

# Synthesis of the Switched Bias Proportional Navigation Guidance Law to Destroy Complexly Maneuvering Targets

Dang Tien Trung<sup>\*1</sup>, Nguyen Van Bang<sup>2</sup>

Submitted: 11/11/2022

Accepted: 12/02/2023

**Abstract:** This paper presents the results of synthesis of guidance law for homing missile to destroy highly and complexly maneuvering targets. The aim is to realize the guidance methods applied to modern homing missiles. The algorithm is simple, highly convergent, stable and has small errors. The efficiency of the algorithm is verified through simulation, the results are reliable. New guidance law for short range flight missiles, based on the theory of variable mode structure. The new guidance law is built around the basic the proportional navigation guidance law with a switched bias added to the guidance law to bring the line of sight angle towards zero, even when the target is maneuvering.

**Keywords:** Guidance law, Homing missile, Maneuver, Proportional navigation, Target.

## 1. Introduction

The maneuver of the target causes the appearance of higher order derivative components of the tracking target coordinates at input of the tracking system. Therefore, the tracking error will increase. The proportional navigation guidance (PNG) law is only optimal for the linearized transfer function of missile, constant speed missiles, and non-maneuvering target models [5]-[16]. For maneuvering targets, the performance of the PNG law significantly decreases [1]-[4].

In order to solve the problem of destroying maneuvering targets, the augmented proportional navigation guidance (APNG) law has been researched and the results are acceptable [2], [3], [18]-[25]. At this time, the missile's command acceleration always has an additional hypothetical component of the target acceleration. However, it is very difficult to measure the actual target acceleration with high accuracy without delay [4], [25]. With complexly maneuvering targets in terms of both intensity and frequency of maneuvers, the performance of the APNG law also decreases [2].

Therefore, the paper proposes a method of synthesizing the switched bias proportional navigation guidance (SBPNG) law to improve the performance of the proportional navigation guidance law. By supplementing the guidance

law with a switched bias component caused by maneuverability of the target (due to target acceleration and unmodeled kinematic structures) on the basis of the application of sliding mode control theory, effective response in the case of high maneuvering target with great acceleration, complex flying trajectory. In the SBPNG law, this additional switched bias component is a function of the line of sight angle rotation speed.

## 2. Synthesis of the Switched Bias Proportional Navigation Guidance Law

The differential equation describing the relative kinematics between the missile and the target, has the form [4], [5]-[13]:

$$\dot{D}_m = V_{mt} \cos(\theta_{mt} - \sigma) - V_p \cos(\theta_p - \sigma) \quad (1)$$

$$D_m \dot{\sigma} = V_{mt} \cos(\theta_{mt} - \sigma) - V_p \cos(\theta_p - \sigma) \quad (2)$$

$$\theta_p = \frac{W_p}{V_p} = \frac{W_c}{V_p} \quad (3)$$

$$\dot{\theta}_{mt} = \frac{W_{mt}}{V_{mt}} \quad (4)$$

Where;

$D_m, \dot{D}_m$  - Relative distance and speed of change of distance between missile and target;

$\theta_p, \theta_{mt}$  - Trajectory inclination angle of missile and target;

$\sigma, \dot{\sigma}$  - The line of sight angle and the line of sight rotation speed;

$V_p, V_{mt}$  - Missile and target velocity;

<sup>1</sup> Faculty of Electrical Engineering, Electric Power University, Ha Noi - 10000, Viet Nam

ORCID ID: 0000-0001-7388-8037

Email: dangtientrung@gmail.com

<sup>2</sup> Air Defense - Air Force Academy, Ha Noi-10000, Viet Nam

ORCID ID: 0000-0002-9354-9283

Email: banghvpkkq@gmail.com

$W_c$  - The command acceleration;

$W_p, W_{mt}$  - Missile normal acceleration and target acceleration.  $|W_{mt}| < \alpha$ , where  $\alpha$  is a positive constant.

Step 1: Select the switch plane

To apply variable structure control (VSC) theory to guidance law design, a switching plane representing the kinetics of the desired system is given. Because the structure of the guidance law and the properties are very much dependent on the switch plane. The purpose of determining the switch plane will make the guiding law simpler, easier to implement and better able to destroy many types of maneuvering targets. The switch plane equation has the following form [2]:

$$S = \dot{\sigma} \quad (5)$$

The purpose of the switch plane selection is to bring the line of sight rotation speed to zero. At that time, the missile is in the target kill zone. Supposedly,  $\dot{\theta} \equiv 0$  can be obtained by a suitable choice.

Step 2: Design the guidance law

Design a guidance law to secure surface  $S = 0$  and slide along that surface. To achieve this, choose a Lyapunov function of the following form:

$$V = \frac{1}{2} S^2 \quad (6)$$

Obviously, this function is always positive. A sufficient condition to ensure that  $\dot{V} = \dot{S}S < 0$  with  $S \neq 0$ . From (1) ÷ (5), we have:

$$\dot{V} = \frac{\dot{\sigma}}{D_m} [-2\dot{D}_m \dot{\sigma} - W_p \cos(\theta_p - \sigma) + W_{mt} \cos(\theta_{mt} - \sigma)] \quad (7)$$

To ensure  $\dot{V} < 0$ , the command acceleration should have the following form:

$$W_c = \frac{1}{\cos(\theta_p - \sigma)} [-2\dot{D}_m \dot{\sigma} + K\dot{\sigma} + W \text{sign}(\dot{\sigma})] \quad (8)$$

Where:  $W \geq \alpha + \mu$  and  $\mu, K > 0$

$$\text{sign}(\dot{\sigma}) = \begin{cases} 1 & \text{when } \dot{\sigma} > 0 \\ 0 & \text{when } \dot{\sigma} = 0 \\ -1 & \text{when } \dot{\sigma} < 0 \end{cases}$$

Substituting equation (8) into (7), we get:

$$\begin{aligned} \dot{V} &= \frac{\dot{\sigma}}{D_m} [-K\dot{\sigma} - W \text{sign}(\dot{\sigma}) + W_{mt} \cos(\theta_{mt} - \sigma)] \\ &\leq \frac{\dot{\sigma}}{D_m} [-K\dot{\sigma} - \mu \text{sign}(\dot{\sigma})] < 0 \end{aligned} \quad (9)$$

Therefore, the value of  $W_c$  given in equation (8) guarantees the switch plane  $S = 0$ .

Step 3: Define parameters  $K$  and  $\mu$ .

Assume that  $K = -K'\dot{D}_m$  and  $\mu = -K'\rho\dot{D}_m$ . From formula (7), for case  $\dot{\sigma} \geq 0$ , we have:

$$\begin{aligned} \ddot{\sigma} &\leq \frac{1}{R} [K'\dot{D}_m \dot{\sigma} + K'\rho\dot{D}_m \text{sign}(\dot{\sigma})] < 0 \\ \frac{\ddot{\sigma}}{\dot{\sigma} + \rho} &\leq \frac{K'\dot{D}_m}{D_m} \end{aligned} \quad (10)$$

Transforming (10) both sides, we have:

$$\dot{\sigma} + \rho \leq [\dot{\sigma}(0) + \rho] \left[ \frac{\dot{D}_m}{D_m(0)} \right]^{1/K'}$$

Similar to the case  $\dot{\sigma} < 0$  and it can be easily shown that  $\dot{\theta} = 0$  will be reached at a relative distance, satisfying the following expression:

$$D_m \leq D_m(0) \left\{ \frac{\rho}{[\dot{\sigma}(0) + \rho]} \right\}^{1/K'} \quad (11)$$

Where,  $K' > 0$  and  $\dot{\sigma}(0), D_m(0)$  the initial values of the line of sight rotation speed and the initial relative distance. Choosing  $K$  and  $\mu$  thus, the guidance law (8) can be rewritten as:

$$W_c = \frac{1}{\cos(\theta_p - \sigma)} [-N\dot{D}_m \dot{\sigma} + W \text{sign}(\dot{\sigma})] \quad (12)$$

In (12) then  $W \geq \alpha - K'\rho\dot{D}_m$  and  $N = K' + 2$ . Here, it is assumed that the initial conditions are chosen so that the guidance law (8) satisfies the condition  $\dot{D}_m < 0$  until the target is met. Choice  $W_c$  is assumed based on the parameters  $\dot{D}_m, \dot{\sigma}$  and  $(\theta_p - \sigma)$ .

The new guidance law given by formula (12) can be considered as PNG law with time varying guidance coefficient, due to the presence of the term  $\cos(\theta_p - \sigma)$  and the switch bias component.

The meaning of the switched bias component can be explained as follows. Assuming that the guidance law can guarantee the slip condition, then the orbital  $\theta$  will slide along the surface  $S \equiv 0$  in the steady state. However, due to the presence of the target acceleration, it is not possible to achieve  $\dot{\theta} = 0$  precisely. Instead,  $\dot{\theta}$  will remain close to zero. In the vicinity of the sliding surface  $\theta \approx 0$  or can be understood:

$$-K'\dot{D}_m \dot{\sigma} - W \text{sign}(\dot{\sigma}) + W_{mt} \cos(\theta_{mt} - \sigma) \approx 0 \quad (13)$$

$$W_{mt} \cos(\theta_{mt} - \sigma) \approx -K'D_m \dot{\sigma} - W \text{sign}(\dot{\sigma}) \quad (14)$$

When the system is at steady state in the sliding mode, the guidance law (12) behaves like the APNG law with the guidance coefficient  $N = \frac{2}{\cos(\theta_p - \sigma)}$ .

The guidance law (12) can be reduced to the PNG law by dropping the term  $\cos(\theta_p - \sigma)$ , assuming its value is approximately equal to 1.

### 3. Simulation Results and Analysis

Compare the performance of the newly synthesized guidance law (12) with the laws PNG, APNG; with the assumption that the target and missile parameters are as follows:

- Missile parameters:

+ Missile velocity:  $V_M = 1300 (m/s)$

+ Distance:  $0(km)$

+ Height:  $0(km)$

+ Missile's orbit tilt angle:  $10^0$

- Target parameters:

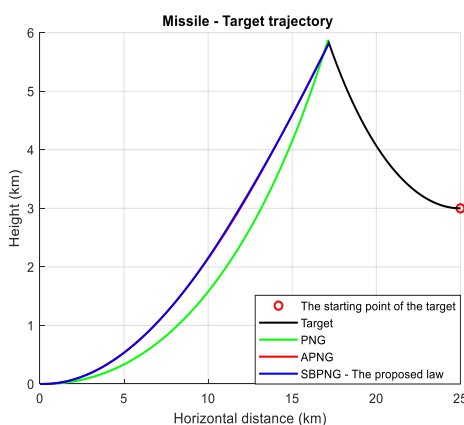
+ Target velocity  $V_T = 600 (m/s)$

+ Distance:  $15(km)$

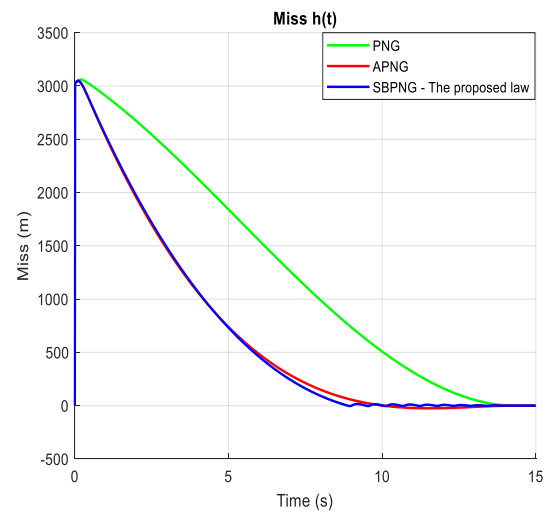
+ Height:  $10(km)$

+ Target's orbit tilt angle:  $10^0$

*The case of a one-sided maneuvering target.*



**Fig. 1:** Target missile trajectory

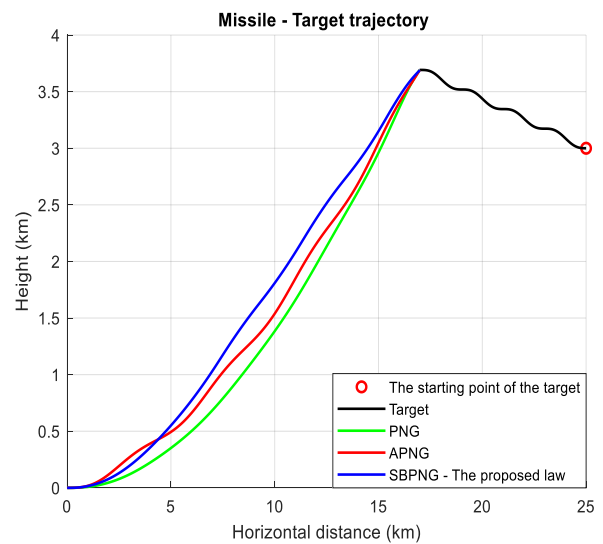


**Fig. 2:** The miss distance between missile and target

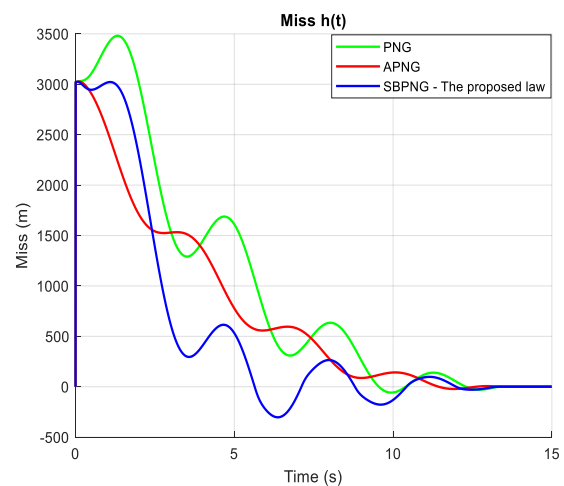
*The case of Snake style maneuvering target.*

Snake maneuvering target with normal acceleration:

$W_T = 10.9,81 \cdot \sin(\omega t)$ , where  $\omega = 0,5 (rad/s)$ .



**Fig. 3:** Target missile trajectory



**Fig. 4:** The miss distance between missile and target

Comments: In both cases.

The missile trajectory is controlled by the proposed guidance law (SBPNG), which is more straight forward than the traditional PN guidance law trajectory in all survey cases. Even when the target maneuvers with different intensity and type of maneuver.

The miss distance at the vicinity of the meeting point when using the new guidance law under all test conditions is always smaller than the traditional PNG law, the APNG law, therefore, the accuracy will be higher.

#### 4. Conclusion

This paper presents a method of synthesizing a new guidance law for short range flight missiles, based on the theory of variable mode structure. The new guidance law is built around the basic PNG law with a switched bias added to the guidance law to bring the line of sight angle towards zero, even when the target is maneuvering.

The new guidance law has a simple structure, fast convergence and stability. Simulation results confirm the superiority of SBPNGs over PNG and APNG. In particular, the adaptability of the guidance law not only reduces the degree of influence associated with VSC based systems, but also helps to reduce the magnitude of the initial transient under command acceleration. In the ideal case, the performance of this method is close to the performance of APNG law.

#### References

- [1] H. Jin Kim and Min-Jea Tahk, "Fast Adaptive Guidance Against Highly Maneuvering Targets", *Korea Advanced Institute of Science and Technology (KAIST) Daejeon, Korea*, 2008.
- [2] K. Ravindra Babu, I.G.Sarma, "Switched bias proportional navigation for homing guidance against highly maneuvering targets", *Journal of Guidance, Control, and Dynamics*, Vol.17, No.6, pp. 1357-1363, 2004.
- [3] Moon, J., Kim, K., and Kim, Y, "Design of missile guidance law via variable structure control", *Journal of Guidance, Control, and Dynamics*, Vol.24, No.4, pp. 659-664, 2015.
- [4] Neil F. Palumbo, Ross A. Blauwkamp, and Justin M. Lloyd, "Modern Homing Missile Guidance Theory and Techniques", *Johns Hopkins APL Technical Digest*, Vol.29, No.1, pp.1367-1385, 2010.
- [5] Paul Zarchan., "Tactical and Strategic Missile Guidance", *Progress in Astronautics and Aeronautics*, American Institute of Aeronautics and Astronautics, Inc., Washington, D.C, Vol.2, 2002.
- [6] Liu T., Xie Y, "A relative navigation algorithm for a chaser tracking a non-cooperative maneuvering target in space". Vol 31, No5, pp. 1338–1344, 2016.
- [7] Jiyuan L., Jun Z., Yingying L, "Applying auto-adaptation filter to tracking of maneuvering target in special relative navigation", *J. Northwest, Polytech. Univ.* 4, 013, 2018.
- [8] Bar S. Y., RongLi X., Kirubarajan T. "Estimation with Applications to Tracking and Navigation. Theory Algorithms and Software" *John Wiley & Sons*, 2001.
- [9] Blackman S., Popoli R, "Design and analysis of modern tracking systems", *Artech House*, 1999.
- [10] Shaofeng M., Xinxi F., Yulei L., Zhang W., Xiaomei Z, "A variable dimension adaptive IMM tracking algorithm", *Electron. Opt. Control* 22. (02), pp. 36-45, 2016.
- [11] Xiu L. H., Jing S. Y, "Curve Model of Adaptive Interaction Model Algorithm Tracking Method", *Applied Mechanics and Materials*, Vol 738-739, pp. 344-349, March 2018.
- [12] Jiangw L.V.Z., Lan Y, "IMM-CKF algorithm based on variable dimension interaction", *Comput. Appl. Softw*, 30(5). 4–6, 2017.
- [13] Nguyen N. T., Nguyen D. T., Nguyen V. B, "Synthesis of remote control law when taking into dynamics and nonlinearity of the missile stage", *Intelligent Systems and Networks (ICISN 2021)*, Springer, pp. 171-180, March 2021.
- [14] Xiong, K., Wei, C, "Spacecraft relative navigation based on multiple model adaptive estimator", *J. Syst. Sci. Math. Sci.* 34(07), pp. 828–837, 2018.
- [15] Le X.R., Kilkov V.P. "A survey of maneuvering target tracking: Approximation techniques for nonlinear filtering", *Proceedings of SPIE conference on signal and data processing of small targets*, pp. 537-550, 2014.
- [16] Li X.R., Jilkov V.P. "Survey of maneuvering target tracking-part V". *IEEE transaction on aerospace and electronic system*, 41 (4), pp. 1255-1321, 2015.
- [17] Xu B. "An adaptive tracking algorithm for bearings-only maneuvering target", *International journal of computer science and network security*, 7 (1), 2017.
- [18] Yang C. C., Tsung T. K, "An interactive dynamic multi-objective programming model to support better land use planning", *Land Use Policy, Elsevier*, Vol.36, pp. 13-22, 2016.
- [19] Qian G.H., Li, Y., Luo, R.J. "One maneuvering frequency and the variance adaptive filtering algorithm for maneuvering target tracking", *J. Radars* 2(6), pp. 258–264, 2017.
- [20] Nguyen V. B., Nguyen T. T., Dang T. T, "Synthesis of parameter recognition algorithm and state evaluation for flight device", *East European Scientific Journal*, Vol 2. No.66, pp. 10-17, 2021.
- [21] Nguyen V. B., Dang C. V. "Synthesis of the maneuver target acceleration determines algorithm", *Journal of natural and technical sciences, Sputnik Publishing*

House, No.2 (153), pp. 145-156, 2021.

- [22] Wu N.E., Youmin Z., Kermin Z. Detection, “Estimation and accommodation of loss of control effectiveness”, *International journal of adaptive control and signal processing*, 14, pp. 775-795, 2010.
- [23] Kim, H.S., Park, J.G., Lee, D, “Adaptive fuzzy IMM algorithm for uncertain target tracking”, *Int. J. Control Autom. Syst.* 7(6), pp. 1001–1008, 2017.
- [24] Jiadong R., Xiaotong Z, “Interactive multi-model target Maneuver tracking method based on the adaptive probability correction”, *International Conference on Swarm Intelligence, ICSI*, pp. 235-245, 2018.
- [25] Ming-Hsiung Hsueh, Chin-I Huang, Li-Chen Fu, “A Differential Game Based Guidance Law for the Interceptor Missiles”, *Industrial Electronics Society, IECON, 33rd Annual Conference of the IEEE*, pp.665-670, 2017.