# Transportation Problem Solver for Drug Delivery in Pharmaceutical Companies using Steppingstone Method 

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#### Abstract

In this paper, several solutions such as initial basic feasible solution (IBFS), optimal solution and degeneracy solution of the transportation problem are given, regarding the drug delivery from drug factories to different warehouses for minimizing the delivery time as well as cost of transportation according to the destination's requirement. In this Pandemic, it is the most essential part of the pharmaceutical marketing to focus in this cost minimization. The cost of production varies from company to company, and the transportation cost from one company drug factory to multiple warehouses also varies. Each drug factory has some specific production capacity and each warehouse has some certain amount of requirement. To verify the efficiency of this problem, we use Vogel's method to find IBFS and compare it with the Stepping stone method for optimization of the cost. In this work, we proposed a case study related to the above problem in which the drug items to be shipped from the drug factories to the warehouses, so that the cost of the transportation is minimized. It also explains the degeneracy in the transportation techniques. From the case study, it is found that the minimum transportation cost is Rs. 212 for both techniques. However, it is observed that the Stepping stone method reduces the degeneracy better than the Vogel's method. For scalability, we have also simulated the methods in MATLAB to observe the results in two cases. From the two cases, it is seen that Stepping stone method shows minimum cost of transportation.


Keywords: Transportation Problem, Drug Delivery, Stepping Stone Method, Vogel's Method, Degeneracy, Cost Minimization

## 1. Introduction

In transportation problem, each source has a multiple destination for transferring products or goods with a minimum cost [1-28]. In this problem, all units (source and destinations) available must be assigned in a particular drug factory. The transportation of goods from several sources to different destinations of
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transportation models is presented by F. L. Hitchcock in 1941 [10]. Any transportation model for shipping products from various sources to multiple destinations contains the capacity of requirement. The capacities and requirements of the source and destinations that satisfy the basic feasible solution should be in the form of ( $\mathrm{m}+\mathrm{n}-1$ ) [1-28], where m and n are the number of rows and columns of supply and demand matrix. Assume that a Company has four drug factories (sources) $\mathrm{O}_{1}, \mathrm{O}_{2}, \mathrm{O}_{3}$ and $\mathrm{O}_{4}$ producing the same product. From these factories, the product is transported to six warehouses (destinations) $\mathrm{D}_{1}, \mathrm{D}_{2}$, $D_{3}, D_{4}, D_{5}$, and $D_{6}$. The problem is to determine the quantity of each manufacturing company should transport to each warehouse in order to minimize total transportation costs. The number of units to be shipped is directly proportional to the cost of shipping from a particular source to a fixed destination. So, in this work we have solved the drug delivery transportation problem using Stepping Stone method and Vogel's method by taking a case study. Several problems have been introduced by applying the concept of various models as well as the empirical framework to solve the transportation problem. Many
research has been done on this area are discussed as follows. Ahmed et al. (2014) [1] studied the transportation problem solving to the minimize the cost. Anam et al. (2012) [2] studied the impact of transportation cost on Potato Price, distribution in Bangladesh. Babu et al. (2013) [3] found an approach to obtain IBFS of transportation model. Babu et al. (2014) [4] described a simple experimental analysis on risk management system. Bierman et al. (1977) [5] done a quantitative analysis for business related decision problems. Das et al. (2014) [7] proposed a sequential development of Vogel's method to study an approach to find solution for various transportation problems. Dykstra et al. (1984) [8] described the behaviour of transportation problem using natural resource management. Garfinkel et al. (1971) [11] studied about the naval transportation problem solving. Hammer et al. (1969) [12] developed an interpretation model to minimize the cost of transportation of perishable goods. Islam et al. (2012) [18] found a basic feasible solution for moderate transportation problem. Khan et al. (2015) [18-19] studied different transportation solving algorithms to find a feasible solution. Nikolic et al. (2007) [20] studied the time minimization in transportation problem for searching various routes. Ravindran et al. (1987) [22] provided suggestions for researchers to solve data analytics problem. Sharma et al. (1977) [24] also pointed out this problem to minimize the time. Szwarc (1971) [25] proposed some basic model on the Transportation Problem. Uddin et al. (2011) [26-27] developed an efficient network model to minimize the cost of transportation. Many such research works are presented in [1-32]. From the above work, it is observed that research has been performed in several transportation techniques. However, very less work has been done in the area of drug delivery in this pandemic situation as per our knowledge which is most important nowadays to reduce the transportation cost.

The major contributions of this research work in this paper are stated as follows:

1) In this work, several solutions such as IBFS, optimal solution and degeneracy solution of the transportation problem are given, regarding the drug delivery for minimizing the cost of transportation.
2) To verify the efficiency of the problem, we use Vogel's method to find IBFS and compare it with the Stepping stone method for optimization of cost.
3) We also proposed a case study related to the above problem in which the drug items to be shipped from 4 drug factories to 6 warehouses as per the supply and demand, so that the cost of the transportation is minimized.
4) It also explains the degeneracy in the transportation techniques and from the case study, it is found that the minimum transportation cost is Rs. 212 for both techniques, however, it is observed that Stepping stone method reduces the degeneracy better than the Vogel's method.
5) For scalability, we have also simulated the methods in MATLAB to observe the results in two cases. From the two cases, it is seen that Stepping stone method shows minimum cost of transportation.
The rest of the paper is described as follows: materials and methods are presented in Section 2, results and discussion is presented in Section 3, and at last in section 4, we concluded the work.

## 2. Materials and Methods

In this section, we have described about the basic transportation model, Vogel's method and Stepping stone method to solve the transportation method. The simple architecture of transportation model is shown in Fig. 1.


Fig. 1: Simple network architecture of network problem.

## Steps of solution for solving the transportation problem [1-28]:

Step-1: Transportation table may be constructed, representing the source and destinations with m-rows and n - columns respectively.
Step-2: Verifying the solution is feasible or not. To find the feasible solution with each independent positions for testing allocations in $(m+n-1)$ cells.
Step-3: Those cells representing allocations called as occupied and other cells are termed as unoccupied.
Step-4: Verify the solution assumed in step (4) is optimal. That can be computed for all unoccupied cells for getting optimal.
Step-5: The solution obtained in step-4 is not optimal, change the transportation cost by introducing that value in the unoccupied cell.
Step-6: Optimal solution is obtained, by using steps (5) and (6) successively.
So, many different methods [1-8] are present to find IBFS are north west corner method, Vogel's method, Modi method, least cost method, Stepping stone method, etc. In this work we consider Stepping stone method for considering the problem and done the comparison with Vogel's method because the problem of drug delivery best suits the taken methods. Firstly, we have discussed Vogel's method and afterwards Stepping stone method.

### 2.1 Vogel's Approximation Method (VAM) [1-28]

The North west-corner method and least cost method in transportation problem, no doubt, provided an optimal solution but the method is tedious specially if the given matrix is a largest one. VAM provides an alternative, which considerably reduces the number of stones to be moved and the number of values of squares to be tested. In other words, we can say that through VAM, the transportation problem can be solved in lesser time. VAM solution may be considered as the approximate solution to the given problem and should be checked for optimum solution, and some does not prove to be optimum, then the optimum solution is worked out by the transportation method. VAM method sometimes referred to as the penalty method in view of the fact that the cost differences it uses are nothing but the penalties of not using any other methods of transportation. Since the objective happens to be the cost minimization, in each iteration that route is selected which involves maximum penalty of not being used.
Steps of VAM: The steps are discussed as follows,
Step (1): From the cost figures in different squares of a given matrix, the difference between the two best cost
figures for each row and each column are found out, and it is put in row difference and column difference (are also termed as penalty respectively in the given matrix).
Step (2): The maximum of all the row and column differences are found out (select either in case of a tie by choosing arbitrarily). The square with lowest cost in the row or column as the case may be, whether the said maximum lies is then ascertained. The assignment is then made for this square (i.e., a stone is put) depending upon the supply and demand conditions
Step (3): The row/column which has zero supply/zero demand is deleted and the corresponding row/column differences are revised. Afterwards, least cost in that cell is identified for allocation of cell.
Step (4): Again, the differences between two cost figures are worked out but the squares crossed out and the squares to which assignments has been made are executed for the purpose of calculating the differences. Then the above (2) and (3) steps are repeated.
Step (5): The above stated process continues till the approximate assignment is obtained under the VAM.

### 2.2 Steps for Stepping stone method [1-28]

This method was developed for testing the optimality of the transportation problems. In this method cell evaluations (for reallocation) are made by path formation, each step of which is a tedious job.
The stepping stone method can be summarized as follows:

1. Before applying Stepping-stone method, use any one method for the transportation problem to find an IBFS.
2. Then check whether the number of allocations is equal to $\mathrm{m}+\mathrm{n}-1$ or not. If the number of allocations is less than $m+n-1$, then their exists degeneracy in the solution ( m - rows and n - columns).
3. Afterwards, starting from the cell a closed path may be formed from occupied cells to the unoccupied cells moving through horizontal and vertical ways. Then the cell at the corner points is considered to be on the closed path alternatively by assigning positive and negative signs to form a closed path. Each unoccupied cell can be evaluated by using a positive sign.
4. Step-3 is executed for all non-allocated cells till no further reduction is possible in the transportation.
There are, in general, two methods for testing optimality by cell evaluation, viz.
(i) The Stepping stone method
(ii) The modified distribution (MODI) method

In this problem we use only the Stepping stone method and compare with Vogel's method to find the better solutions.

## 3. Results and Discussion

In result section the optimality test of two methods such as Vogel's method and Stepping stone method is performed using a drug delivery scenario as discussed below. Also, at last a computer simulation is done to test the efficiency by considering two cases.
Problem: A pharmaceutical company has four drug factories and six warehouses. The warehouse altogether has a surplus of 30 units of a given commodity, divided among them follows:

Drug factories: $\begin{array}{lllll}\mathrm{O}_{1} & \mathrm{O}_{2} & \mathrm{O}_{3} & \mathrm{O}_{4}\end{array}$
$\begin{array}{llllll}\text { Surplus: } & 7 & 8 & 4 & 11\end{array}$
The six drug warehouses altogether need 30 units of the commodity. Individual requirements at warehouses $D_{1}$, $D_{2}, D_{3}, D_{4}, D_{5}$, and $D_{6}$ are $6,6,8,2,6$, and 2 units respectively. The problem is to minimize the cost of transportation by considering the case taken in Table 1.

Table 1: Data table for solving the transportation problem; O: Origin, D: Destination.

| Origin | D | D | D | D | D | D | Surplus/ <br> supply |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Destination |  |  | 2 | 3 | 4 | 5 | 6 |
| a | 1 | 1 | 1 | 8 | 1 | 1 | 7 |
| $\mathrm{O}_{2}$ | 9 | 5 | 1 |  | 1 | 2 |  |
| $\mathrm{O}_{3}$ | 8 | 7 | 1 | 1 | 5 | 1 | 4 |
| $\mathrm{O}_{4}$ | 8 | 1 | 1 | 4 | 4 | 1 | 11 |
| Requiremen <br> t/demand | 6 | 6 | 8 | 2 | 6 | 2 | $30 / 30$ |

### 3.1 Solution of the Model using Vogel's Method

First, we use Vogel's method to find IBFS, which involves a transportation model (in the form of a matrix), performing optimality test. The steps are discussed as follows:
Step-1: Calculate row \& column penalties from Table 3.

Table 2: Allocating penalties in the least cost cell
$\mathrm{O}_{2} \mathrm{D}_{6}$.

|  | D <br> 1 | D <br> 2 | D <br> 3 | D <br> 4 | D <br> 5 | D <br> 6 | Surplu <br> s/suppl <br> y | Ro <br> w <br> Pen <br> alty |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{O}_{1}$ | 1 | 1 | 1 | 8 | 1 | 1 | 7 | 3 |
| 1 | 4 | 1 |  | 1 | 2 |  |  |  |
| $\mathrm{O}_{2}$ | 9 | 5 | 9 | 9 | 7 | 7 <br> $(2$ | $8(8-$ <br> $2=6)$ <br> $\mathrm{O}_{3}$ | 8 |

Step-2: Calculate highest penalties among all penalties. Highest penalties are present in column-6. First allocate minimum of supply and demand of (2) units in the least cost cell $\mathrm{O}_{2} \mathrm{D}_{6}$ in column 6 corresponding to highest penalty 5 . Balance supply is 6 and balance demand is 0 and the column 6 is cancelled.

Table 3: Allocating penalties in the least cost cell

| $\mathrm{O}_{4} \mathrm{D}_{4}$. |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \mathrm{D} \\ & 1 \end{aligned}$ | $\begin{array}{\|l} \hline \mathrm{D} \\ 2 \end{array}$ | $\begin{aligned} & \mathrm{D} \\ & 3 \end{aligned}$ | $\begin{array}{\|l\|l} \hline \mathrm{D} \\ 4 \end{array}$ | $\begin{aligned} & \hline \mathrm{D} \\ & 5 \end{aligned}$ | Surplus/ supply | $\begin{aligned} & \hline \text { Ro } \\ & \mathrm{w} \\ & \text { Pen } \\ & \text { alty } \\ & \hline \end{aligned}$ |
| $\mathrm{O}_{1}$ | $\begin{aligned} & \hline 1 \\ & 1 \end{aligned}$ | $\begin{aligned} & \hline 1 \\ & 4 \end{aligned}$ | $\begin{aligned} & \hline 1 \\ & 1 \end{aligned}$ | 8 | $\begin{aligned} & \hline 1 \\ & 1 \end{aligned}$ | 7 | 3 |
| $\mathrm{O}_{2}$ | 9 | 5 | 9 | 9 | 7 | 6 | 2 |
| $\mathrm{O}_{3}$ | 8 | 7 | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | $\begin{array}{\|l\|} \hline 1 \\ 3 \end{array}$ | 5 | 4 | 2 |
| $\mathrm{O}_{4}$ | 8 | $\begin{aligned} & \hline 1 \\ & 0 \end{aligned}$ | $\begin{aligned} & 1 \\ & 3 \end{aligned}$ | $\begin{aligned} & \hline 4 \\ & \text { 2) } \end{aligned}$ | 4 | $\begin{aligned} & 11(11- \\ & 2=9) \end{aligned}$ | 0 |
| Requiremen t/demand | 6 | 6 | 8 | 2 | 6 | 28/28 |  |
| Column <br> Penalty | 0 | 2 | 2 | 4 | 1 |  |  |

Step-3: Calculate Row penalties again. In second allocation, minimum of supply and demand (2) units is made in the least cost cell $\mathrm{O}_{4} \mathrm{D}_{4}$ of columns 4 corresponding to the highest penalty of 4 . Balance
supply is 9 and balance demand is 0 and column 4 is also cancelled.

Table 4: Allocating penalties in the least cost cell

| $\mathrm{O}_{4} \mathrm{D}_{5}$ |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{O}_{1}$ | D <br> 1 | D <br> 2 | D <br> 3 | D <br> 5 | Surplus/s <br> upply | Row <br> Pena <br> lty |
| $\mathrm{O}_{2}$ | 4 | 1 | 1 | 7 | 3 |  |
| $\mathrm{O}_{3}$ | 8 | 5 | 9 | 7 | 6 | 2 |
| $\mathrm{O}_{4}$ | 8 | 1 | 1 | 1 | 4 | 4 |
| 1 | 9 | $9(9-6=3)$ | $4^{*}$ |  |  |  |
| Requirement/ <br> demand | 6 | 6 | 8 | 6 | $26 / 26$ |  |
| Column <br> Penalty | 0 | 2 | 2 | 1 |  | 2 |

Step-4: Calculate row penalties again. Third allocation of (6) units is made in the least cost cell $\mathrm{O}_{4} \mathrm{D}_{5}$ of row 4. Balance supply is 3 and balance demand is 0 and column 5 is also cancelled

Table 5: Allocating penalties in the least cost cell

| $\mathrm{O}_{2} \mathrm{D}_{2}$. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{D}$ | $\begin{aligned} & \mathrm{D} \\ & 2 \end{aligned}$ | $\begin{aligned} & \hline \mathrm{D} \\ & 3 \end{aligned}$ | Surplus/su pply | Row <br> Pena <br> lty |
| $\mathrm{O}_{1}$ | $\begin{aligned} & \hline 1 \\ & 1 \end{aligned}$ | $\begin{aligned} & 1 \\ & 4 \end{aligned}$ | 1 1 | 7 | 3 |
| $\mathrm{O}_{2}$ | 9 | $\begin{aligned} & \hline 5 \\ & 6 \\ & \hline \end{aligned}$ | 9 | 6 | 4* |
| $\mathrm{O}_{3}$ | 8 | 7 | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | 4 | 1 |
| $\mathrm{O}_{4}$ | 8 | $\begin{aligned} & \hline 1 \\ & 0 \end{aligned}$ | $\begin{aligned} & 1 \\ & 3 \end{aligned}$ | 3 | 2 |
| Requirement/d emand | 6 | 6 | 8 | 20/20 |  |
| Column <br> Penalty | 0 | 2 | 2 |  |  |

Step-5: Calculate row penalties again. Fourth allocation of (6) units is made in the least cost cell $\mathrm{O}_{2} \mathrm{D}_{2}$ in row 2 corresponding to the highest penalty of 4 . The balance supply as well as demand are 0 each and row 2 as well as column 2 are cancelled. The solution to be obtained will be degenerate.

Table 6: Allocating penalties in the least cost cell

| $\mathrm{O}_{4} \mathrm{D}_{1}$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{O}_{1}$ | $\mathrm{D}_{1}$ | $\begin{array}{l}\mathrm{D} \\ 3\end{array}$ | $\begin{array}{l}\text { Surplus/sup } \\ \text { ply }\end{array}$ | $\begin{array}{l}\text { Row } \\ \text { Penal } \\ \text { ty }\end{array}$ |
| $\mathrm{O}_{3}$ | 11 | $\begin{array}{l}1 \\ 1\end{array}$ | 7 | 0 |
| $\mathrm{O}_{4}$ | $8^{(3)}$ | $\begin{array}{l}1 \\ 1\end{array}$ | 4 | 3 |
| 3 |  |  |  |  |\(\left.| \begin{array}{l}6(6 <br>

- <br>
3= <br>
3\end{array}\right)\)

Step-6: Calculate row as well as column penalties again. Fifth allocation of (3) units is made in the least cost cell $\mathrm{O}_{4} \mathrm{D}_{1}$ in row 4 corresponding to the highest penalty of 5 . The balance demand is 3 , balance supply is 0 and row 4 is cancelled.

Table 7: Allocating penalties in the least cost cell

| $\mathrm{O}_{3} \mathrm{D}_{1}$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{O}_{1}$ | $\begin{array}{l}\mathrm{D} \\ 1\end{array}$ | $\begin{array}{l}\mathrm{D} \\ 3\end{array}$ | $\begin{array}{l}\text { Surplus/sup } \\ \text { ply }\end{array}$ | $\begin{array}{l}\text { Row } \\ \text { Penal } \\ \text { ty }\end{array}$ |
| $\mathrm{O}_{3}$ | $\begin{array}{l}8 \\ 3)\end{array}$ | $\begin{array}{l}1 \\ 1\end{array}$ | 7 | $4(4-3=1)$ |
| 1 |  |  |  |  |$)$

Step-7: Calculate column penalties again. There is tie among the highest penalties 3 for row 3 as well as column 1. Corresponding either, the lowest cost cell is $\mathrm{O}_{3} \mathrm{D}_{1}$ with unit cost of 3 . Sixth allocation of (1) is made in the cell $\mathrm{O}_{3} \mathrm{D}_{1}$. Balance supply is 1 , balance demand is 0 and column 1 is also cancelled.

Table 8: Allocating penalties in the least cost cell $\mathrm{O}_{1} \mathrm{D}_{3}$ and $\mathrm{O}_{3} \mathrm{D}_{3}$.

|  | $\mathrm{D}_{3}$ | Surplus/supp <br> ly | Row (a) <br> Penalt <br> y |
| :--- | :--- | :--- | :--- |
| $\mathrm{O}_{1}$ | $11^{(7}$ <br> $)^{(7}$ | 7 |  |
| $\mathrm{O}_{3}$ | $11^{(1}$ <br> $)^{\prime}$ | 1 |  |
| Requirement/dema <br> nd | 8 | $8 / 8$ |  |
| Column Penalty | 0 |  |  |

Step 8: Now column penalty is 0 but row penalty cannot be calculated. Remaining allocations are made as per least cost method. Since the unit cost of either of the cells is 7, cell $\mathrm{O}_{1} \mathrm{D}_{3}$ is chosen arbitrarily and (7) units are allocated to this cell. Balance supply is 0 and balance demand is 1 .
Lastly (1) units are allocated to the cell $\mathrm{O}_{3} \mathrm{D}_{3}$. Balance supply as well as demand is 0 . Now no surplus unit is left and also the demand of all the six stores is met with.

The total transportation cost associated with the above problem gives:
$\mathrm{Z}=$ Rs. $(11 \mathrm{x} 7+5 \mathrm{x} 6+7 \times 2+8 \times 3+11 \mathrm{x} 1+8 \times 3+4 \times 2+4 \mathrm{x} 6)$
$=$ Rs. $(77+30+14+24+11+24+8+24)=$ Rs 212
In Vogel's method when solving problem is not satisfying optimality then we check the optimality with other method such as Stepping stone method.

Table 9: Final allocation table.

|  | $\mathrm{D}$ | $\begin{aligned} & \hline \mathrm{D} \\ & 2 \\ & \hline \end{aligned}$ | $\mathrm{D}$ | $\begin{aligned} & \mathrm{D} \\ & 4 \\ & \hline \end{aligned}$ | $\begin{aligned} & \mathrm{D} \\ & 5 \\ & \hline \end{aligned}$ | $\begin{aligned} & \mathrm{D} \\ & 6 \end{aligned}$ | Surplus/ supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{1}$ | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | $\begin{aligned} & 1 \\ & 4 \end{aligned}$ | $\begin{array}{\|l\|} \hline 11 \\ (7) \end{array}$ | 8 | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | 7 |
| $\mathrm{O}_{2}$ | 9 | $5$ | 9 | 9 | 7 | 7 <br> 2) | 8 |
| $\mathrm{O}_{3}$ | $\begin{aligned} & \hline 8 \text { ( } \\ & \text { 3) } \end{aligned}$ | 7 | $\begin{aligned} & 11 \\ & (1) \end{aligned}$ | $\begin{aligned} & 1 \\ & 3 \end{aligned}$ | 5 | $\begin{aligned} & 1 \\ & 3 \end{aligned}$ | 4 |
| $\mathrm{O}_{4}$ | $\begin{aligned} & \hline 8 \text { ( } \\ & \text { 3) } \end{aligned}$ | $\begin{aligned} & 1 \\ & 0 \end{aligned}$ | 13 | $\begin{aligned} & \hline 4 \\ & 2) \end{aligned}$ | $\begin{aligned} & \hline 4 \\ & 6 \text { ( } \end{aligned}$ | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | 11 |
| Requiremen t/demand | 6 | 6 | 8 | 2 | 6 | 2 | 30/30 |

### 3.2 Finding Optimal Solution using Stepping Stone Method

To find optimality of a transportation problem we use Stepping-stone method for an opportunity cost to find whether the obtained feasible solution is optimal or not.

An optimality test can, be obtained based on the feasible solution in which:
a) Number of allocations in the given problem is $m+n-1$. In the given situation $m=4, n=6$ therefore, $m+n-1=4+6-$ $1=9$. Now the number of allocations $=8(<9)$. Therefore, we cannot apply optimality test, as in such cases degenerate solutions arises.
Next step to fix these $m+n-1$ allocations must be in its corresponding independent positions to check optimality.
In the present example, violating the row and column restrictions in each independent positions are impossible for any allocation without either changing the positions to increase or decrease. Since the number of allocations is 8 , there is need for making one infinitesimal allocation. Out of the unoccupied cells, cell $\left(\mathrm{O}_{3} \mathrm{D}_{5}\right)$ has the least cost of Rs. 5. The infinitesimal allocation should be made in this cell.

Table 10: Choosing $\Delta$ in the corresponding table.

|  | $\begin{array}{\|l\|l\|} \hline \mathrm{D} \\ \hline \end{array}$ | D 2 | $\begin{array}{\|l\|} \hline \mathrm{D} \\ 3 \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline \mathrm{D} \\ 4 \\ \hline \end{array}$ | $\begin{aligned} & \hline \mathrm{D} \\ & 5 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \mathrm{D} \\ & 6 \\ & \hline \end{aligned}$ | Surplus/ supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{1}$ | $\begin{aligned} & 1 \\ & \hline 1 \\ & 1 \end{aligned}$ | 1 4 | ${ }_{(7)}^{11}$ | 8 | 1 | 1 2 | 7 |
| $\mathrm{O}_{2}$ | 9 | $\begin{array}{\|l\|} \hline 5 \\ \hline 6 \end{array}$ | 9 | 9 | 7 | $\begin{aligned} & \hline 7 \\ & \hline 2) \end{aligned}$ | 8 |
| $\mathrm{O}_{3}$ | $\begin{aligned} & \hline 8^{\prime} \\ & 3) \end{aligned}$ | 4 | $\begin{array}{\|c\|} \hline 11 \\ (1) \end{array}$ | $\begin{aligned} & \hline 1 \\ & 3 \end{aligned}$ | $\begin{aligned} & 5( \\ & \Delta) \end{aligned}$ | $\begin{aligned} & 1 \\ & 1 \\ & \hline \end{aligned}$ |  |
| $\mathrm{O}_{4}$ | $\begin{aligned} & \hline 8^{\prime} \\ & 3) \end{aligned}$ | $\begin{array}{\|l\|} \hline 1 \\ \hline 0 \\ \hline \end{array}$ | 13 | $\begin{array}{\|l\|} \hline 4! \\ \hline 2) \\ \hline \end{array}$ | $-\frac{4!}{6!}$ | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | 11 |
| Requiremen t/demand | 6 | 6 | 8 | 2 | 6 | 2 | 30/30 |

In present case condition (a) is not satisfying. To fulfil this condition, we allocate an infinitesimally small positive quantity $\Delta$ (delta) to the suitable cell. As $\Delta$ is infinitesimally small $(\Delta \rightarrow 0)$, its effect can be ignored when it is added or subtracted from a positive value. It does not affect the physical nature of the original set of allocations but does helps in carrying out further iterations. Now keep $\Delta$ in respective positions in the respective unoccupied cells. Let us think to put it in the cell with the least cost, i.e., cell $\left(\mathrm{O}_{3} \mathrm{D}_{5}\right)$ which has the least cost of 5 units.
But we cannot do this because a closed path will be formed of cells $(3,5)(3,1)(4,1) \&(4,5)$ so that allocations in these cells do not remain in independent position and condition (b) will be violated, as shown in Table 12. Hence, no allocation in the cell $(3,5)$. There are two next higher cost cells viz. cell $(2,5)(3,2)$ each with a cost of 7 units. Allocation in either of these cells does not result in a closed path. Hence, $\Delta$ can be put in
either of the cells. Let us put in cell $(2,5)$, as shown in Table 11.

Table 11: Choosing $\Delta$ from replaced cells.

|  | $\mathrm{D}_{1}$ | $\begin{array}{\|l\|l} \hline \mathrm{D} \\ 2 \\ \hline \end{array}$ | $\begin{aligned} & \mathrm{D} \\ & 3 \end{aligned}$ | $\begin{array}{\|l\|} \hline \mathrm{D} \\ 4 \\ \hline \end{array}$ | $\mathrm{D}_{5}$ | $\begin{array}{\|l\|} \hline \mathrm{D} \\ 6 \\ \hline \end{array}$ | Surplus <br> /supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{1}$ | 11 | $\begin{aligned} & 1 \\ & 4 \end{aligned}$ | $\begin{aligned} & \hline 1 \\ & 1( \\ & 7 \text { ( } \end{aligned}$ | 8 | 11 | $\begin{aligned} & \hline 1 \\ & 2 \end{aligned}$ | 7 |
| $\mathrm{O}_{2}$ | 9 | 5 <br> 16 <br> $)$ | $9^{\Delta}$ |  | $\begin{array}{\|l\|} \hline 7( \\ \hline \Delta-4 \\ \Delta) \end{array}$ | $\begin{aligned} & \hline 7 \\ & 12 \\ & 1 \end{aligned}$ | 8 |
| $\mathrm{O}_{3}$ | $\begin{gathered} 8^{(3)} \\ +\Delta) \end{gathered}$ | $7$ | $\begin{array}{\|l\|} \hline 1 \\ \hline 1 \\ 1- \\ 1- \\ \Delta) \\ \hline \end{array}$ | $\begin{aligned} & 1 \\ & 3 \end{aligned}$ |  | $\begin{array}{r} -1 \\ 3 \end{array}$ | 4 |
| $\mathrm{O}_{4}$ | $\begin{aligned} & 8^{(3)} \\ & -\Delta) \end{aligned}$ | $\begin{array}{\|l\|} \hline 1 \\ 0 \end{array}$ | $\begin{array}{\|l\|} \hline 1 \\ 3 \\ \hline \end{array}$ | $\begin{array}{\|l} \hline 4 \\ \hline \\ \hline \end{array}$ | $\begin{aligned} & 4^{(6)} \\ & +\Delta) \end{aligned}$ | $\begin{array}{\|l\|} \hline 1 \\ 2 \end{array}$ | 11 |
| Requireme nt/demand | 6 | 6 | 8 | 2 | 6 | 2 | 30/30 |

Apply the Stepping stone method for optimal solution, firstly we have to calculate the unoccupied cells:

Table 12: Finding unoccupied cells.

| Unoccupied Cells | Net Cost | Optimal <br> Cost=Net <br> Cost |
| :---: | :---: | :---: |
| $\mathrm{O}_{1} \mathrm{D}_{1}$ | $\begin{aligned} & \mathrm{O}_{1} \mathrm{D}_{1}-\mathrm{O}_{1} \mathrm{D}_{3}+\mathrm{O}_{4} \mathrm{D}_{3}- \\ & \mathrm{O}_{4} \mathrm{D}_{1}=11-11+11-8=3 \end{aligned}$ | 3 |
| $\mathrm{O}_{1} \mathrm{D}_{2}$ | $\begin{aligned} & \mathrm{O}_{1} \mathrm{D}_{2}-\mathrm{O}_{2} \mathrm{D}_{2}+\mathrm{O}_{2} \mathrm{D}_{5}- \\ & \mathrm{O}_{4} \mathrm{D}_{5}+\mathrm{O}_{4} \mathrm{D}_{1}-\mathrm{O}_{3} \mathrm{D}_{1}+ \\ & \mathrm{O}_{3} \mathrm{D}_{3}-\mathrm{O}_{1} \mathrm{D}_{3}=14- \\ & 5+7-4+8-8+11-11=12 \end{aligned}$ | 12 |
| $\mathrm{O}_{1} \mathrm{D}_{4}$ | $\begin{aligned} & \mathrm{O}_{1} \mathrm{D}_{4}-\mathrm{O}_{4} \mathrm{D}_{4}+\mathrm{O}_{4} \mathrm{D}_{1}- \\ & \mathrm{O}_{3} \mathrm{D}_{1}+\quad+\quad \mathrm{O}_{3} \mathrm{D}_{3-} \\ & \mathrm{O}_{1} \mathrm{D}_{3}=8-4+8-8+11- \\ & 11=4 \end{aligned}$ | 4 |
| $\mathrm{O}_{1} \mathrm{D}_{5}$ | $\begin{aligned} & \mathrm{O}_{1} \mathrm{D}_{5}-\mathrm{O}_{4} \mathrm{D}_{5}+\mathrm{O}_{4} \mathrm{D}_{1-}- \\ & \mathrm{O}_{3} \mathrm{D}_{1} \quad+\quad \mathrm{O}_{3} \mathrm{D}_{3}- \\ & \mathrm{O}_{1} \mathrm{D}_{3}=11-4+8-8+11- \\ & 11=7 \end{aligned}$ | 7 |
| $\mathrm{O}_{1} \mathrm{D}_{6}$ | $\begin{aligned} & \mathrm{O}_{1} \mathrm{D}_{6}-\mathrm{O}_{2} \mathrm{D}_{6}+\mathrm{O}_{2} \mathrm{D}_{5}- \\ & \mathrm{O}_{4} \mathrm{D}_{5}+\mathrm{O}_{4} \mathrm{D}_{1}-\mathrm{O}_{3} \mathrm{D}_{1} \\ & +\mathrm{O}_{3} \mathrm{D}_{3}-\mathrm{O}_{1} \mathrm{D}_{3}=12- \\ & 7+7-4+8-68+11-11=8 \end{aligned}$ | 8 |
| $\mathrm{O}_{2} \mathrm{D}_{1}$ | $\begin{aligned} & \mathrm{O}_{2} \mathrm{D}_{1}-\mathrm{O}_{2} \mathrm{D}_{5}+\mathrm{O}_{4} \mathrm{D}_{5}- \\ & \mathrm{O}_{4} \mathrm{D}_{1}=9-7+4-8=-2 \end{aligned}$ | -2* |
| $\mathrm{O}_{2} \mathrm{D}_{3}$ | $\begin{aligned} & \mathrm{O}_{2} \mathrm{D}_{3}-\mathrm{O}_{2} \mathrm{D}_{5}+\mathrm{O}_{4} \mathrm{D}_{5}- \\ & \mathrm{O}_{4} \mathrm{D}_{1}+\quad+\quad \mathrm{O}_{3} \mathrm{D}_{1-} \\ & \mathrm{O}_{3} \mathrm{D}_{3}=9-7+4-8+8- \\ & 11=-5 \end{aligned}$ | -5* |


| $\mathrm{O}_{2} \mathrm{D}_{4}$ | $\mathrm{O}_{2} \mathrm{D}_{4}-\mathrm{O}_{2} \mathrm{D}_{5}+\mathrm{O}_{4} \mathrm{D}_{5}-$ <br> $\mathrm{O}_{4} \mathrm{D}_{4}=9-7+4-4=2$ | 2 |
| :--- | :--- | :--- |
| $\mathrm{O}_{3} \mathrm{D}_{2}$ | $\mathrm{O}_{3} \mathrm{D}_{2}-\mathrm{O}_{3} \mathrm{D}_{1}+\mathrm{O}_{4} \mathrm{D}_{1}-$ <br> $\mathrm{O}_{4} \mathrm{D}_{5} \quad+\quad \mathrm{O}_{2} \mathrm{D}_{5} \quad-$ <br> $\mathrm{O}_{2} \mathrm{D}_{2}=7-8+8-4+7-5=5$ |  |
| $\mathrm{O}_{3} \mathrm{D}_{4}$ | $\mathrm{O}_{3} \mathrm{D}_{4}-\mathrm{O}_{4} \mathrm{D}_{4}+\mathrm{O}_{2} \mathrm{D}_{1}-$ <br> $\mathrm{O}_{3} \mathrm{D}_{1}=13-4+8-8=9$ | 9 |
| $\mathrm{O}_{3} \mathrm{D}_{5}$ | $\mathrm{O}_{3} \mathrm{D}_{5}-\mathrm{O}_{4} \mathrm{D}_{5}+\mathrm{O}_{4} \mathrm{D}_{1}-$ <br> $\mathrm{O}_{3} \mathrm{D}_{1}=5-4+8-8=1$ | 1 |
| $\mathrm{O}_{3} \mathrm{D}_{6}$ | $\mathrm{O}_{3} \mathrm{D}_{6}-\mathrm{O}_{2} \mathrm{D}_{6}+\mathrm{O}_{2} \mathrm{D}_{5}-$ <br> $\mathrm{O}_{4} \mathrm{D}_{5} \quad+\quad \mathrm{O}_{4} \mathrm{D}_{1} \quad-$ <br> $\mathrm{O}_{3} \mathrm{D}_{1}=13-7+7-4+8-$ <br> $8=9$ | 9 |
| $\mathrm{O}_{4} \mathrm{D}_{2}$ | $\mathrm{O}_{4} \mathrm{D}_{2}-\mathrm{O}_{2} \mathrm{D}_{2}+\mathrm{O}_{2} \mathrm{D}_{5}-$ <br> $\mathrm{O}_{4} \mathrm{D}_{5}=10-5+7-4=8$ | 8 |
| $\mathrm{O}_{4} \mathrm{D}_{3}$ | $\mathrm{O}_{4} \mathrm{D}_{3}-\mathrm{O}_{3} \mathrm{D}_{3}+\mathrm{O}_{3} \mathrm{D}_{1}-$ <br> $\mathrm{O}_{4} \mathrm{D}_{1}=13-11+8-8=2$ | 2 |
| $\mathrm{O}_{4} \mathrm{D}_{6}$ | $\mathrm{O}_{4} \mathrm{D}_{6}-\mathrm{O}_{2} \mathrm{D}_{6}+\mathrm{O}_{2} \mathrm{D}_{5}-$ <br> $\mathrm{O}_{4} \mathrm{D}_{5}=12-7+7-4=8$ | 8 |

(*) Optimal cost is negative for $\mathrm{O}_{2} \mathrm{D}_{1}$ and $\mathrm{O}_{2} \mathrm{D}_{3}$ this is we have to modify it.
Suppose, we are selecting $\mathrm{O}_{2} \mathrm{D}_{1}$ there is no effect for $\mathrm{O}_{2} \mathrm{D}_{3}$ so, we can get the optimal cost to be positive for $\mathrm{O}_{2} \mathrm{D}_{3}$. Therefore, we have to select $\mathrm{O}_{2} \mathrm{D}_{3}$ only and select the neighbor allocation at the place of $\mathrm{O}_{2} \mathrm{D}_{3}$ and balance the problem (in the path).

Table 13: Stepping stone method forming the closed path.

|  | 1 | 2 | 3 | 4 | 5 | 6 | Surplus/ supply |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 9 |  | 9 5) 5 | 6 | 9 | 1 | 5 |
| 2 | 7 | $3$ | 7 | 7 | $5($ $\Delta)$ | $\begin{aligned} & 5( \\ & 2 \text { ( } \end{aligned}$ | 6 |
| 3 | $\begin{aligned} & \hline 6 \\ & \hline 1) \end{aligned}$ | 5 | 9 $1)$ 1 | 1 | 3 | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | 2 |
| 4 | $\begin{aligned} & 6( \\ & \text { 3) } \end{aligned}$ | 8 | 1 1 | $\begin{aligned} & 2( \\ & 2) \end{aligned}$ | 2 4) | $\begin{aligned} & 1 \\ & 0 \end{aligned}$ | 9 |
| Requiremen t/demand | 4 | 4 | 6 | 2 | 4 | 2 | 30/30 |

Now we can consider $\mathrm{O}_{2} \mathrm{D}_{5}$ as a non-allocated cell and $\mathrm{O}_{2} \mathrm{D}_{3}$ is an allocated cell. Now, again apply the stepping stone method for optimal checking, as shown in Table 14.

Table 14: Stepping stone method for optimality checking.

| Unoccupied Cells | Net Cost | Optimal Cost=Net Cost |
| :---: | :---: | :---: |
| $\mathrm{O}_{1} \mathrm{D}_{1}$ | $\begin{aligned} & \mathrm{O}_{1} \mathrm{D}_{1}-\mathrm{O}_{1} \mathrm{D}_{3}+\mathrm{O}_{2} \mathrm{D}_{3}- \\ & \mathrm{O}_{2} \mathrm{D}_{1}=11-11+9-9=0 \end{aligned}$ | 0 |
| $\mathrm{O}_{1} \mathrm{D}_{2}$ | $\begin{array}{\|l} \hline \mathrm{O}_{1} \mathrm{D}_{2}-\mathrm{O}_{1} \mathrm{D}_{3}+\mathrm{O}_{2} \mathrm{D}_{3}- \\ \mathrm{O}_{2} \mathrm{D}_{2}=14-11+9-5=7 \\ \hline \end{array}$ | 7 |
| $\mathrm{O}_{1} \mathrm{D}_{4}$ | $\begin{aligned} & \mathrm{O}_{1} \mathrm{D}_{4-}-\mathrm{O}_{4} \mathrm{D}_{4}+\mathrm{O}_{4} \mathrm{D}_{1}- \\ & \mathrm{O}_{3} \mathrm{D}_{1} \quad+\quad \mathrm{O}_{3} \mathrm{D}_{3-} \\ & \mathrm{O}_{1} \mathrm{D}_{3}=8-4+8-9+9- \\ & 11=1 \end{aligned}$ | 1 |
| $\mathrm{O}_{1} \mathrm{D}_{5}$ | $\begin{aligned} & \mathrm{O}_{1} \mathrm{D}_{5}-\mathrm{O}_{4} \mathrm{D}_{5}+\mathrm{O}_{4} \mathrm{D}_{1}- \\ & \mathrm{O}_{2} \mathrm{D}_{1} \quad+\quad+\quad \mathrm{O}_{2} \mathrm{D}_{3-} \\ & \mathrm{O}_{1} \mathrm{D}_{3}=11-4+8-9+9- \\ & 11=4 \end{aligned}$ | 4 |
| $\mathrm{O}_{1} \mathrm{D}_{6}$ | $\begin{aligned} & \mathrm{O}_{1} \mathrm{D}_{6}-\mathrm{O}_{2} \mathrm{D}_{6}+\mathrm{O}_{2} \mathrm{D}_{3}- \\ & \mathrm{O}_{1} \mathrm{D}_{3}=12-7+9-11=3 \end{aligned}$ | 3 |
| $\mathrm{O}_{2} \mathrm{D}_{1}$ | $\begin{aligned} & \mathrm{O}_{2} \mathrm{D}_{1}-\mathrm{O}_{2} \mathrm{D}_{5}+\mathrm{O}_{3} \mathrm{D}_{3}- \\ & \mathrm{O}_{3} \mathrm{D}_{1}=9-9+11-8=3 \end{aligned}$ | 3 |
| $\mathrm{O}_{2} \mathrm{D}_{4}$ | $\begin{aligned} & \mathrm{O}_{2} \mathrm{D}_{4}-\mathrm{O}_{4} \mathrm{D}_{4}+\mathrm{O}_{4} \mathrm{D}_{1}- \\ & \mathrm{O}_{3} \mathrm{D}_{1}+\quad+\quad \mathrm{O}_{3} \mathrm{D}_{3}- \\ & \mathrm{O}_{2} \mathrm{D}_{3}=9-4+8-8+11- \\ & 9=7 \end{aligned}$ | 7 |
| $\mathrm{O}_{2} \mathrm{D}_{5}$ | $\begin{aligned} & \mathrm{O}_{2} \mathrm{D}_{5}-\mathrm{O}_{4} \mathrm{D}_{5}+\mathrm{O}_{4} \mathrm{D}_{1}- \\ & \mathrm{O}_{3} \mathrm{D}_{1}+\mathrm{O}_{3} \mathrm{D}_{3}-\mathrm{O}_{2} \mathrm{D}_{3}- \\ & =7-4+8-8+11-9=5 \end{aligned}$ | 5 |
| $\mathrm{O}_{3} \mathrm{D}_{2}$ | $\begin{aligned} & \mathrm{O}_{3} \mathrm{D}_{2}-\mathrm{O}_{3} \mathrm{D}_{3}+\mathrm{O}_{2} \mathrm{D}_{3}- \\ & \mathrm{O}_{2} \mathrm{D}_{2}=7-11+9-5=0 \end{aligned}$ | 0 |
| $\mathrm{O}_{3} \mathrm{D}_{4}$ | $\begin{array}{\|l} \hline \mathrm{O}_{3} \mathrm{D}_{4}-\mathrm{O}_{4} \mathrm{D}_{4}+\mathrm{O}_{4} \mathrm{D}_{1}- \\ \mathrm{O}_{3} \mathrm{D}_{1}=13-4+8-8=9 \\ \hline \end{array}$ | 9 |
| $\mathrm{O}_{3} \mathrm{D}_{5}$ | $\begin{aligned} & \mathrm{O}_{3} \mathrm{D}_{5}-\mathrm{O}_{4} \mathrm{D}_{5}+\mathrm{O}_{4} \mathrm{D}_{1}- \\ & \mathrm{O}_{3} \mathrm{D}_{1}=5-4+8-8=1 \end{aligned}$ | 1 |
| $\mathrm{O}_{3} \mathrm{D}_{6}$ | $\begin{aligned} & \mathrm{O}_{3} \mathrm{D}_{6}-\mathrm{O}_{3} \mathrm{D}_{3}+\mathrm{O}_{2} \mathrm{D}_{3}- \\ & \mathrm{O}_{2} \mathrm{D}_{6}=13-11+9-7=4 \end{aligned}$ | 4 |
| $\mathrm{O}_{4} \mathrm{D}_{2}$ | $\begin{aligned} & \hline \mathrm{O}_{4} \mathrm{D}_{2}-\mathrm{O}_{4} \mathrm{D}_{1}+\mathrm{O}_{3} \mathrm{D}_{1}- \\ & \mathrm{O}_{3} \mathrm{D}_{3}+\quad \mathrm{O}_{2} \mathrm{D}_{3-} \\ & \mathrm{O}_{2} \mathrm{D}_{2}=10-8+8-11+9- \\ & 5=3 \\ & \hline \end{aligned}$ | 3 |
| $\mathrm{O}_{4} \mathrm{D}_{3}$ | $\begin{aligned} & \mathrm{O}_{4} \mathrm{D}_{3}-\mathrm{O}_{3} \mathrm{D}_{3}+\mathrm{O}_{3} \mathrm{D}_{1}- \\ & \mathrm{O}_{4} \mathrm{D}_{1}=13-11+8-8=2 \end{aligned}$ | 2 |
| $\mathrm{O}_{4} \mathrm{D}_{6}$ | $\begin{aligned} & \mathrm{O}_{4} \mathrm{D}_{6}-\mathrm{O}_{4} \mathrm{D}_{1}+\mathrm{O}_{3} \mathrm{D}_{1}- \\ & \mathrm{O}_{3} \mathrm{D}_{3}+\quad \mathrm{O}_{2} \mathrm{D}_{3-} \\ & \mathrm{O}_{2} \mathrm{D}_{6}=12-8+8- \\ & 11+9=3 \end{aligned}$ | 3 |

We got the all-primal costs to be positive values. Therefore, the solution is an optimal solution ( $\Delta$ is a very small value and it can be neglected).

Optimal solution cost =
$9 \times 5+3 \times 4+5 \times 2+6 \times 1+9 \times 1+6 \times 3+2 \times 2+2 \times 4=$ Rs. 212
This is the minimal cost we found from Stepping stone method. Further for two more cases as shown in Table 15 and Table 16 the minimum cost is calculated to show the scalability of the Stepping stone model. The transportation models are simulated using MATLAB tool and the following results are mentioned in Fig. 2. The cases taken are random values of supply and demand.

### 3.3 Comparative Study using Computer Simulation

 In this study, MATLAB tool is used to implement two methods to test the optimality for drug delivery using two cases as discussed below.Case 1: In this case the number of destinations is 4 and sources are 2 and the supply and demands are shown in Table 15 as follows. From the results it is observed that Vogel method shows a cost of 3900 and Stepping stone method shows a minimum cost of 3650 (Fig. 2).

Table 15: Case 1 for simulation.

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 10 | 13 | 16 | 13 | 150 |
| $S 2$ | 15 | 18 | 13 | 9 | 200 |
| Demand | 100 | 125 | 175 | 150 |  |

Case 2: In this case the number of destinations is 7 and sources are 5 and the supply and demands are shown in Table 16 as follows. From the results it is observed that Vogel method shows a cost of 12965 and Stepping stone method shows a minimum cost of 11740 , it is also well represented in Fig. 2.

Table 16: Case 2 for simulation.

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | $D 5$ | $D 6$ | $D 7$ | Suppl <br> y |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 10 | 13 | 16 | 13 | 18 | 20 | 31 | 150 |
| $S 2$ | 15 | 18 | 13 | 9 | 14 | 14 | 15 | 200 |
| $S 3$ | 23 | 12 | 13 | 14 | 15 | 16 | 17 | 250 |
| $S 4$ | 12 | 13 | 23 | 24 | 25 | 26 | 27 | 265 |
| $S 5$ | 23 | 28 | 28 | 29 | 12 | 11 | 7 | 235 |


| Deman <br> d | 10 <br> 0 | 12 | 17 | 15 | 16 | 15 | 13 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 5 | 0 | 5 | 0 | 0 |  |  |  |



Fig. 2: Simulation of above two cases using MATLAB to compute the minimum transportation cost.

## 4. Conclusion

In this work, we use Vogel's method and Stepping Stone method as a case study of different drugs to be shipped from the drug factories to the warehouses, so that cost of the transportation is minimum. It also explains the degeneracy in the transportation techniques which are fixed in an optimized way by giving the input data. We analyze the total effectiveness of transportation problem by considering a case where there are four drug factories and six warehouses. It is solved using the above methods. We found the optimal cost to be Rs 212, however the degeneracy is reduced in Stepping stone method. So, from the results and discussion it is found that Stepping stone method will be a better solution for solving the transportation problems of drug delivery or other products delivery

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## Conflict of Interest

There is no conflict of interest.

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