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Implement Fuzzy-PID Controllers for Trajectory Tracking of an Underactuated Surface Vessel

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Abstract: Controlling of underactuated unmanned surface vessels (USVs) has been a research topic of interest in recent decades. However, control methods proposed in most studies have great complexity and are difficult to implement. This paper introduces guidance laws for the underactuated USV to track desire trajectories. Two control guidance laws will be established, which are the control of surge velocity and yaw angle. With the guidance laws, the controllers design process is simplified and easy to implement. PID controller is popular for its simplicity and ease of implementation. To maintain stability and reduce tracking error in various trajectories, a Fuzzy controller is introduced. The Fuzzy controller is designed to tune the gains of the PID controller. The Fuzzy-PID controller is designed for the USV to follow guidance laws, maintain stability and reduce tracking error. The proposed control method is implemented in simulations and experiments to verify its performance. The simulation and experiment show that this method can track different trajectories with small disturbances.

Keywords: *Fuzzy, PID, Trajectory tracking, USV, Unmanned Surface Vessel.*

1. Introduction

A number of researches on unmanned surface vessels (USV) has been implemented and control of USVs is a challenging problem. Many particular methods have been established for the USV to track desire paths including Path-Following, Target-tracking, Trajectory tracking,… Unlike Path-Following and Target-tracking, the aim of trajectory tracking is to control the vehicle to follow a time-varying trajectory. The Vessel in this paper is underactuated, which is more difficult to design controllers since the control in sway direction is not available.

In previous trajectory tracking control researches on USV, in [4], Ashrafiuon et al. (2008) proposed sliding mode controllers (SMC) for their vessel. These controllers are set to control the desire surge and sway velocity. To improve controllers in [4], Yu et al. (2012) designed their controllers to track desire surge and sway velocity. Two guidance law of surge and sway velocity are established to correct the convergence of the along-track error and cross-track error [5]. In [2], H.Huang et al. concluded that the correct heading of the USV is difficult to guarantee since the sway velocity can't be controlled directly. Motivated from other methods, H.Huang et al. proposed a simpler control method. Their controllers are set to track desire surge velocity and desire yaw angle. This is more suitable since yaw angle can be controlled directly by the thruster of the USV. Motivated from H.Huang, Xuan-Dung et al. proposed a guidance law for surge velocity and yaw angle [6]. In our previous research, our guidance law hasn't been verified for the convergence of along-track error and cross-tracking error in spite of our simulation results show the convergence of the USV to the desire trajectory. Motivated from H.Huang and our previous research, this paper introduce a similar control method from [2], [6] and use PID and Fuzzy controllers to control the USV to track desire pseudo-surge velocity and pseudo-yaw angle.

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2. Kinematics and Dynamics Model

The USV's kinematic and dynamic model are described by the following equations in [1]:

$$\dot{\boldsymbol{\eta}} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \boldsymbol{\nu} \quad (1)$$

$$(\mathbf{M_{RB}} + \mathbf{M_A})\dot{\boldsymbol{\nu}} + (\mathbf{C_{RB}} + \mathbf{C_A} + \mathbf{D})\boldsymbol{\nu} = \boldsymbol{\tau} \quad (2)$$

Where $\boldsymbol{\eta} = [\mathbf{x}\ \mathbf{y}\ \boldsymbol{\psi}]^{\mathbf{T}}$ represents the inertial coordinates and the yaw angle of the USV. $\boldsymbol{\nu} = [\mathbf{u}\ \mathbf{v}\ \mathbf{r}]^{\mathbf{T}}$ represents the surge velocity, sway velocity and yaw rate. $\boldsymbol{\tau} = [\mathbf{F_x} + \Delta\mathbf{F_x}\ \ \mathbf{F_y} + \Delta\mathbf{F_y}\ \ \mathbf{F_z} + \Delta\mathbf{F_z}]^{\mathbf{T}}$ represents the forces, moments and small pertubations applied on the USV. $[\mathbf{M_{RB}}]$ is the rigid-body mass matrix, $[\mathbf{M_A}]$ is the added mass matrix, $[\mathbf{C_{RB}}]$ is the rigid-body Coriolis and centripetal matrix, $[\mathbf{C_A}]$ is the linear hydrodynamic Coriolis and centripetal matrix, $[\mathbf{D}]$ is the linear damping matrix.

$$[\mathbf{M_{RB}}] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & mx_g & I_z \end{bmatrix}$$

$$[\mathbf{M_A}] = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}} \\ 0 & -N_{\dot{v}} & -N_{\dot{r}} \end{bmatrix}$$

$$[\mathbf{C_{RB}}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & mU \\ 0 & 0 & mx_gU \end{bmatrix}$$

$$[\mathbf{C_A}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -Y_{\dot{v}}U \\ 0 & 0 & -Y_{\dot{r}}U \end{bmatrix}$$

$$[\mathbf{D}] = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -N_v & -N_r \end{bmatrix}$$

Where m is the USV's mass, x_g is the x_b – axis coordinate of the USV's center of gravity (CG). X_u, Y_v, Y_r, N_v, N_r, $X_{\dot{u}}$, $Y_{\dot{v}}$, $Y_{\dot{r}}$, $N_{\dot{v}}$, $N_{\dot{r}}$ are the hydrodynamic derivatives. U is the cruise speed, about which $[\mathbf{C_A}]$ and $[\mathbf{D}]$ matrices are linearized.

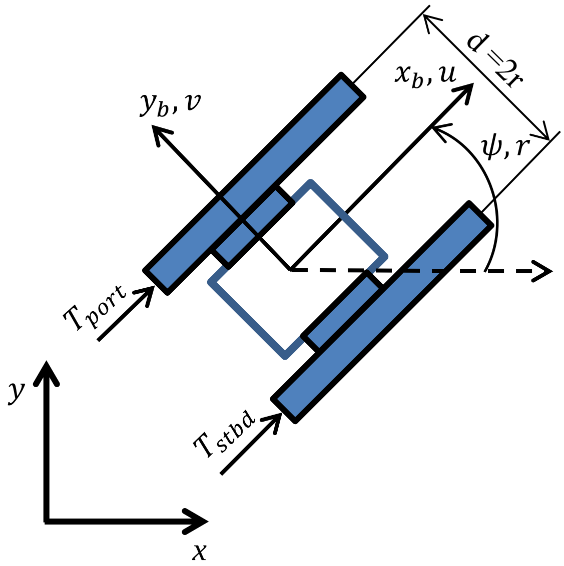

Fig. 1. The USV's configuration

3. Trajectory Tracking Guidance law

The along-track error and the cross-track error are defined as follows:

$$\begin{bmatrix} x_e \\ y_e \end{bmatrix} = \begin{bmatrix} \cos(\psi_d) & \sin(\psi_d) \\ -\sin(\psi_d) & \cos(\psi_d) \end{bmatrix} \begin{bmatrix} x - x_d(t) \\ y - y_d(t) \end{bmatrix} \quad (3)$$

Where $\psi_d = \text{atan2}\left(\frac{\dot{y_d}(t)}{\dot{x_d}(t)}\right) \in [-\pi, \pi]$ denotes the tangent angle of the trajectory.

$$U_d = \dot{x_d}(t)\cos(\psi) + \dot{y_d}(t)\sin(\psi) = \sqrt{\dot{x_d}(t)^2 + \dot{y_d}(t)^2}.$$

$U = \sqrt{u^2 + v^2}$ is the total speed of the USV.

$\beta = \text{atan}\left(\frac{v}{u}\right)$ is the sideslip angle between the USV heading and the orientation of the velocity vector.

From the work of H.Huang in [2]:

$$\begin{cases} \dot{x_e} = U\cos(\psi - \psi_d + \beta) - U_d + y_e\dot{\psi_d} \\ \dot{y_e} = U\cos(\psi - \psi_d + \beta) - x_e\dot{\psi_d} \end{cases} \quad (4)$$

The proposed pseudo-surge velocity and pseudo-yaw angle are designed as follows:

$$\begin{cases} U_{pseudo} = \frac{(U_d - f(x_e))\sqrt{y_e^2 + \Delta^2}}{\Delta} \\ \psi_{pseudo} = \psi_d - \beta + \arctan\left(\frac{-y_e}{\Delta}\right) \end{cases} \quad (5)$$

Where $\Delta > 0$ is the look-ahead distance.

In order to proof the convergence of x_e and y_e, consider the following Lyapunov function:

$$V = \frac{1}{2}(x_e^2 + y_e^2) \quad (6)$$

Differentiate V with respect to time, yields:

$$\dot{V} = x_e\dot{x_e} + y_e\dot{y_e}$$

$$= x_e[U\cos(\psi - \psi_d + \beta) - U_d + y_e\dot{\psi_d}]$$

$$+ y_e[U\cos(\psi - \psi_d + \beta) - x_e\dot{\psi_d}]$$

$$= x_eU\cos\left[\arctan\left(\frac{-y_e}{\Delta}\right)\right] + y_eU\sin\left[\arctan\left(\frac{-y_e}{\Delta}\right)\right] - U_dx_e$$

$$= x_e\left(U\frac{\Delta}{\sqrt{\Delta^2 + y_e^2}} - U_d\right) - \frac{Uy_e^2}{\sqrt{\Delta^2 + y_e^2}}$$

$$= -x_ef(x_e) - \frac{Uy_e^2}{\sqrt{\Delta^2 + y_e^2}} \quad (7)$$

To create $\dot{V} < 0$, $f(x_e)$ is considered being selected since the USV's performance depends on $f(x_e)$. In this paper, $f(x_e)$ is chosen as: $f(x_e) = \frac{k.x_e}{\sqrt{a^2 + x_e^2}}$, thus:

$$\dot{V} = -\frac{k.x_e^2}{\sqrt{a^2 + x_e^2}} - \frac{Uy_e^2}{\sqrt{\Delta^2 + y_e^2}} \leq 0 \quad (8)$$

In [2], H.Huang et al. take $f(x_e) = x_e$, this could cause $|U_{pseudo}|$ be arbitrary large since $|x_e|$ is also large. $f(x_e) = \frac{k.x_e}{\sqrt{a^2 + x_e^2}}$ can saturate U_{pseudo} for the USV to track. In this paper, k and a are considered to be 1.2 and 200 respectively.

4. Design of Fuzzy PID controllers

4.1. Structure of Fuzzy PID controllers

A PID controller is designed to control thrust and moment of the USV as follows:

$$\begin{cases} T_{control} = K_{p_u} e_u + K_{d_u} \cdot \frac{d(e_u)}{dt} + K_{i_u} \int e_u dt \\ N_{control} = K_{p_\psi} e_\psi + K_{d_\psi} \cdot \frac{d(e_\psi)}{dt} + K_{i_\psi} \int e_\psi dt \end{cases} \quad (9)$$

Where $e_u = u - U_{pseudo}$, $e_\psi = \psi - \psi_{pseudo}$.

To achieve the convergence of e_u and e_ψ, a Fuzzy controller is added to change PID controller coefficient. This paper proposed a Fuzzy controller to change K_p coefficients by the operating time of the USV. K_p coefficients can change whereas the errors change.

For the format of the Fuzzy reason is: if the magnitude of the error is high, the K_p coefficient is low to reduce the instability. If the magnitude of the error is low, the K_p coefficient is high.

Thrust on the starboard-side and port-side motor are calculated as follows:

$$\begin{cases} T_{stbd} = \frac{T_{control}}{2} + \frac{N_{control}}{r} \\ T_{port} = \frac{T_{control}}{2} - \frac{N_{control}}{r} \end{cases} \quad (10)$$

Where r is the distance from each motors to x_b-axis coordinate. The thrust on each motor are saturated by the motor's maximum thrust T_{max}. The force and moments acting on the USV are:

$$\tau = \begin{bmatrix} F_x + \Delta F_x \\ F_y + \Delta F_y \\ N + \Delta N \end{bmatrix} = \begin{bmatrix} T_{stbd} + T_{port} + \Delta F_x \\ \Delta F_y \\ (T_{stbd} - T_{port}) \frac{r}{2} + \Delta N \end{bmatrix} \quad (11)$$

4.2. Membership design of Fuzzy controller

In this paper, two Fuzzy-sugeno controllers will be implemented to control surge velocity and yaw angle of the USV. The input of two Fuzzy controllers are the same: {NB (Negative Big), M (Medium), PB (Positive Big)}. The basic domain of e_u is [-4,4], e_ψ is $[-\pi, \pi]$. Considering the design should be simple and meet real-time requirements, triangular membership function are implemented. **Fig. 2** and **3** show the membership function of input variable e_u and e_ψ.

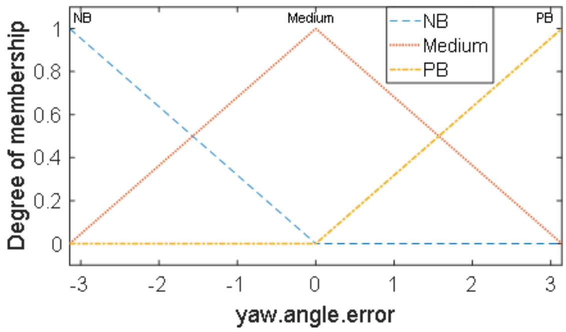

Fig. 2. Input variable of $\mathbf{e_\psi}$

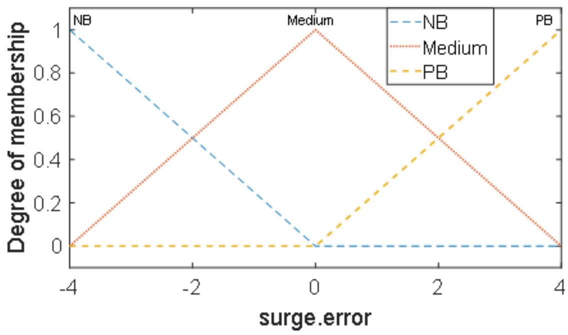

Fig. 3. Input variable of $\mathbf{e_u}$

4.3. Design of Fuzzy Rules and defuzzication

Fuzzy Rules are designed as follows:

Table 1. Fuzzy rule table for $\mathbf{e_u}$

$\mathbf{e_u}$	NB	M	PB
$\mathbf{K_{p_u}}$	$K_{p_{u_{low}}}$	$K_{p_{u_{high}}}$	$K_{p_{u_{low}}}$

Table 2. Fuzzy rule table for $\mathbf{e_\psi}$

$\mathbf{e_\psi}$	NB	M	PB
$\mathbf{K_{p_\psi}}$	$K_{p_{\psi_{low}}}$	$K_{p_{\psi_{high}}}$	$K_{p_{\psi_{low}}}$

Where $K_{p_{u_{high}}} = 100$, $K_{p_{u_{low}}} = 40$, $K_{\psi_{u_{high}}} = 10$, $K_{\psi_{u_{low}}} = 5$

From the Fuzzy-sugeno controller, the overall K_{p_u} and K_{p_ψ} are calculated as follows:

$$K_{p_u} = \frac{\sum_{i=1}^{3} \mu_{i_u} f_{i_u}}{\sum_{i=1}^{3} \mu_{i_u}} \quad (12)$$

$$K_{p_\psi} = \frac{\sum_{i=1}^{3} \mu_{i_\psi} f_{i_\psi}}{\sum_{i=1}^{3} \mu_{i_\psi}} \quad (13)$$

Where μ_{i_u}, μ_{i_ψ} are the degree of membership in Fig. 2 and Fig. 3.
f_{i_u}, f_{i_ψ} are calculated from the Fuzzy rule table 1 and table 2.

5. Simulation Results

This section presents simulation results to validate the effectiveness of the proposed controller. The USV model is selected with parameters specified in [3]. The proposed method is tested with different types of trajectories. The small external disturbances $[\Delta\mathbf{F_x}\ \Delta\mathbf{F_y}\ \Delta\mathbf{N}]$ are assumed to be: $\Delta\mathbf{F_x} = 0.5\sin(0.2t)$, $\Delta\mathbf{F_y} = 0.5\sin(0.2t)$, $\Delta\mathbf{N} = 0.02\sin(0.2t)$. The control parameters are: $\Delta = 40$, $K_{d_u} = 0.02$, $K_{i_u} = 5$, $K_{d_\psi} = 0.02$, $K_{i_\psi} = 0.02$.

The desire trajectories and initial conditions are as follows:

Circle trajectory:

$$\begin{cases} x_d(t) = 50\sin(0.015t) \\ y_d(t) = -10 + 50\sin(0.015t) \\ [u\ v\ r]_{t=0} = [0\ 0\ 0] \\ [x\ y\ \psi] = [0\ 0\ \frac{\pi}{4}] \end{cases}$$

Sinusoidal trajectory:

$$\begin{cases} x_d(t) = 0.8t + 60 \\ y_d(t) = 60\sin(0.015t) \\ [u\ v\ r]_{t=0} = [0\ 0\ 0] \\ [x\ y\ \psi]_{t=0} = [0\ 0\ \frac{\pi}{4}] \end{cases}$$

Straight trajectory:

$$\begin{cases} x_d(t) = 50 + t \\ y_d(t) = -30 + t \\ [u\ v\ r]_{t=0} = [0\ 0\ 0] \\ [x\ y\ \psi]_{t=0} = [0\ 0\ \frac{\pi}{4}] \end{cases}$$

In **Fig. 4**, although e_u converges to zero, the USV takes time to converge x_e and y_e to zero because of the initial condition. This is because of $f(x_e)$ is a factor that constrain U_{pseudo} when x_e and y_e are large. The contraints U_{pseudo} is to meet the USV's configurations. The error of yaw angle e_ψ converges to zero and yaw rate has a steady state value of 0.015 rad/s after t = 190 seconds. The USV tracks the trajectory after t = 200 seconds with the cross-track and along-track error converge to a steady state value of 0.5m and 0.12m.

In **Fig. 5**, the USV takes shorter time to converge x_e and y_e to zero since the initial condition is benefical to the trajectory and can track the trajectory with the cross-track and along-track error converge to zero. In the beginning, the USV steers to track the desire trajectory. In steady state, the largest cross-track and along-track error is 1.5m and 1.2m when the USV is trying to steer at t = 460 seconds.

In **Fig. 6**, the USV can track the straight trajectory, the cross-track and along-track error converge to a steady state value of 0.048m and 0.1m. At t = 10 seconds, after the USV operates, the error of yaw angle e_ψ is -0.5 rad. This is because the USV is trying to track the desire pseudo-yaw angle, which leads the USV to steer in order to track the straight trajectory. After t = 10 seconds, the USV tracks the straight trajectory but still oscillating. After 200th seconds, the USV tracks the straight trajectory.

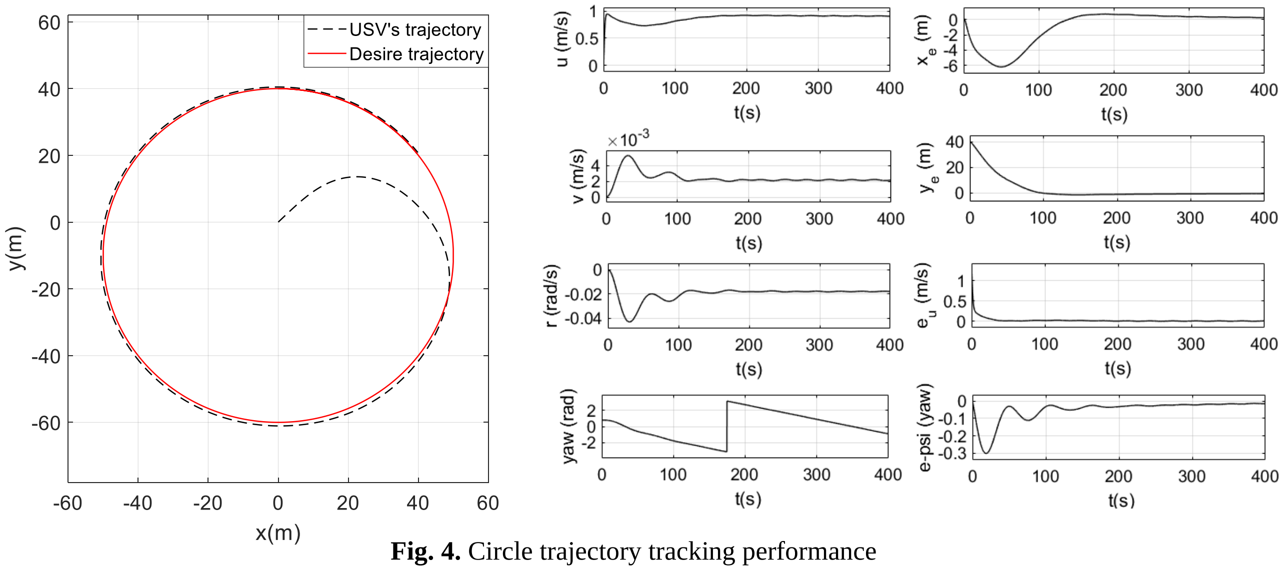

Fig. 4. Circle trajectory tracking performance

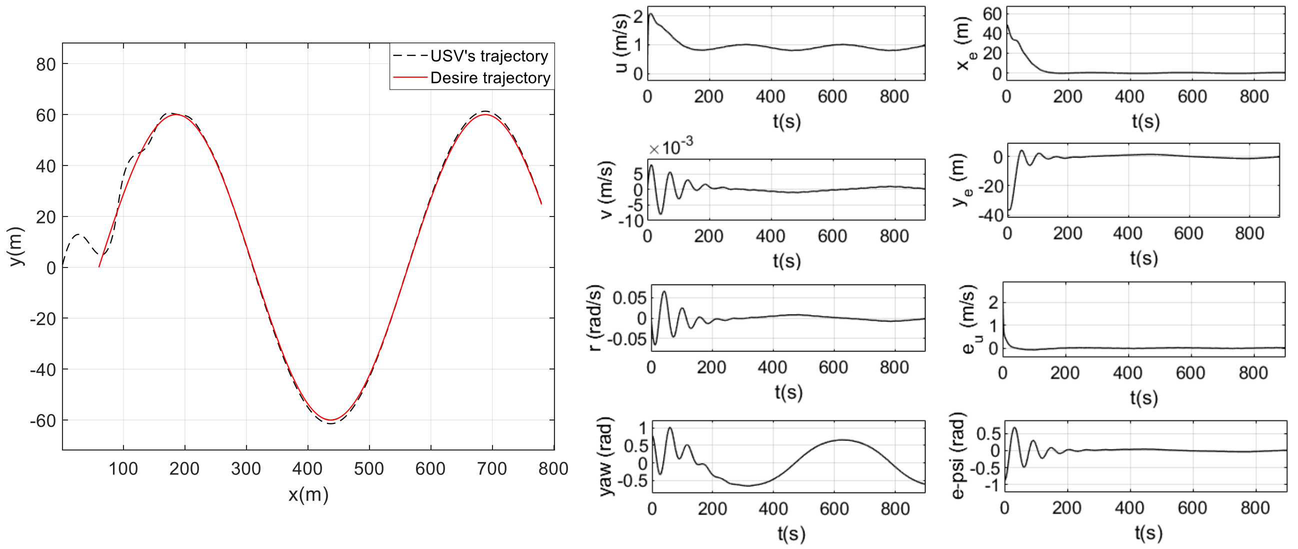

Fig. 5. Sinusoidal trajectory tracking performance

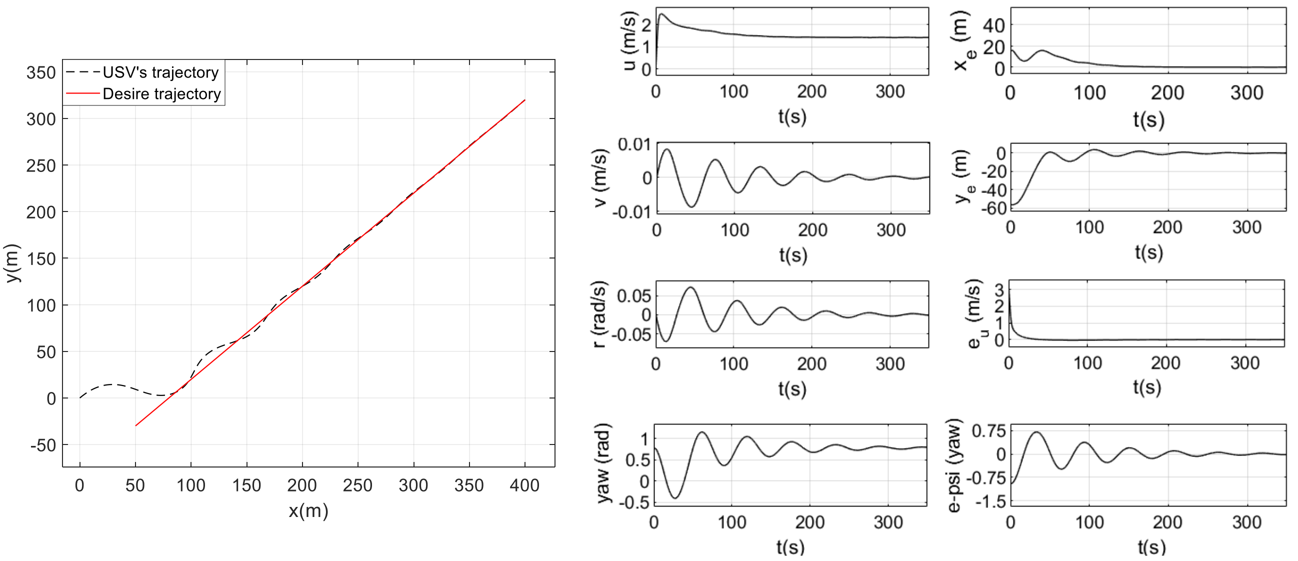

Fig. 6. Straight trajectory tracking performance

In this paper, we only consider 3 type of trajectories. Some trajectories can't be written in form of parametric equations. To track any desire trajectories, trajectory tracking problem can be converted to target tracking problem. While USV tries to track a trajectory, the USV instead can track multiple of steady targets on the trajectory.

6. Experiment Results

In this section, our control law for the experimental USV model and experimental results will be presented.

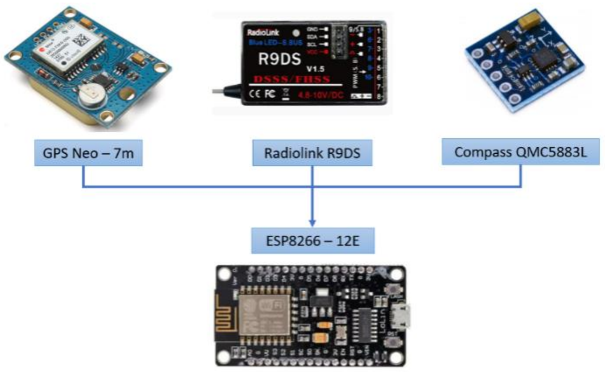

Fig. 7. Electronic module to determine state of the USV

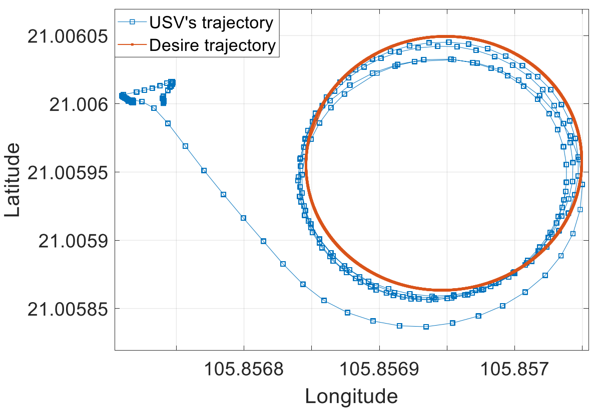

Fig. 8. Circle trajectory tracking performance in experiment

Fig 8. shows the performance of the experimental USV. In this model, only the Proportional term joins in the PID controller, the Integral-Derivative terms are neglected. From the proof of the guidane law, the convergence of x_e and y_e depend on the convergence of ψ_{pseudo}, which is the control of the yaw angle. The fuzzy controller is implemented only for ψ_{pseudo}. Instead of using $\psi_{pseudo} = \psi_d - \beta + \arctan(\frac{-y_e}{\Delta})$, after multiple experiments since our vessel has no IMU sensors, we considered $\psi_{pseudo} = \arctan\left(\frac{y(t)-y_d(t)}{x(t)-x_d(t)}\right)$ for small circle trajectories.

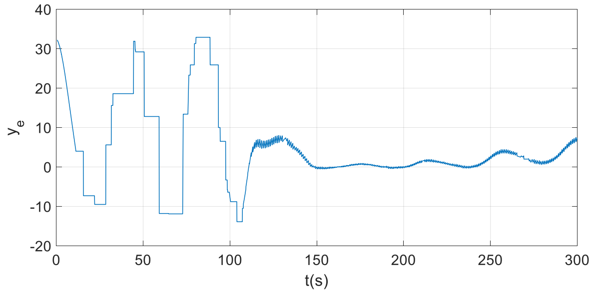

Fig. 9. Cross-track error in circle trajectory performance

The USV is attached to a cable to prevent failure in the middle of the lake. This affects the quality of convergence to the trajectory. Even so, the vessel can still track the trajectory with an error of x_e and y_e about 3m and 1.2m after 150th seconds. **Fig. 9** and **Fig. 10** show the convergence of

Fig. 10. Along-track error in circle trajectory tracking performance

is twisted, this affects the USV's tracking performance.

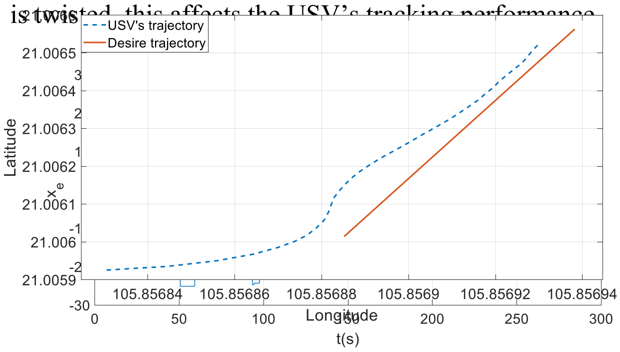

Fig. 11. Straight trajectory tracking performance in experiment

Fig. 11 shows the performance of the experimental USV with straight trajectory. Although the initial condition of the USV is not benifical, the USV tracks the straight trajectory. **Fig. 12** and **Fig.13** show the convergence of $\mathbf{x_e}$ and $\mathbf{y_e}$. But as mentioned, we were using $\boldsymbol{\psi_{pseudo} = \psi_d - \beta + \arctan(\frac{-y_e}{\Delta})}$. This causes the USV take longer time to converge $\mathbf{y_e}$.

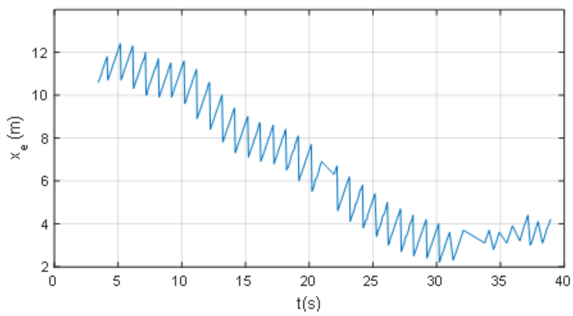

Fig. 12. Along-track error in straight trajectory performance

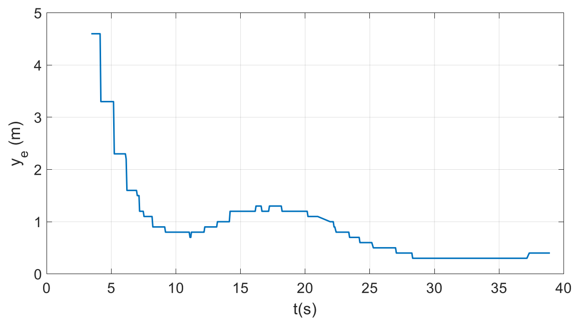

Fig. 13. Cross-track error in straight trajectory performance

Fig. 14. The experiment environment

7. Conclusion

From the guidance law of H.Huang and the proposed PID controller of Xuan-Dung, this paper proposed a Fuzzy PID controllers to track the guidance law of H.Huang and improve Xuan-Dung controllers. The simulation results show that the simple Fuzzy PID controllers can track trajectories. The experimental results show the effective of the proposed guidance law. While our previous controller in [6] can't perform some of the sinusoidal trajectories, our new controller can reduce overshooting and instability.

8. Acknowledgment

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