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Developing a Methodology for the Formation of a System of Attributes of Pathological Vascular Changes in the Fundus

Aslan Adal'bievich Tatarkanov*[1], Abas Khasanovich Lampezhev[2], Ruslan Khalitovich Tekeev[3], Dmitry Alekseevich Marenkov[4], Leonid Mikhailovich Chervyakov[5]

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Abstract: This research aimed to develop a methodology for extracting data from diagnostic images of the fundus blood vessels and methods for their high-precision evaluation, focused on ensuring the diagnostic process standardization, reducing the time of examination and its cost within the framework of evidence-based medicine. Timely and competent diagnosis plays an important role in obtaining an optimal result for treating vascular pathologies. This research evaluated the effectiveness of existing approaches to the analysis of the geometric diagnostic attributes of the fundus vascular system state reflected in the images, which are necessary for identifying pathological vascular changes. A technique was developed for the formation of an optimal system of geometric diagnostic attributes according to the criterion of separability. It was shown that the most effective method for solving this problem is the discriminant analysis method, which, in the presence of a strong connection between certain groups of attributes, makes it possible to decide whether it is expedient to use them and reduce the dimension of the attribute space. Reducing the dimension can significantly reduce the number of calculations. The results of using full-scale images of the fundus with active medical support confirmed the effectiveness of the developed technique.

Keywords: *classification methods, clustering methods, discriminant analysis, evidence-based medicine, vascular pathology diagnosis*

1. Introduction

The creation of a wide range of information systems capable to collect, store and process large amounts of various potentially useful medical information about patients, in the presence of uncertainties, is one of the promising areas of medicine development [1], [2]. The intellectual analysis technologies implemented within the framework of such systems using digital methods for modeling various medical and technological processes are aimed at preparing a database for making medical decisions [3].

For treating any disease, the most significant interrelated tasks are:

– timely diagnosing the disease, based on the processing of a large amount of input information obtained as a result of medical research (to identify potential pathologies) of the anatomical features of patient's specific organs, and a patient survey;

– treating productively the identified disease (pathology), based on an objective analysis of the input information about the patient's condition, and further monitoring of patient's medical state.

The human vascular system is the most important part in localizing the anatomical features of a particular organ to identify potential pathologies [4]. All organs of the human body are permeated and surrounded by circulatory systems. Most diseases, from common cold to complex cancerous tumors, affect the characteristics of the vessels in one way or another [5]. Diseases affect changes in vessel calibers, their number, curvature radius and branching frequency [6].

All similar changes in the vascular system morphology are squite clearly reflected in such a separate area of the body as the fundus (and its image), in which the vascular system can be subject to non-invasive viewing and study [7].

Isolation and monitoring of the fundus vascular pattern are actively used in medicine for assessing and preventing glaucoma, diabetic retinopathy, cardiovascular diseases,

[1] Institute of Design and Technology Informatics of RAS, Moscow – 127055, RUSSIA
ORCID ID: 0000-0001-7334-6318
[2] Institute of Design and Technology Informatics of RAS, Moscow – 127055, RUSSIA
ORCID ID: 0000-0003-1796-0748
[3] Institute of Design and Technology Informatics of RAS, Moscow – 127055, RUSSIA
ORCID ID: 0000-0003-0605-1370
[4] Institute of Design and Technology Informatics of RAS, Moscow – 127055, RUSSIA
ORCID ID: 0000-0003-0812-8802
[5] Institute of Design and Technology Informatics of RAS, Moscow – 127055, RUSSIA
ORCID ID: 0000-0002-2310-8992
** Corresponding Author Email: as.tatarkanov@yandex.ru*

etc. [8]. Traditional monitoring of image data based on "manual" multi-stage processing is very laborious and does not exclude errors [9]. To exclude the subjective human factor, it is relevant to develop computerized systems for studying the fundus, aimed at improving the accuracy and objectivity of diagnosis in the initial phase, and a significant reduction in the diagnostic time of [10]. Automating the process of studying the fundus will make it possible to standardize the diagnosis and ensure the objectivity of measurements [11].

In view of the above, this research aims to develop a methodology for extracting data from diagnostic images of the fundus blood vessels and methods for their high-precision evaluation aimed at ensuring the diagnostic process standardization, reducing the examination time and cost in the framework of evidence-based medicine.

2. State of the Problem of Non-Invasive Diagnostics of Eye Vessels

The arteries and veins of the fundus form a tree-like structure of its vascular system. Visible changes in this structure make it possible to accurately diagnose various diseases in the early stages of a non-invasive study [12]. This is related to the fact that a violation of microcirculation in the retinal and optic nerve vasculature is a common feature of a massive number of completely different diseases (see Table 1).

Table 2. Basic distinctive properties of changes in the circulatory system structure in various diseases

Disease	*Baseline vascular changes*	*References*
Atherosclerosis	The retinal veins are enlarged, twist in different directions, their calibration is uneven. The arteries remain virtually unchanged.	[13]
Arterial hypertension	Non-linear decrease in arterial diameter and non-linear increase in venous diameter, a higher degree of arterial system tortuosity, changes in the ratio of the width of arteries and veins.	[14]
Hypertonic disease	Reduction and enlargement of arteries and veins, non-linearity of the arterial system calibration of the, corkscrew squirming of the veins. Many patients have artery constrictions.	[15]
Diabetes	Non-linearity of the venous and capillary system width, vein tortuosity, changes in the vascular branching angle.	[16]
Age-related senile changes	Reduction in the arterial system diameter, arterial straightening, uneven arterial caliber.	[17]
Optic nerve atrophy	Reduction in the diameter of the vascular system and its components	[18]
Retinal periphlebitis	Enlargement, tortuosity, and non-linearity of the venous system.	[19]
Leukemia	Retinal vein enlargement and angiopathy	[20]

Thus, in the images of the fundus, the vasculature brightness parameter and vessel orientation change smoothly. However, simultaneously, indicators of the vasculature size, shape and local brightness can have significant differences. Possible pathological changes that can be seen in the fundus are caused by damage to the optic nerve, retina or choroid. With such lesions, artery narrowing, vein dilation, hemorrhages along the entire length of the veins, etc. can be seen. Such lesions are visible in the images: areas of black and white spots, excessive branching (tortuosity) of the vessels, extensive branching of the capillary structures [21]. Fundus cameras are the most accessible hardware for conducting this kind of clinical research [22], [23]. A fragment of the analyzed image obtained using a fundus camera is shown in Figure 1 [24].

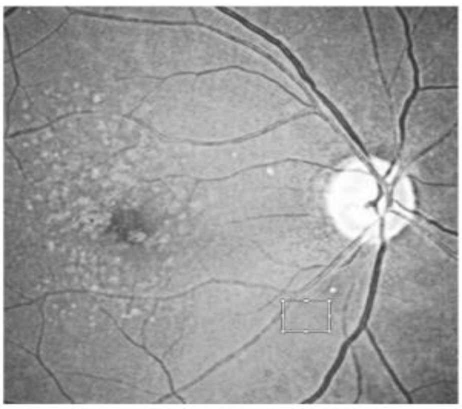

Fig. 1. A fragment of the analyzed image obtained using a fundus camera.

However, the use of fundus cameras in studying the fundus vasculature microcirculation to determine pathologies is complicated by numerous peculiarities [25]. For example, the normal range of vessel size is extremely wide. Moreover, the vascular system has a tree-like structure, and as the vessels approach the root of the tree, they thicken. In recent years, the methods for quantitative assessment of diagnostic attributes of the vasculature state based on rheological and geometric models have been used to improve the objectivity, efficiency, and accuracy of diagnosing vascular pathologies [26], [27]. The results of a comparative analysis of their capabilities are shown in Table 2.

Table 2. Models for the study of blood flow microcirculation

Models	***Rheological***	***Geometric***
Basic measurement parameter	Linear and volumetric blood flow velocity	Geometry of the vessel parameters
Advantages	Uniformity of interpretation	Unified equipment
Disadvantages	Complex, expensive and monofunctional equipment sensitive to fluctuations and accuracy of diametrical measurement is required	Indirect correlation of the studied parameter and the blood flow physiology
Vessel width examination	In the section passing through the point of the velocity measurement during the measurement	As in all preset sections with a non-uniformity function, including along their length
Advantages	Accuracy in the laser scanning process	The absolute clarity of the diameter measurement is not very important, its relative deviation is critical
Measurement scales	No borders between norm and pathology	No quantitative boundaries of norm or pathology
Advantages	A normal value can be set	Normal values are determined by an expert method
Disadvantages	Not a high range of values	The expert’s influence on the value of the norm
Using parameters in practice	Previously not used in daily diagnostic practice	There are clinically valid, understandable qualitative descriptions of the studied parameters in a wide range of diseases

Quantitative monitoring of the state of the fundus retina in the context of studying the vascular network is carried out by analyzing the course and caliber of blood vessels, monitoring the blood flow velocity and strength, and determining the level of blood filling with oxygen. An assessment method based on the analysis of the geometry and position of the vessels, such as vessel tortuosity, caliber non-linearity, venous system clarity, vessel straightness, etc., is the most effective method for assessing the condition of the fundus retinal vessels [28]. This assessment method involves the analysis of digital images of the fundus vessels. Back in 2003, a review of imaging technologies for diabetic retinopathy) was published by Sharp et al. [29]. In that study, a numerical analysis was conducted according to the stages of digital image processing.

When analyzing digital images of the fundus vessels, the quality of the digital image, determined by such indicators as opacity, directivity, and uneven illumination, is of great importance. Light unevenness correction, color equalization, contrast enhancement, and extraneous noise filtering are widely used to improve the quality of digital images. Another important factor in the analysis of digital images of the fundus vessels is the identification and selection of vessels in the image. Currently, two main algorithms are used to determine and single out vessels: segmentation and tracing. Segmentation makes it possible to determine the entire tree of vessels through an iterative approach, and tracing involves tracking individual vessels. When developing segmentation algorithms, the following types of approaches can be distinguished: image reading with and without training, matched filtering teams, mathematical morphology, and simulation. To conduct vascular tracing, it is required to define it between two points, which implies the use of local information. A systemic analysis of the methods used in practice for solving the tracing problem [30] showed that wavelet transform and local fan transform algorithms are most effective, as well as their modified versions, which provide high resistance to noise and invariance regarding the vessel brightness and orientation [31], [32].

The following parameters of the state of the retinal vascular network with a tree structure are of main interest from the standpoint of medical professional diagnostics:

- geometrical characteristics of the central line of the vascular system – routes and direction of vessels: their length, level of tortuosity, actual curvature, etc.;

- distribution of the measured parameter of a particular vessel thickness along the length of the route and its direction;

- branching angles of the investigated vessels.

From a mathematical viewpoint, the reflection of the tree-like form of the vascular system in the image can be described by the standard function for calculating the brightness *A(x, y)*, as a function of describing the degree of vessel brightness under consideration inside the area s, and the function $A_f(x, y)$, which is a description of the background around the vascular network in question.

The tree-like image model of the fundus vascular system [33] in a generalized form should include a set of data presented in Figure 2.

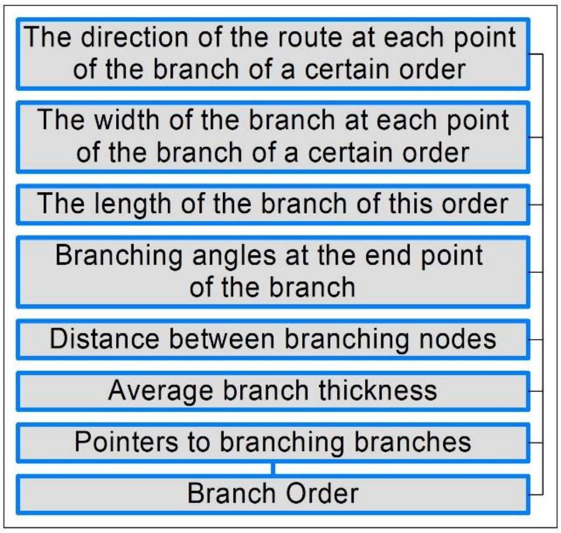

Fig. 2. Structural composition: a vasculature model.

In a mathematical context, any vessel branch can be represented by a set of such functions:

$$x = x(t), y = y(t), r = r(t), 0 \le t \le L_V, \quad (1)$$

where *x(t)* and *y(t)* are simple functions of a differentiable trace type; *r(t)* is the general radius function of the studied vessel; t is the interval from the initial trace point; L_v is the total length of the route.

This set of functional features determines the corresponding properties of the physical nature of blood vessels (Figure 3):

− The function of the studied direction of the route at the points of interest *φ(t)*.

− Single height function *f(t)* from the starting route point to the selected projection on the line *L*;

− The boundaries of the studied vessel, called walls:

$$x_{в}^{1} = x_{в}^{1}(t),\ y_{в}^{1} = y_{в}^{1}(t),\ x_{в}^{2} = x_{в}^{2}(t),\ y_{в}^{2} = y_{в}^{2}(t),\ 0 \le t \le L_v \quad (2)$$

The above attributes are classified as local characteristics formed from an image of the vascular system during the procedure for their tracing.

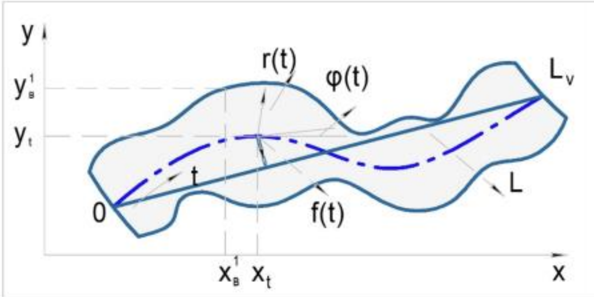

Fig. 3. Geometric model of a separate branch of a tree-like object.

In case when for the measured parameter t there are several chosen directions *φi* at once, at a particular point of the route (*x(t)*, *y(t)*) the presence of branching should be recorded.

The central line of the studied blood vessel is a set of points equidistant from the boundaries of the vessel. It can be represented as a regular curved line with pre-set natural input parameters:

$$\gamma(t) = (x(t), y(t)), \gamma(t) \in \mathbb{R}^2, t \in [t_1, t_2]. \quad (3)$$

where:

$$\|\gamma(t_1) - \gamma(t_2)\| > r(t_1) + r(t_1), \forall t_1, t_2 \in [0, L]. \quad (4)$$

The mathematical representation of the central line of the vessel displayed in the image has a discrete form:

$$\Gamma = \{t_t, x_t, y_t\}\Big|_{t=1}^{N} = \{t_t, \vec{x}_t\}\Big|_{t=1}^{V}, \quad (5)$$

where $\vec{x}_t \in \mathbb{R}^2$ denotes the established coordinates of the selected points of the vessel median line, defined in a discrete form, N is the number of points selected for the discrete representation.

Despite some progress in the domain of quantitative analysis of digital images of the fundus vessels, it is still urgent to develop and implement novel, more efficient models in the field of information technology that would allow us to assess the morphological features of blood vessels based on diagnostic images. A systemic analysis of practical methods for tracing the fundus vessels [30] showed that the wavelet transforms, local fan transforms, and their modified versions, which provide high resistance to noise and invariance regarding the vessel brightness and orientation, are the most effective algorithms.

3. Selection of an Effective Set of Geometric Diagnostic Attributes of the Fundus Vasculature State, Necessary for Identifying Pathological Changes in Blood Vessels by their Images

According to the structure of the analyzed vasculature, shown in the image of the fundus, directly within the tracing procedure, a vector of local geometric parameters of a single vessel can be formed, which characterize the direction of the route at each point, local height and curvature of vessels. These are the most important parameters of the diagnostic plan, based on the analysis of which procedures are carried out to assess the degree of the pathology that has arisen, and to analyze the probability of the occurrence of new diseases. High-resolution retinal images are required for diagnostics based on them. However, even the presence of high-quality images of the retina does not fully solve the problem of insufficient accuracy of their assessment by classical numerical methods. This is associated with the discrete representation of the images under study, quantization noise and mathematical distortions that appear in the input devices.

The accuracy of the calculations is extremely important in diagnostics, since elements with extremely small dimensions are investigated. Therefore, it is necessary to conduct comprehensive studies aimed at:

- developing methodological approaches that are more effective from the viewpoint of increasing the accuracy of assessing diagnostic attributes when tracing the image of the vessel;

- substantiating and forming an effective set of noise-resistant geometric diagnostic attributes of the fundus vasculature state, necessary for the identification of pathological changes in blood vessels by their images;

- developing a methodology for choosing the optimal, from a technical and economic standpoint, composition of noise-resistant geometric diagnostic attributes of the fundus vasculature state, necessary for identifying pathological changes in blood vessels by their images.

The improvement of the accuracy of monitoring diagnostic attributes requires to use averaged integral values obtained from a larger amount of initial data: the average diameter of the vessel, its straightness, the vascular branching angle, a clear image of the vessel under study, the amplitude of fluctuations in vascular density, the tortuosity of the route and the frequency of its oscillations.

The average diameter is calculated, through the averaged value of the radius:

$$\bar{D} = 2\bar{r} = \frac{2}{N}\sum_{n=1}^{N} r(t_n) \qquad (6)$$

here N is the number of measurements of the local radius r along the length of the vessel.

The clarity of the vessel S image demonstrates the uneven thickness of the vessel. Exploring a certain diametral distribution as a way to implement a stationary ergodic optional process, the dynamics of the vascular section width transformation along the route (Figure 4) appears in the role of a relative root mean square value, developed on the basis of monitoring the mean and variance:

$$S = \sqrt{\overline{r^2} - \bar{r}^2} \,/\, \bar{r} \qquad (7)$$

where $\bar{r}$ is a mean, $\overline{r^2}$ is a mean square.

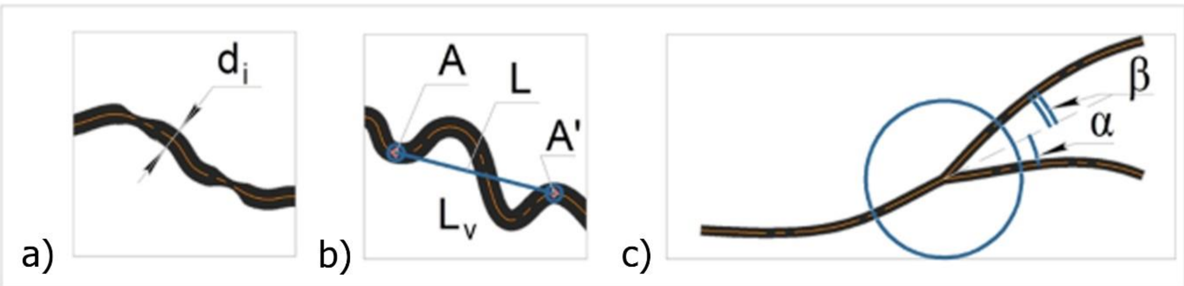

Fig. 4. Geometric characteristics of the vessel: (a) width of the vascular section d_i; (b) the length of the median vascular line L_v; (c) branching angles α and β, which are revealed during vessel tracing.

The straightness of the vessel P_r represents its deviation from the rectilinear course, calculated according to the formula below. If we consider it in the context of the ratio of length L_v of the middle vascular line relative to the length of the segment connected by trace points (Figure 4b).

$$\Pr = \frac{L_v}{L} = \frac{\sum_{n=1}^{N-1}\sqrt{(x_n - x_{n+1})^2 + (y_n - y_{n+1})^2}}{\sqrt{(x_1 - x_N)^2 + (y_1 - y_N)^2}} \qquad (8)$$

The branching angle is defined in the nodes of the tree-like structure. The creation of two angles α and β, which are revealed during vessel tracing, is a full description of trace branching (Figure 4c).

The maximum vascular thickness m_0 is determined by the expression:

$$m_0 = \arg\max_{1<m<N} R(m), R(m) = \left| \sum_{n=0}^{N-1} r(t_n) \exp\left(-i \frac{2\pi nm}{N} \right) \right| \tag{9}$$

The tortuosity of the route $I_1 = A_1 \omega_1$ is determined using the expressions:

$$\Pr = \frac{2}{\pi} \sqrt{1+I_1^2} \cdot E(k), k = \frac{I_1}{\sqrt{1+I_1^2}} \tag{10}$$

where *Pr* is the vessel straightness, *E(k)* is the complete elliptic integral of the second kind.

The oscillation amplitude of the vascular route A1, and the previously mentioned level of its tortuosity, clearly demonstrates the degree of the existing deviation of the vascular route and is determined by the expressions:

$$A_1 = 2\bar{f} \cdot E(k) \Big/ \left(1 + \mathrm{In}\left(I_1 + \sqrt{1+I_1^2} \right) \Big/ I_1 \sqrt{1+I_1^2} \right) \tag{11}$$

$$\bar{f} = N^{-1} \sum_{n=1}^{N} |f(t_n)| \tag{12}$$

where *f(t)* is a function and $\bar{f}$ is the mean for the vessel.

The oscillation frequency of the vascular route ω1 (or the tortuosity mentioned in the article) shows the number of changes in the route direction per unit length of the route and is calculated by the formula: $\omega_1 = I_1 / A_1$.

The experience regarding the impact of noise shows a persistent lability of techniques for assessing a large-scale complex of geometric vascular attributes. Cluster monitoring shows a clear possibility of using the mentioned attributes in the context of the diagnostic properties of the current state of the vessels.

4. The Results of Assessing the Route Attributes by the Typical Images

Synthetic routes with different values of amplitudes and frequencies were analyzed to calculate the error in assessing the vessel trace attributes by typical images (Table 3).

Table 3. The results of assessing the route attributes by the typical images

Image	***Route oscillation amplitude A_1, pixels***			***Route oscillation frequency ω_1, reverse pixels***		
	Original	***Calculation***	***Error***	***Original***	***Calculation***	***Error***
	60	60.18	0.006	0.1	0.108	0.16
	60	60.54	0.018	0.126	0.128	0.032
	60	59.78	0.008	0.178	0.186	0.09
	60	60.98	0.032	0.252	0.266	0.11
	30	29.42	0.038	0.1	0.11	0.02
	30	28.92	0.072	0.126	0.13	0.062
	30	29.5	0.032	0.178	0.192	0.158
	30	30.48	0.032	0.252	0.258	0.046

The average relative error in determining the route oscillation amplitude was 1.5%, and in determining the route oscillation frequency the error is 5.4%. The average relative error for the thickness difference amplitude is 2.3%, and in determining the frequency of oscillations average relative error was 4.5%. Calculations were obtained with an accuracy of up to 95% due to a computational experiment at an acceptable sampling level.

The influence of the Gaussian blur value and the noise/signal value in the case of additive noises was analyzed to assess the attribute resistance to distortions (Figures 5 and 6).

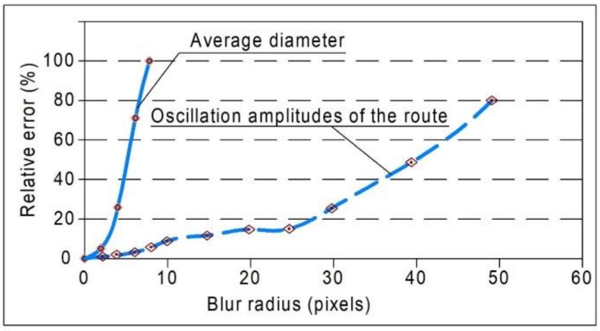

Fig. 5. Influences of the Gaussian blur value

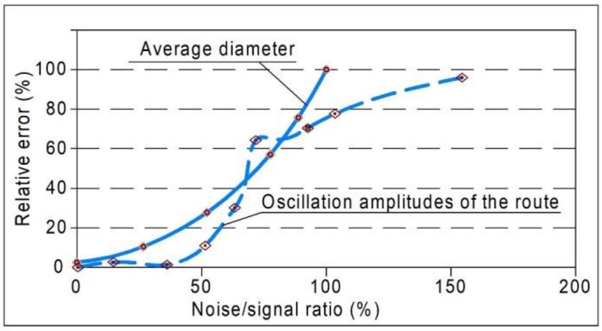

Fig. 6. Influence of the noise/signal values in the case of additive noises

Summarizing the results of the conducted research, it can be concluded that the trace function is characterized by those attributes that are more resistant to image blur compared to those that serve to describe the thickness function. Figure 5 shows how systematically the transformation occurs when the noise level changes.

5. Development of a Method for the Formation of an Optimal System of Geometric Diagnostic Attributes According to the Criterion of Separability, Used in Determining the Degree of Vascular Pathology

The method for creating an optimal separation system of mathematical diagnostic attributes used in calculating the degree of vascular pathology is based on the analysis of their properties and division into norm and deviation.

During the study, the discriminant monitoring, which belongs to the group of machine learning methods, turned out to be most effective. Machine learning methods are a set of methodologies and algorithms that make it possible to extract knowledge from data sets while constantly improving their capabilities, learning from experience (i.e., from data accumulating over time) [34]. The discriminant monitoring method involves ensuring a stable relationship between the studied attributes, while the question of the rationality of using these attributes becomes relevant. A powerful statistical relationship between attributes raises the question of reducing the proportionality of the attribute space or abandoning the whole group in the direction of a certain specific sign. It is also allowed to form an aggregate attribute that combines their characteristics. Dimensionality reduction can significantly reduce the number of calculations, especially in case of abandoning the attributes that are most difficult for this procedure. The selection of the most effective attribute in terms of the subsequent algorithm for its use is considered an optimal solution. Combining attributes into groups makes it possible to obtain a more effective attribute for classification, based on the algorithm for the formation of new attributes.

The discriminant analysis method basically implies the formation of class separability criteria using the within-class-scatter matrix, and the between-class-scatter matrix.

The generalized separability criterion can be expressed by the following formula:

$$\eta = \eta_B(B) / \eta_W(W) \tag{13}$$

where ηB is the value of scattering between clusters, B is the between-cluster-scatter matrix, ηW is the scattering value within one cluster, W is the within-cluster-scatter matrix.

The main procedures for the formation of the optimal method by the separability criterion η, within which there are g groups and the object of consideration in the general case are p geometric diagnostic attributes used to determine the degree of vascular pathology, are presented below.

1. Assessing the initial information, which makes it possible to characterize the differences between objects located in a point space within the framework of the study, is determined by a number of groups:

$$T = \sum_{k=1}^{K} \left(X_k - \bar{x} \right) \left(X_k - \bar{x} \right)' \tag{14}$$

where X_k is a sample corresponding to the k-class from the common p-dimensional sample of all classes: $X = \left| X_1 X_2 ... X_g \right|, X_k = \{ x_{ikm} \}$ p-dimensional sample of the k-class, x_{ikm} is a value of the i-th characteristic for the m-th investigated event in the k-th class. The elements of the scatter matrix form a complete set of information regarding the point distribution of spatial attributes.

2. Monitoring global mathematical vascular attributes, there are only nine of them (p=9) in this research.

3. Forming and determining the correlation matrix between these attributes (***R***), whose elements are equal:

$$r_{ik} = \sum_{k-1}^{p} \left(w_{ik} \Big/ \sqrt{w_{ii} w_{kk}} \right) \tag{15}$$

4. Forming and determining the within-group (W) and between-group (B) scattering matrices:

$$W = \sum_{k=1}^{g} \left(X_k - \bar{x}_k \right) \left(X_k - \bar{x}_k \right)' \quad (16)$$

5. Calculating criteria (J_1, J_2, J_3) for separability of classes, whose values should increase with an increase in between-class-scatter or with a decrease in within-class-scatter:

$$J_1 = tr\left(T^{-1}B\right);$$
$$J_2 = In\left|W^{-1}T\right| = In\left\{|T|/|W|\right\}; \quad (17)$$
$$J_3 = trB/\ \ trT.$$

6. Iterating over all initial attributes to select the most effective attributes for the algorithm of their subsequent use.

7. Selecting specific attributes for further classification, forming and determining the classification error.

8. Calculating eigen-values λ_i and vectors v_i, which ensure the implementation on the basis of the original attribute vector x = $[x_1 x_2 \ldots x_p]^T$ of a new space of effective attributes y = $[y_1 y_2 \ldots y_m]^T$ (m<p, $\lambda_i, v_i, i = 1, 2, \ldots, p$), maximizing the separability criterion.

9. Recalculating new attributes.

10. Forming and determining the correlation and scatter matrix of calculated attributes (R, W, B).

11. Calculating the separability criteria for classes J_1, J_2, J_3 relative to the obtained new attributes, and conducting a comparative analysis with their values for the previous attributes.

12. Classifying all received and generated attributes, accurately calculating the classification error, and comparing with past results for the original attributes.

In the research, a computational experiment was conducted, which was based on a test set of images. Its implementation required the generation of routes with different parameters, which would have to be classified into one of the groups according to their formal attributes. The research was based on 150 objects, with exactly 50 objects belonging to each category. Also, the improvement of the efficiency of studying the classification quality required to form a training and test data sets. Pairwise results of correlation coefficients are presented in Table 5. After analyzing the available data, several groups can be formed. Separate attention deserves the fact that each group of attributes with a strong correlation is based on the attributes that have a characteristic corresponding to the vessel route and thickness distribution. Discriminant analysis was conducted for the selected nine attributes.

Table 5. Paired coefficients

Attribute	D_{av}	***P***	***S***	A_0	ω_0	I_0	A_1	ω_1	I_1
D_{av}	1	0.83292	0.06864	0.6666	0.40458	0.67331	0.76791	0.39578	0.83523
P	0.83292	1	0.30118	0.70983	0.45045	0.76098	1.00848	0.52184	1.0989
S	0.06864	0.30118	1	0.88781	0.1408	0.79904	0.26917	0.14619	0.29876
A_0	0.6666	0.70983	0.88781	1	0.33495	1.0571	0.63965	0.34848	0.7073
ω_0	0.40458	0.45045	0.1408	0.33495	1	0.58311	0.44187	0.17479	0.45661
I_0	0.70664	0.10098	0.79904	1.0571	0.58311	1	0.69641	0.36223	0.7601
A_1	0.76791	1.00848	0.26917	0.63965	0.44187	0.69641	1	0.10879	1.01255
ω_1	0.39578	0.52184	0.14619	0.34848	0.17479	0.36223	0.10879	1	0.52063
I_1	0.83523	1.0989	0.29876	0.7073	0.45661	0.76043	1.01255	0.52063	1

It is also worth paying attention to the fact that the values of the criteria J_1 and J_2 turned out to be similar, because matrices T and B are scalars. After analyzing the available data, it can be concluded that the attributes I_1, I_0, P, A_0 have the highest separability value. At the same time, S, ω1 and ω0 should be referred to the least effective parameters (Table 6, Figure 7). Based on the existing attributes, an updated set was defined.

Table 5. Paired coefficients

Attribute	J_1	J_2	J_3
D_{av}	1.41	2.4418	1.41
P	1.6356	3.4058	1.6356
S	0.191	0.2008	0.191
A_0	1.0356	1.459	1.0356
ω_0	0.417	0.4676	0.417

I_0	1.164	1.7446	1.164
A_1	1.4258	2.4958	1.4258
ω_1	0.4398	0.4966	0.4398
I_1	1.6432	3.4478	1.6432

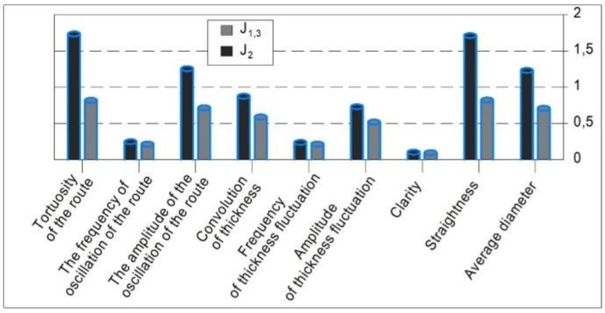

Fig. 7. The significance of different criteria.

The results of the data obtained from the discriminant analysis of new attributes are shown in Figure 8. These new attributes include eigen-values, structure coefficients, and eigen-vector components. If we turn to Figure 9, we can find that the updated attributes have higher separability criteria than the original ones. Thus, a conclusion can be drawn that the classification quality has improved.

Parameter		p_1	p_2
λ		0,8884	0,0128
D_{AV}	β	21,7591	-1,9707
	s	0,7758	-0,0613
P	β	65,2402	-11,943
	s	1,0632	-0,1967
A_0	β	26,9443	26,9443
	s	0,5174	0,5081
β_0		-364,4861	3,3945

a)

Parameter		p_1	p_2
λ		0,8550	0,0829
I_0	β	6,2334	6,0192
	s	0,4282	0,3829
A_1	β	0,0502	-4,1500
	s	0,5682	-0,8006
I_1	β	2639443	26,9443
	s	0,7821	-0,1908
β_0		-129,6810	15,2867

b)

Fig. 8. Total discriminant analysis in determining new generalized attributes p1 and p2 using the original attributes: a) D_{av}, P, A_0, b) I_0, A_1, I_1.

Attribute	J_1	J_2	J_3
p_1	0,8884	2,1933	0,8884
p_2	0,0128	0,0129	0,0128

a)

Attribute	J_1	J_2	J_3
p_1	0,8550	1,9313	0,8550
p_2	0,0828	0,0829	0,0829

b)

Fig. 9. Criteria for attributes based on a) D_{av}, P, A_0, b) I_0, A_1, I_1.

The results of a number of experiments using full-scale images of the fundus with sufficiently active medical support confirmed the effectiveness of the method for forming an optimal system of geometric diagnostic attributes in terms of separability criterion used in determining the degree of vascular pathology.

6. Results and Discussion

The article proposes a new approach to the study of blood vessels, based on the construction of estimates of the geometric attributes of blood vessels in the fundus.

Methods for assessing the noise immunity of integral geometric vascular attributes were studied, which are significant for finding diagnostic attributes. The experimental impact of additive and impulse noise in the diagnostic process, the grouping of spatial attributes in model images, monitoring the influence of noise on the number of clustering errors demonstrated the stability of the described assessing methods. Cluster analysis showed the possibility of rational use of medical information as diagnostic indicators of the state of the patient's vessels.

The proposed method for the formation of an optimal system of geometric diagnostic attributes by the criterion of separability, used in determining the degree of vascular pathology, is based on the analysis of their characteristics and the use of the efficiency criterion for classifying vessels into two main groups: normal and pathological. The results of experiments using full-scale images of the fundus confirmed its effectiveness. The use of new characteristics for vascular groups formed within the framework of the method optimized the level of separability of the vasculature photographs by diseases corresponding to different stages of retinopathy: for the first – by 32%, for the second – by 14%, for the third – by 26%, and for the fourth group by almost 16%.

7. Conclusion

The computer analysis of medical images is one of the modern applied medical methods. Comprehensive studies of the state of the circulatory systems are essential for the diagnosis of various diseases, primarily such as: the cardiac vasculature and the vessels of the eye.

Possible pathological changes in the vascular system can manifest themselves in the fundus, for example, in the form of lesions such as arteriostenosis, venous dilation, hemorrhages, etc. They are visible in the images and can be assessed visually. However, such an approach to studying the fundus vasculature microcirculation to determine pathologies is quite subjective, since the vascular system has a tree-like structure, thickening towards the root of the tree, and the range of normal vessel sizes is extremely wide.

The methods for quantitative assessment of diagnostic attributes of the vasculature state, based on the geometric model of the vessel and recognition procedures by tracing its central line in the image are the most effective way to increase the objectivity, operativity and accuracy of diagnostics.

The accuracy of the calculations is the most important requirement to the procedures for the diagnostic examination of the fundus vessels: indeed, elements that are extremely small in size are being examined. Therefore, the complex of studies performed became quite obvious, as a result of which:

- an effective set of noise-resistant geometric diagnostic attributes of the fundus vasculature state, necessary for the identification of pathological changes in blood vessels by their images was substantiated and formed;

- a methodology was developed for choosing the composition of noise-resistant geometric diagnostic attributes of the fundus vasculature state, necessary for identifying pathological changes in blood vessels by their images.

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Author contributions

Aslan Tatarkanov: Methodology, Writing - Reviewing and Editing **Abas Lampezhev:** Field study, Validation **Ruslan Tekeev:** Formal Analysis, Draft Preparation **Dmitry Marenkov:** Visualization, Software **Name3 Chervyakov:** Conceptualization.

Conflicts of interest

The authors declare no conflicts of interest.

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