. Then the map A from $\hat{\mathbf{G}}$ **-** DPMS to itself is called a graph preserving map if (v ,w) in E($\hat{\mathbf{G}}$) implies (A(v), A(w)) in E($\hat{\mathbf{G}}$).

Definition 2.9. Suppose that (V,E) = $\hat{\mathbf{G}}$ represents the graph where E($\hat{\mathbf{G}}$) contains loops ,

$V(\hat{G}) = \mathring{A}$, $\mathring{A} \neq \phi$. A graph preserving mapping on $\hat{G}$ - DPMS ($\mathring{A}$, D) is termed as $\hat{G}$ - Convergence Comparison Property ($\hat{G}$ - CCP) if

(1) A sequence $\{ v_n \}$ which converges to v such that (v_n , v) in $E(\hat{G})$.

(2) $D(v,v) \leq D(A(v),A(v))$, where (v,v), (A(v), A(v)) in $E(\hat{G})$.

Example 2.10 Consider $\mathring{A} = \mathbb{R}$ and $D:\mathring{A}\times\mathring{A}\rightarrow \mathbb{R}$ defined as $D(v,w) = \max\{v,w\}$ endowed with a graph $\hat{G} = (V, E)$ where $E(\hat{G})$ is $\{(v,w) \in \mathring{A}\times\mathring{A}\}$ is a $\hat{G}$ - DPMS. We observe A is a graph preserving map. Let $\{ v_n = \frac{1}{n} - 2$, n is natural number $\}$, we observe that $\{ v_n \}$ converges to -2 .

Let $A(v) = e^{v}$ whenever (v,v) in $E(\hat{G})$. Now $D(-2,-2) \leq D(A(v),A(v))$. Thus A satisfies all the conditions of $\hat{G}$ - CCP.

The following lemma is useful for our upcoming results.

Lemma 1[16].

(1) A (DPMS) ($\mathring{A}$, D) is complete $\Leftrightarrow$ metric space ($\mathring{A}$, ℓ_D^*) is complete.

(2) A sequence v_n in $\mathring{A}$ converges to $s \in \mathring{A}$ w.r.t $\mho(\ell_D^*)$ iff $D(v_m ,v_n) = D(v,v) = D(v ,v_n)$ as $n,m \rightarrow \infty$.

3 .Main Results

Theorem 3.1. Consider a complete $\hat{G}$ - DPMS ($\mathring{A}$, D) along with a graph $\hat{G} = (V,E)$, $\mathring{A} = V(\hat{G})$, $E(\hat{G})$ possesses loops also, A,B be self mappings on ($\mathring{A}$, D) satisfies the conditions given below:

(1)The pair (A, B) satisfies $\hat{G}$ - CCP property.

(2)The pair (A , B) preserves the edges of $E(\hat{G})$. For a $\lambda \in [0,1)$ such that $\left|D(A(v),B(v))\right| \leq \lambda \max\{\left|D(w,B(w))\right|\}$ $\forall$ $v,w \in \mathring{A}$ and (v,w) in $E(\hat{G})$ then A , B possess the fixed point.

Proof. Consider a sequence v_n and an initial term v_0 with (v_n , v_0) in $E(\hat{G})$ with $v_n = A(v_{n-1})$ true for any natural number n. We also suppose that this assumption is also true for the map B.If $\exists$ $v_{n_0} = v_{n_0+1} = A(v_{n_0}) = B(v_{n_0})$ implies v_{n_0} satisfies $A(v_{n_0}) = B(v_{n_0}) = v_{n_0}$. Now we consider $v_n \neq v_{n+1}$ such that $(v_n, v_{n+1}) \in E(\hat{G})$

By contraction condition (2), we obtain

$|D(v_n,v_{n+1})| =$
$\left|D(A(v_{n-1}),B(v_n)\right) \leq \left|\lambda\max\{\left|D(v_{n-1},A(v_{n-1}))\right|,\left|D(v_n,B(v_n\right.\right.$
$\}$
$=$
$\lambda\max\{|D(v_{n-1},v_n)|,|D(v_n,v_{n+1})|\}$
(3.1)

Thus $|D(v_n,v_{n+1})| \leq \lambda\max\{|D(v_{n-1},v_n)|,|D(v_n,v_{n+1})|\}$
(3.2)

If $|D(v_n,v_{n+1})| = \max\{|D(v_{n-1},v_n)|,|D(v_n,v_{n+1})|\}$
(3.3)

Then we observe that from (3.2), we get contradiction. Therefore we must have

$|D(v_n,v_{n+1})| \leq \lambda|D(v_{n-1},v_n)|$ (3.4)

$|D(v_{n-1},v_n)| = \left|D(A(v_{n-2}),B(v_{n-1}))\right| \leq \lambda\max\{|D(v_{n-2},v_{n-1})|,|D(v_{n-1},v_n)|\}$ (3.5)

Continuing in this way,

$\max\{|D(v_{n-2},v_{n-1})|,|D(v_{n-1},v_n)|\} = |D(v_{n-2},v_{n-1})|$ (3.6)

We observe (3.4) leads to

$|D(v_n,v_{n+1})| \leq \lambda^2|D(v_{n-2},v_{n-1})|$ (3.7)

In similar way , we obtain

$|D(v_n,v_{n+1})| \leq \lambda^n|D(v_0,v_1)|$ (3.8)

Now consider

$|D(v_n,v_n)| = \left|D(A(v_{n-1}),B(v_{n-1}))\right| \leq \lambda\max\{|D(v_{n-1},v_n)|,|D(v_{n-1},v_n)|\}$ (3.9)

By (3.8) we get

$|D(v_n,v_n)| \leq \lambda^n|D(v_0,v_0)|$ (3.10)

We use the condition $\ell_D(v,w) = D(v,w) - D(v,v)$, we obtain

$\ell_D(v_n,v_{n+1}) + D(v_n,v_n) \leq |D(v_n,v_{n+1})|$ (3.11)

From (3.8) and (3.10) we get

$\ell_D(v_n,v_n) \leq 2\lambda^n|D(v_0,v_0)|$ (3.12)

For m> n , we get

$$\begin{aligned}\ell_D(v_n,v_m) &\leq \ell_D(v_n,v_{n+1}) + \ell_D(v_{n+1},v_{n+2}) + \cdots.. + \ell_D(v_{m-1},v_m)\\ &\leq 2\lambda^n|D(v_0,v_1)| + 2\lambda^{n+1}|D(v_0,v_1)| + \cdots. + 2\lambda^{m-1}|D(v_0,v_1)|\\ &\leq 2(\lambda^n + \cdots + \lambda^{m-1})|D(v_0,v_1)|\\ &\leq 2\frac{\lambda^n(1-\lambda^{n-m})}{(1-\lambda)} \qquad (3.13)\end{aligned}$$

Now letting $n,m \longrightarrow \infty$ implies $\ell_D^*(v,w) = \max$

{ ℓ_D(v,w),ℓ_D(w,v)} tends to zero , implies { v_n } is a sequence which is cauchy in (Å, ℓ_D^*) . Also , (Å, D) is a $\hat{G}$ - DPMS which is complete so by using (1) of Lemma1 (we use this lemma because proof of this lemma is also obtained on the structure of graph and proof is same as given in Lemma1) (Å, ℓ_D^*) is $\hat{G}$ - complete metric space. So ∃ v in (Å, ℓ_D^*) with { v_n } → v as n → ∞ i.e $\ell_D(v_n$,v) = 0 and by using (2) of Lemma1, inequalities (3.10) and ℓ_D(v,w) = D(v,w) - D(v,v), we write

$$D(v,v) = D(v_n,v) = D(v_n,v_m) = 0 \quad (3.14)$$

as n and m → ∞ .Thus v_n is a sequence which is cauchy also converges to v .Now we show here v = A(v) = B(v). Using contraction condition of this theorem

$$|D(v_n,B(v))| = |D(A(v_{n-1}),B(v))| \leq \lambda \max\{|D(v_{n-1},A(v_{n-1}))|, |D(v,B(v))|\} \quad (3.15)$$

for n → ∞ $|D(v_n,B(v))| \leq |D(v,B(v))|$ ⇒D(v, B(v)) = 0 and B has $\hat{G}$ - CCP , we obtain

$$0 = D(v,v) \leq \lambda D(B(v),B(v)) + D(B(v),v) - D(B(v),B(v)) \quad (3.16)$$

By using D(4) property of $\hat{G}$ - DPMS, D(v,v)≤ D(v ,B(v)) - D(B(v),v) - D(B(v) ,B(v)), we obtain

$$D(B(v),B(v)) \leq 0 \qquad (3.17)$$

From (3.16) and (3.17) D(B(v),B(v)) = 0 and D(v, B(v)) = D(B(v) , B(v)) = D(v,v).
This implies v = B(v) i.e s is fixed point for B .
Now we show that v = A(v). Now consider,

$$|D(A(v),v_n)| = |D(A(v),B(v_{n-1}))| \leq \lambda\max\{|D(v,A(v))|, |D(v_{n-1},B(v_{n-1})|\} \quad (3.18)$$

As we have used the steps for fixed point for B ,similarly it is used for mapping A. 1 .

Consequently we get v = A(v) . Thus A(v) = B(v) = v. Now uniqueness let t be another fixed point for A and B and D(w,w) = 0 and A(w) = B(w) = w.
$|D(v,w)| \leq \lambda \max\{|D(v,A(v))|, |D(w,B(w))|\}$ ⇒ D(v,w) = D(v,v) = D(w,w) = 0 ⇒ v = w.

Corollary 3.2. Consider a $\hat{G}$ - complete partial metric space (Å, D) occupied a graph (V, E) = $\hat{G}$, Å = V($\hat{G}$) , A, B are two self maps from (Å, D) to itself then D(A(v),B(w)) ≤ λ max{D(v,A(v)), D(w,B(w))} for all v,w ∈Å, λ ∈ [0,1) and (v,w) in E($\hat{G}$) implies A , B possess a fixed point.

Proof. If we restrict $\hat{G}$ - DPMS ranges from set of real numbers to non - negative numbers then $\hat{G}$ - complete dualistic partial metric space becomes $\hat{G}$ - complete partial metric space consequently proof of Corollary 3.2 is obtained by same steps of Theorem 3.1 .

Example 3.3. Let (V,E) = $\hat{G}$ be a graph choose [0,1] = Å, V($\hat{G}$) = [0,1] and E($\hat{G}$)= {(v,w)∈ [0,1] × [0,1]}, D:Å×Å → ℝ given as D(v,w) = max{v, w} is a $\hat{G}$ -complete DPMS. Now we define the maps as A(v) = $\frac{v}{2}$ and B(v) = $\frac{v}{3}$, both these maps are graph preserving and satisfies $\hat{G}$ -CCP for the sequence { v_n } = $\frac{1}{n}$ converges to 0, (v_n ,0) in E($\hat{G}$) and also satisfies contraction condition of above Theorem 3.1 for λ = $\frac{3}{4}$. Thus 0 = A(0) = B(0), see in figure

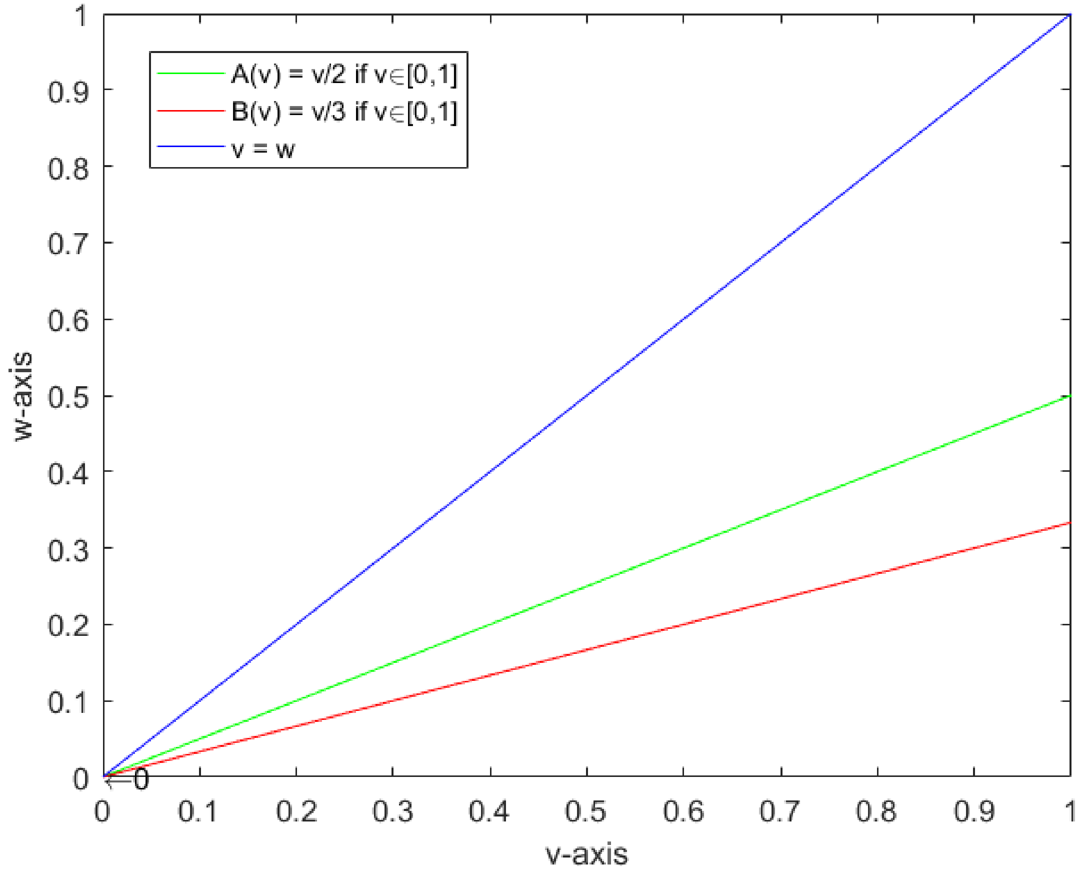

Figure 1: Graph of a fixed point of A and B

Example 3.4. Suppose (-∞, 0] = $\mathbf{\AA}$ and (V,E)= $\hat{\mathbf{G}}$ is a graph , V($\hat{\mathbf{G}}$) = (-∞, 0] where E($\hat{\mathbf{G}}$) = {(v,w): ∈ $\mathbf{\AA}\times\mathbf{\AA}$ }. A map D: $(-\infty,0] \times (-\infty,0] \to \mathbb{R}$ defined as,

$$D(v, w) = \begin{cases} |v - w|, & v \neq w \\ v \wedge w, & v = w \end{cases}$$

Let ∧ denotes the supremum of v and w. So ($\mathbf{\AA}$, D) is $\hat{\mathbf{G}}$ - dualistic partial metric space as well complete .Consider the maps A and B as , B is a zero map and A is defined as

$$A(v) = \begin{cases} -1, & v \in (-\infty, -4] \\ 0, & v \in (-4, 0] \end{cases}$$

Also both the maps are graph preserving and satisfies $\hat{\mathbf{G}}$ - CCP property . For this let us consider the sequence { $v_n = \frac{-1}{n}$}. Thus { v_n } is cauchy sequence which converges to 0 in ($\mathbf{\AA}$, D) and (v_n ,0) in E($\hat{\mathbf{G}}$). Also we have D(A(0),A(0)) ≥ D(0,0) and D(B(0),B(0)) ≥ D(0,0), both the maps satisfies $\hat{\mathbf{G}}$ -CCP. For the contraction condition of our theorem , let $\lambda = \frac{3}{4}$. We discuss the following cases hold

Case (a). (1) v ≠w and v,w ∈ (−4,0] we get $|D(A(v), B(w))| = 0$

(2) v ≠w and v,w ∈ $(-\infty, -4]$, $|D(A(v), B(w))| = 1 \leq \lambda\max\{|D(v, A(v))|, |D(w, B(w))|\}$

(3) if we take v ≠w and w ∈ (−4,0] and v ∈ $(-\infty, -4]$ and conversely the contraction condition is also hold.

Case (b). (1) for v = w ∈ (−4,0] then $|D(A(v), B(w))| = 0$ and when v = w ∈ $(-\infty, -4]$, contraction condition also holds. we observe that w = 0 is fixed point for the map A and B. Now we represent a table for $\lambda = \frac{3}{4}$ which evaluates the contraction condition of theorem

Contraction table for A and B and $\lambda = \frac{3}{4}$		
(v, w)	|D(A(v) , B(w))|	λ max{ |D(v , A(v))| ,|D(w , B(w))|}
(0,0)	0	0
(-1, -1)	0	$\frac{3}{4} . 1$
(-1, -2)	0	$\frac{3}{4} . 2$
(-10 , -10)	0	$\frac{3}{4} . 10$
(-4 ,-3)	1	$\frac{3}{4} . 3$
(-2 , -1)	0	$\frac{3}{4} . 2$
(-1, -3)	0	$\frac{3}{4} . 3$
(-1, - 0 .1)	0	$\frac{3}{4} . 1$
(-3 , -3)	0	$\frac{3}{4} . 3$

Theorem 3.5. Consider a $\hat{\mathbf{G}}$ - Complete dualistic partial metric space ($\mathbf{Å}$, D) and (V,E) = $\hat{\mathbf{G}}$ represents a graph with V($\hat{\mathbf{G}}$) = $\mathbf{Å}$, E($\hat{\mathbf{G}}$) contains loops and A, B are self mappings on ($\mathbf{Å}$, D) met the following postulates:
(1)The pair (A,B) satisfies $\hat{\mathbf{G}}$ - CCP property.
(2)The pair (A,B) preserves the edges of E($\hat{\mathbf{G}}$).
If there is $0 \leq c,e,i$ and $c+e+i < 1$ such that $\left|D\left(A(v), B(w)\right)\right| \leq c\ |D(v,w)| + e \left|D\left(w, A(w)\right)\right| + i\left|D\left(v, B(v)\right)\right|$ $\forall$ v,w $\in \mathbf{Å}$ and (v,w) in E($\hat{\mathbf{G}}$) implies A , B possess the fixed point.

Proof. Consider a sequence v_n and an initial term is v_0 with (v_n , v_0) in E($\hat{\mathbf{G}}$) with $v_n = A(v_{n-1})$ true for any natural number n. We also suppose this assumption also true for the map B.If there $\exists$ $v_{n_0} = v_{n_0+1} = A(v_{n_0}) = B(v_{n_0})$ implies v_{n_0} satisfies $A(v_{n_0}) = B(v_{n_0}) = v_{n_0}$. Now we consider
$v_n \neq v_{n+1}$ such that (v_n, v_{n+1}) $\in$ E($\hat{\mathbf{G}}$) .By contraction condition (2) we obtain

$$|D(v_n, v_{n+1})| = \left|D\left(A(v_{n-1}), B(v_n)\right)\right| \leq c|D(v_{n-1}, v_n)| + e\left|D\left(v_{n-1}, A(v_{n-1})\right)\right| + i\left|D\left(v_n, B(v_n)\right)\right|$$

$$= |D(v_n, v_{n+1})| \leq \frac{c+e}{1-i}|D(v_{n-1}, v_n)| \qquad (3.19)$$

Let $\lambda = \frac{c+e}{1-i}$ so that $0 \leq \lambda < 1$ and repeating the above procedure , we get

$$|D(v_n, v_{n+1})| \leq \lambda^n |D(v_0, v_1)| \qquad (3.20)$$

For self distance,we obtain

$$|D(v_n, v_n)| \leq c|D(v_{n-1}, v_{n-1})| + (e+i)|D(v_{n-1}, v_n)| \qquad (3.21)$$

with the help of (3.18), we write

$$|D(v_n, v_n)| \leq c|D(v_{n-1}, v_{n-1})| + (e+i)\lambda^{n-1}|D(v_0, v_1)| \qquad (3.22)$$

$$\left|D\left(A(v_{n-2}), B(v_{n-2})\right)\right| \leq c|D(v_{n-2}, v_{n-2})| + e\left|D\left(v_{n-2}, A(v_{n-2})\right)\right| + i\left|D\left(v_{n-2}, B(v_{n-2})\right)\right|$$

$$= c|D(v_{n-1}, v_{n-1})| + (e+i)\lambda^{n-2}|D(v_{n-1}, v_n)|$$

(3.23)

Where $\left|D\left(A(v_{n-2}),B(v_{n-2})\right)\right| = |D(v_{n-1},v_{n-1})|$. From (3.22) we obtain

$$\left|D\left(A(v_{n-2}),B(v_{n-2})\right)\right| \le c^n|D(v_0,v_0)| + (c^{n-1}+c^{n-1}\lambda+\cdots+\lambda^{n-1})(e+i)|D(v_0,v_1)| \quad (3.24)$$

$$\left|D\left(A(v_n),B(v_n)\right)\right| \le c^n|D(w_0,w_0)| + \frac{\lambda^n-c^n}{\lambda-c}(e+i)|D(v_0,v_1)| \quad (3.25)$$

We use the condition ℓ_D(v,w) = D(v,w) - D(v,v),we get

$$\ell_D(v_n,v_{n+1}) \le |D(v_n,v_{n+1})| - D(v_n,v_n) \le |D(v_n,v_{n+1})| + |D(v_n,v_n)| \quad (3.26)$$

Also,$|D(v_n,v_{n+1})|$+$|D(v_n,v_n)| \le \lambda^n|D(v_0,v_1)|$+c ^{n}+$|D(v_0,v_1)|$+$\frac{\lambda^n-c^n}{\lambda-c}$(e+i) $|D(v_0,v_1)|$

And, $\frac{\lambda^n-c^n}{\lambda-c}$(e+i)$|D(v_0,v_1)| \le$($\lambda^n$+(e+i)$\frac{\lambda^n-c^n}{\lambda-c}$)$|D(s_0,v_1)|$+ c $^n|D(v_0,v_0)|$

on solving the last above inequalities we write

(λ^n+λ^{n-1}+cλ^{n-2}+...+c $^{n-1}$)$|D(v_0,v_1)|$ +s $^{n-1}|D(v_0,v_1)|$

Let $\gamma^n = \lambda^n+\lambda^{n-1}+c\lambda^{n-2}+\cdots+c^{n-1}$ (3.27)

then we obtain

$$|D(v_n,v_{n+1})| \le \gamma^n + |D(v_0,v_1)| + u^n|D(v_0,v_1)| \quad (3.28)$$

For m > n . let us solve the inequality

$$\ell_D(v_n,v_m) \le \ell_D(v_n,v_{n+1}) + \ell_D(v_{n+1},v_{n+2}) + \cdots..+\ell_D(v_{m-1},v_m)$$
$$\le \gamma^n|D(v_0,v_1)| + c^n|D(v_0,v_1)| + \gamma^{n+1}|D(v_0,v_1)|+..+c^{n-1}|D(v_0,v_1)|$$
$$\le (\gamma^n + \gamma^{n+1}+..+\gamma^{m-1}+..)|D(v_0,v_1)| + (c^n + c^{n+1}+..+c^{m-1}+..)|D(v_0,v_1)|$$
$$\le \left(\frac{\gamma^n}{(1-\gamma)}+\frac{c^n}{(1-c)}\right)|D(s_0,s_1)| \quad (3.29)$$

Letting m,n → ∞ , implies $\ell_D^*(v_n,v_m)$ = 0.Also , (**Å**, D) is a **Ĝ** - dualistic partial metric space and complete also so by using (1) of Lemma1 (we use this lemma because proof of this lemma is also obtained on the structure of graph and proof is similar as given in Lemma1) (**Å**, ℓ_D^*) is **Ĝ** - complete metric space . So ∃ s in (**Å**, ℓ_D^*) with v $_n$ converges to v i.e $\ell_D(v_n$,v) = 0,as n → ∞ along with (v $_n$,v) in E (**Ĝ**). So by (2) of Lemma1,using (3.25) and ℓ_D(v,w) = D(v,w) - D(v,v),we can write, D(v,v) = D(v $_n$, v)= D(v $_n$, v $_m$) = 0 as m, n → ∞.

$$\left|D\left(v_n,B(v)\right)\right| = \left|D\left(v_{n-1},B(v)\right)\right| \le c|D(v_{n-1},s)| + e|D(v_{n-1},A(v_{n-1}))| + i|D(v,B(v))| \quad (3.30)$$

Taking limit as n → ∞ , and B has **Ĝ** - CCP,we obtain $\left|D\left(v,B(v)\right)\right|$ = 0.

$$0 = D(v,v) \le D\left(B(v),B(v)\right) \quad (3.31)$$

Now we use (D2) property

$$D\left(B(v),B(v)\right) \le D\left(v,B(v)\right) \quad (3.32)$$

From 3.31 and 3.32 we obtain D(B(s),B(s)) = 0 .

$$D\left(v,B(v)\right) = D\left(B(v),B(v)\right) = D(v,v) \quad (3.33)$$

Now by (D1) we obtain v = B(v).

$$|D(A(v),v_n)| = \left|D\left(A(v),B(v_{n-1})\right)\right| \le c|D(v,v_{n-1})| + e|D(v,A(v))| + i|D(v_{n-1},B(v_{n-1}))| \quad (3.34)$$

Taking limit as n → ∞ and A has **Ĝ** -CCP,we obtain $\left|D\left(v,B(v)\right)\right|$ = 0.

As we have used the steps for fixed point for B ,similarly it is used for mapping A. Consequently v is also fixed point for A. This implies v = A(v) = B(v).

Corollary 3.6. Consider a **Ĝ** - complete partial metric space (**Å**, D) occupied a graph (V, E) = **Ĝ**, **Å** = V(**Ĝ**) , A,B are two self maps from (**Å**, D) to itself then D(A(v),B(w)) ≤ c D(v,w)+ e D(v,A(v))+i D(w,B(w)) ∀ v,w ∈**Å**, 0 ≤ c,e,i and c+e+i < 1 , (v,w) in E(**Ĝ**) implies B , A possess a fixed point.

Proof. As if we restrict **Ĝ** -DPMS ranges from set of real numbers to non - negative numbers then **Ĝ** -complete dualistic partial metric space becomes **Ĝ** - complete partial metric space consequently proof of this Corollary is obtained by same steps of Theorem 3.5 .

Example 3.7 Suppose that (V,E) = **Å** is a graph and **Å** = [0,1],V(**Ĝ**) = [0,1] further E(**Ĝ**)= { (v,w)∈ **Å**×**Å** } consider a map D: [0,1] × [0,1] → ℝ defined as D(v,w) =

max{v,w}
∀ v,w ∈[0,1] is a $\hat{\mathbf{G}}$ -complete DPMS. Now we define the maps on [0,1] to itself as A(v) = $\frac{v}{3}$ and B(v) = $\frac{v}{6}$, both these maps are graph preserving and satisfies $\hat{\mathbf{G}}$ -CCP for the sequence { v_n } = $\frac{1}{n}$ converges to 0,(v_n ,0) in E($\hat{\mathbf{G}}$).Now $\left|D\left(A\left(\frac{v}{3}\right),B\left(\frac{v}{6}\right)\right)\right| \leq c|D(v,w)|+ e\left|D\left(v,A\left(\frac{v}{3}\right)\right)\right|+i \quad \left|D\left(w,B\left(\frac{w}{6}\right)\right)\right|= \quad \frac{v}{3} \leq$ $\frac{1}{4}v+\frac{1}{5}v+\frac{1}{6}w$.

Thus we observe that $\frac{v}{3} \leq \frac{1}{4}v+\frac{1}{5}v+\frac{1}{6}w$, since $\frac{v}{3}$ is greater than $\frac{v}{6}$ and v and w behave same.Thus, A and B satisfies contraction condition of above theorem for some c,e,i which is non -negative and their sum is less than one. So , A and B has a fixed point say zero.We have a graphical representation of this example which is given in figure 2 on next page

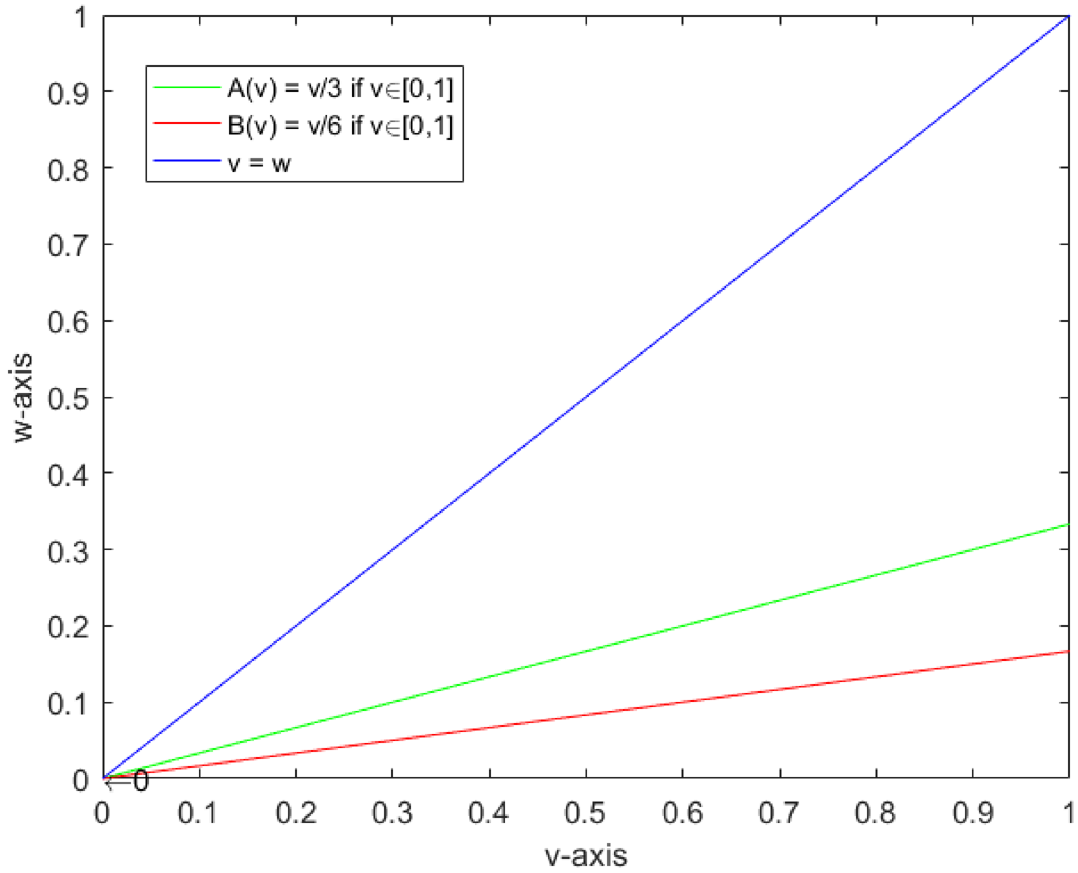

Figure 2: Graph of a fixed point for A and B

Example 3.8 Suppose {0, −1, −0.3, −0.1, −2, −3} = $\mathbf{\mathring{A}}$ along with a graph (V,E)= $\hat{\mathbf{G}}$,
$\mathbf{\mathring{A}}$ = V($\hat{\mathbf{G}}$), {(v,w): v,w∈ $\mathbf{\mathring{A}}$ ×$\mathbf{\mathring{A}}$ }= E($\hat{\mathbf{G}}$). Let us define the map D:$\mathbf{\mathring{A}}$×$\mathbf{\mathring{A}}$ → ℝ as we define in Example 3.4 .Consider the map A: $\mathbf{\mathring{A}}$→$\mathbf{\mathring{A}}$ defined as A(v) =

$$\begin{cases} -1, & v \in \{0, -1, -0.3, -2, -3\} \\ -0.3, & v = -2 \\ -0.1, & v = -3 \end{cases}$$

and B: $\mathbf{\mathring{A}}$→$\mathbf{\mathring{A}}$ defined as B(v) =

$$\begin{cases} -1, & v \in \{0, -1, -0.3, -0.1 - 2, -3\} \\ -0.3, & v = -2 \\ -0.3, & v = -3 \end{cases}$$

Both the maps are graph preserving and satisfies $\hat{\mathbf{G}}$ -CCP property i.e for a convergent sequence v_n converges to v due to completeness of ($\mathbf{\mathring{A}}$,D) and (v_n , v) in E($\hat{\mathbf{G}}$). So we write D(A(v),A(v)) ≥ D(v,v) and D(B(v),B(v)) ≥ D(v,v). The table given below evaluates the contraction condition for two mappings.

Contraction table for two maps A and B		
(v, w)	|D(A(v) , B(w))|	C|D(v ,w)| +e |D(v ,A(v))|+ i | D(w ,B(w)|
(0,0)	0	0
(0 ,-1)	0	c+i
(0 , -2)	0.3	2c+ 1.7i
(0 , -3)	0.3	3c+ 2.7i
(0 , - 0.1)	0	c.0.1+0.1i
(- 1 , - 2)	0 .3	c+ e+1.7i
(-1, -3)	0.3	2c+e+2.7i
(-1, - 0.1)	0	0.9c+e+0.1i
(-1 , - 0.3)	0	0.7c+e+0.3i
(-1 , -1)	0	c+e+i
(-2 ,-2)	0.3	2c+1.7e+1.7i
(-2 , -3)	0.3	c+1.7e+2.7i
(-2 , - 0.3)	0.3	1.7c+1.7e+0.3i
(-2, - 0.1)	0.3	1.9c+1.7e+0.1i
(-3 ,-3)	0.2	3c+2.9e+2.7i
(-3 ,- 0.1)	0.1	2.9c+2.9e+0.1i
(-3 , - 0.3)	0.1	2.7c+ 2.9e+ 0.3i
(-0.1 , - 0.1)	0	0.1(c+e+i)
(-0.3 , - 0.3)	0	0.1(c+e+i)

Theorem 3.9 Consider a $\hat{\mathbf{G}}$ - Complete dualistic partial metric space ($\mathbf{Å}$, D)along with a graph (V,E)= $\hat{\mathbf{G}}$ with $\mathbf{Å}$ = V($\hat{\mathbf{G}}$) , loops are contained by E($\hat{\mathbf{G}}$) and A,B are self mappings on ($\mathbf{Å}$,D) satisfies the conditions given below:
(1)The pair (A,B) satisfies ($\hat{\mathbf{G}}$ -CCP) property.
(2)The pair (A,B) preserves the edges of E($\hat{\mathbf{G}}$).
If there is, 0 ≤c,e,i and c+e+i < 1 such that $|D(A(v)),B(w))| \leq c\,\frac{|D(v,A(v)).D(w,B(w)|}{|D(v,w)|} + e\left|D\big(w,B(w)\big)\right| + i|D(v,w)|$ ∀ v, w∈ $\mathbf{Å}$ and (v,w) in E($\hat{\mathbf{G}}$) then B , A possess a fixed point

Proof. Consider a sequence v_n and an initial term v_0 with (v_n , v_0) in E($\hat{\mathbf{G}}$) with $v_n = A(v_{n-1})$ true for any natural number n. We also suppose that this supposition is also true for the map B.If ∃ $v_{n_0} = v_{n_0+1} = A(v_{n_0}) = B(v_{n_0})$ implies v_{n_0} satisfies $A(v_{n_0}) = B(v_{n_0}) = v_{n_0}$ .Now we consider
$v_n \neq v_{n+1}$ such that (v_n, v_{n+1}) ∈E($\hat{\mathbf{G}}$) . By contraction condition (2)

$$|D(v_n,v_{n+1})| = \left|D\big(A(v_{n-1}),B(v_n)\big)\right| \leq c\,\frac{|D(v_{n-1},A(v_{n-1}))|.|D(v_n,B(v_n))|}{|D(v_{n-1},v_n)|} + e|D(v_n,B(v_n))| + i|D(v_{n-1},v_n)| \leq \frac{i}{1-c-e}|D(v_{n-1},v_n)| \quad (3.35)$$

Now we choose $\wp = \frac{i}{1-c-i}|D(v_{n-1},v_n)| < 1$

then we get
$|D(v_n, v_{n+1})| \leq \wp|D(v_{n-1}, v_n)| \leq \wp^2|D(v_{n-2}, v_{n-1})| \leq \cdots \leq \wp^n|D(v_0, v_1)|$ (3.36)

$|D(v_n, v_n)| = |D(A(v_{n-1}), B(v_{n-1})|$
$= c\frac{|D(v_{n-1},v_n)|.|D(v_{n-1},v_n)|}{|D(v_{n-1},v_{n-1})|} + e|D(v_{n-1}, v_n)| + i|D(v_{n-1}, v_{n-1})|$ (3.37)

We observe $\frac{D(v_{n-1},v_n)}{D(v_{n-1},v_{n-1})} \geq 1$, so we write
$|D(v_n, v_n)| \leq c|D(v_{n-1}, v_n)| + e|D(v_{n-1}, v_n)| + i|D(v_{n-1}, v_{n-1})|$
$= (c + e)\wp^{n-1} + i|D(v_{n-1}, v_{n-1})|$ (3.38)

Continue in this manner we obtain
$|D(v_{n-1}, v_{n-1})| \leq (c + i)\wp^{n-2}|D(v_0, v_1)| + i|D(v_{n-2}, v_{n-2})|$ (3.39)

From (3.38) we write
$|D(v_n, v_n)| \leq (c + i)(\wp^{n-1} + \wp^{n-2})|D(v_0, v_1)| + i^2|D(v_{n-2}, v_{n-2})|$ (3.40)

Proceeding in this way
$|D(v_n, v_n)| \leq (c + e)(\wp^{n-1} + i\wp^{n-2} + .. + i^{n-1})|D(v_0, v_1)| + i^n|D(v_0, v_0)|$
$\leq (c + e)\left(\frac{i^{n-1}-\wp^{n-1}}{i-\wp}\right)|D(v_0, v_1)| + i^n|D(v_0, v_0)|$ (3.41)

By using this relation i.e ℓ_D(v,w) = D(v,w) - D(v,v), we obtain
$\ell_D(v_n, v_{n+1}) \leq |D(v_n, v_{n+1})| + |D(v_n, v_n)|$
$\leq \wp^n|D(v_0, v_1)| + (c + e)\left(\frac{i^{n-1}-\wp^{n-1}}{i-\wp}\right)|D(v_0, v_1)| + i^n|D(v_0, v_0)|$
$\leq (\wp^n + \wp^{n-2} + i\wp^{n-3} + \cdots + i^{n-2})|D(v_0, v_1)| + i^n|D(v_0, v_0)|$ (3.42)

$\ell_D(v_n, v_{n+1}) \leq \gamma^n|D(v_0, v_1)| + i^n|D(v_0, v_0)|$ (3.43)

. When m ≥ n we obtain
$\ell_D(v_n, v_m) \leq \ell_D(v_n, v_{n+1}) + \cdots . + \ell_D(v_{m-1}, v_m)$
$\leq \gamma^n|D(v_0, v_1)| + i^n|D(v_0, v_0)| + \gamma^{n+1}|D(v_0, v_1)| + \cdots + i^{m-1}|D(v_0, v_0)|$
$\leq (\gamma^n + \gamma^{n+1} + \cdots + \gamma^{m-1})|D(v_0, v_1)| + (i^n + i^{n+1} + \cdots + i^{m-1})|D(v_0, v_0)|$(3.44)

We conclude that as m,n $\to \infty$ we get $\ell_D(v_n, v_m)$and $\ell_D(v_m, v_n) = 0$. Thus { v_n } is cauchy sequence in (**Å**, ℓ_D^*).The remaining part of this theorem is done by same as Theorem 3.5 .

Corollary 3.10. Consider a **Ĝ** - complete partial metric space (**Å**, D) occupied a graph (V, E) = **Ĝ** , **Å** = V(**Ĝ**) and A, B are two self maps from (**Å**, D) to itself then
D(A(v),B(w)) ≤c $\frac{D(v,A(v)).D(w,A(w))}{D(v,w)}$+ e D(w,B(w))+iD(v,w) ∀ v,w ∈ **Å**, 0 ≤c,e,i,and c+e+i < 1 and (v,w) in E(**Ĝ**) and then A , B possess a fixed point.

Proof. Since if we restrict ranges of **Ĝ** - DPMS from set of real numbers to non - negative numbers then **Ĝ** - complete dualistic partial metric space becomes **Ĝ** - complete partial metric space and proof of this Corollary is obtained by same steps of Theorem 3.9 .

Example 3.11. Suppose (V,E) = **Ĝ** is a graph choose [1,4]= **Å**, [1,4] = V(**Ĝ**) and E(**Ĝ**) = {(v,w)∈ [1,4] × [1,4]}, consider the function D:**Å**×**Å** → ℝ defined by D(v,w) = max{v,w} is a **Ĝ** -complete DPMS. Now we define the maps on [1,4] to itself as A(v) = $\sqrt{v}$ and B(v) = v^2 both these maps are graph preserving and satisfies **Ĝ** -CCP for the sequence { v_n } = $\frac{1}{n} + 1$ converges to 1,(v_n ,1) in E(**Ĝ**).
$|D(A(v), B(w))| \leq c\frac{|(D(v,A(v)).D(w,B(w))|}{|D(v,w)|} + e|D(w, B(w))| + i|D(v, w)|$ (3.45)

Now,
$|D(\sqrt{v}, w^2)| \leq \frac{1}{2}\frac{|(D(v,\sqrt{v}).D(v,w^2)|}{|D(v,w)|} + \frac{1}{4}|D(w, w^2)| + \frac{1}{5}|D(v, w)|$ (3.46)

Since all the elements are non - negative and v^2 greater than $\sqrt{v}$ on [1,4] , v and w behave same on [1,4] and solving (3.46) we obtain $v^2 \leq \frac{1}{2}w^2 + \frac{1}{4}w^2 + \frac{1}{5}w$.Thus A and B fulfill entire conditions of theorem(3.10) so A(1)= B(1) = 1, see in figure 3 .

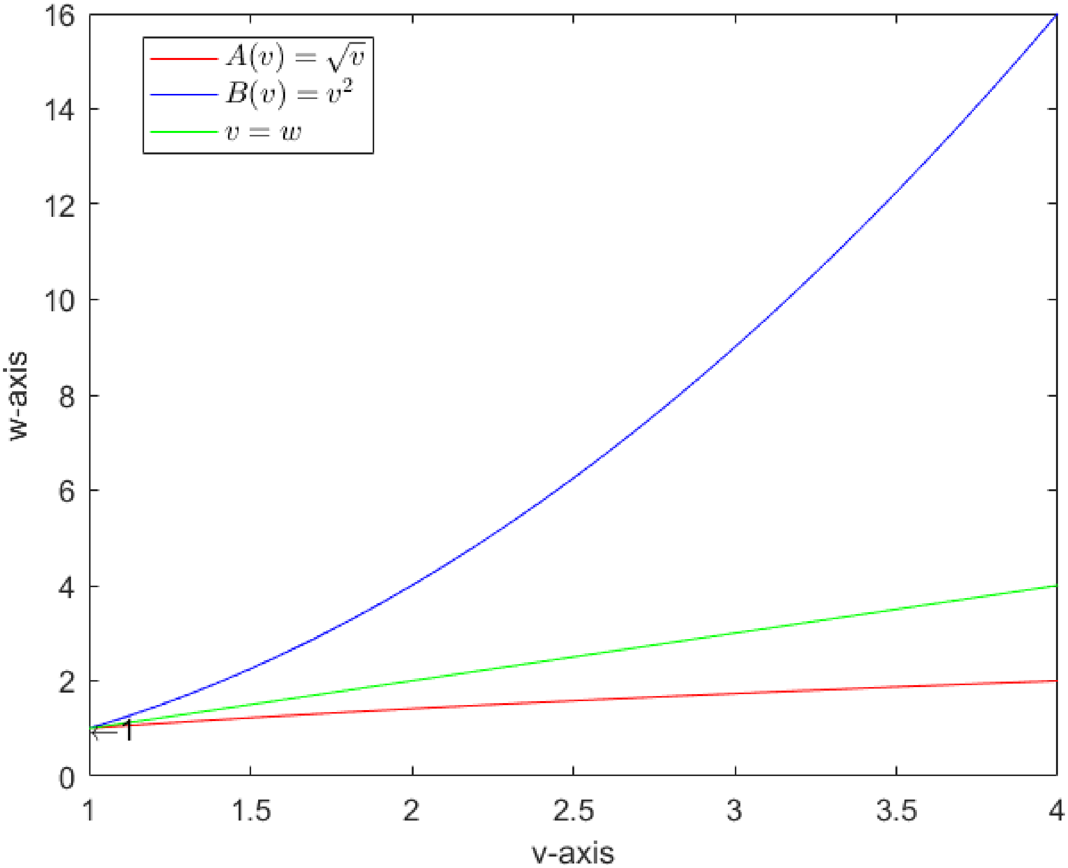

Figure 3: Graph of a fixed point for A and B

4 .Application

This portion includes an application of Theorem 3.1 i.e with the help of Theorem 3.1 we find a solution for boundary value problem defined on [0,1] given below

$$y''(w) = h\big(w, v(w)\big), v(0) = v(1) = 0 \quad (4.1)$$

$$y''(w) = j\big(w, v(w)\big), v(0) = v(1) = 0 \quad (4.2)$$

where $h, j: [0,1] \times \mathbb{R} \to \mathbb{R}$ are continuous map. A green function for the boundary value problem (4.1)and (4.2) is given by i.e denoted as G and defined as

$$G(t, a) = \begin{cases} w(1-a), & 0 \le w \le a \le 1 \\ a(1-w), & 0 \le a \le w \le 1 \end{cases}$$

A space of all continuous functions on [0,1] is represented by C[0,1] . Consider **Å** = (C[0,1],ℝ),V(**Ĝ**) = **Å**, E(**Ĝ**) = { (v,w) ∈ **Å**×**Å** } along with a mapping D:**Å**×**Å** → ℝ is given by

$$D(v, p) = ||v - p||_\infty + k = \sup_{w \in [0,1]} |v(w) - p(w)| + k, k \in \mathbb{R} \quad (4.3)$$

We observe that (**Å**, D) is **Ĝ** - DPMS which is complete and the mappings A,B: **Å**→**Å** given by

$$Av(w) = \int_0^1 G(w,a)h\big(a, v(a)\big)da \quad (4.4)$$

$$Bv(w) = \int_0^1 G(w,a)j\big(a, v(a)\big)da \quad (4.5)$$

where w in [0,1], and (4.1) and (4.2) possess a solution ⇔ A and B possess a fixed point.

Theorem 4.1. Consider **Å** = (C[0,1],ℝ),V(**Ĝ**) = **Å**, E(**Ĝ**)= {(v ,w) ∈**Å**×**Å** } and the two maps which are graph preserving and satisfies **Ĝ** - CCP, A, B: **Å**→**Å** defined as

$$Av(w) = \int_0^1 G(w,a)h\big(a, v(a)\big)da \quad (4.6)$$

$\int_0^1 G(w,a)h\big(a,v(a)\big)da = Av(w)$ (4.7)

where $h, j: [0,1] \times$ **Å**$\rightarrow \mathbb{R}$ are continuous map and v(w) defined in such way D(v(w),v(w)) ≤D(A(v(w)),B(v (w))) also these two mappings satisfy the inequalities

$|h(w,v) - j(w,p)| \leq 8\left(\log\frac{e^{nb}}{\pi}\right)$ (4.8)

for any w in [0,1], v,a in Å and b = $\{|D(v,A(v)),D(p,B(p))|\} \Longrightarrow$ (4.1) and (4.2) has a solution.

Proof. Choose z(w) in (C^2[0,1], $\mathbb{R}$) as a solution for (4.1) and (4.2) ⇔ z(t) in **Å** a solution for (4.6) and (4.7) . Solution for (4.6) and (4.7) are obtained through fixed point of A and B.Choose s, p in **Å** and w in [0,1] we get ,

$$|Av(w) - Ba(w)| = |\int_0^1 G(w,a)[h(a,v(a)) - j(a,y(a))]da|$$

$$\leq \int_0^1 G(w,a)[|h(a,v(a)) - j(a,p(a))|da]$$

$$\leq 8\left(\int_0^1 G(w,a)\left(\log\frac{e^{nb}}{\pi}\right)da\right)$$

$$= 8\left(\log\frac{e^{nb}}{\pi}\right)\left(\sup_{w\in[0,1]}\left[\int_0^1 G(w,a)da\right]\right) \quad (4.9)$$

Since $\int_0^1 G(w,a) = \frac{v}{2} - \frac{v^2}{2} = \frac{1}{8}$ for any v in [0,1] , we obtain

$$\big|D\big(A(p)\big) - D\big(B(p)\big)\big| = \sup_{w\in[0,1]}|Av(w) - Bp(w)| + k \leq sb$$

$$= s(\max(|D(v,A(v),D(p,B(p))|)) \quad (4.10)$$

Where k = logπ.Thus by with the help of Theorem 3.1, A and B has atleast one fixed point say z(w) and also a solution of (4.6) and (4.7)

5 .Conclusion

Recently, a concept for fixed points of contractive type mappings in DPMS is explored by Nazam et al.[9] by the help of Nazam et al.[9],we established many theorems in **Ĝ** - DPMS in which we use the **Ĝ** - CCP property , graph preserving mappings on the structure of graph and give illustration of the theorems.The beauty of this article will bring a lot of returns in the fixed point theory because we observe that many results that are not true in partial metric space i.e due to restrictions of non - negative real numbers are true in **Ĝ** -DPMS.

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