

Improving of Swing up Motion Control Parameters for a Gymnastics Robot Using the Gray Wolf Algorithm

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Abstract: This paper focuses on controlling the swing of a robot consisting of three joints that features free movement of the first joint that causes the system to be both nonlinear system and to develop complex motion. The resulting robot gymnast thus simulates a human trying to swing smoothly from a stable (lower) position to an unstable (upper) position by feeding power to the shoulder and hip joints. The main purpose of this paper is to determine how best to adjust the control signal input to the DC motors in the robot's "shoulder" and "hip" in order to move these joints into a vertical balanced plane. The Gray Wolf Algorithm (GWO) was adopted as a novel optimisation technique to calculate the optimal values for simulation of the behaviour of the robot in the swing phase, and the ensuing experimental and simulation results suggested that this was successful in managing the robot's swing.

Keywords: *Gymnastic Robot, Gray Wolf Optimization, Inverted Pendulum, Swinging-up Control.*

1. Introduction

This paper deals with the problem of controlling swing in a three-link gymnastic robot. In this case, the first link is negative (non-powered), while the other links are powered [1, 2, 3]. In [4], it proposes the use of a hybrid non-linear controller in such cases, which involves integrating the controller to swing the inverted pendulum upward to the vertical position. In [5], it is proposed expounds the obtain an optimal strategy for controlling a two-degree-of-freedom planar robot with a single actuator by exploring a two-joint planar robot equipped with a magnetic handle designing to add energy to the system to facilitate movement of the robot parts. And other scholars have studied the design a control unit for a non-linear inverted pendulum that would enable the pendulum to swing freely and then be fixed at the equilibrium point [6, 7]. As more and more researchers academic understand the properties of non-linear inverted pendulum control strategies and operating limitations, and ways to control the swing of double or triple inverted pendulum without much external interference and ensure the reach a balanced vertical position [8- 11]. João M. Lopes. [12] used a model for a three-jointed non-linear inverted pendulum, using Lagrangian equations and dynamics to achieve significant

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motion. In [13- 15], it is proposed the non-linear inverted double pendulum swing models, with attempts to stabilise these in the vertical plane made by using Lagrangian dynamics and equations to achieve the swing motion. In [16], proposes to determine a control scheme for a sequential upward inverted double pendulum across three steps, first step is swing the first pendulum, second step is swing of second pendulum accompanied by the fixation of the first one, and the third step is the fixation of all pendulums around the point of origin. In [17, 18], the problem of inverted pendulums and their frequently changing symbolic systems has encouraged the development of new ways of modelling and controlling such systems. In [19], it is proposed the three-link robot swinging upwards and attempting to determine the optimal input control by using PSO technique to find the optimal parameters of swing control, and the main strength of the study was the short time taken to reach the upper position (9.25V). Dung-Han Lee. [20] used an simulation and experimental technology to determine the parameters required and the dynamics needed to move the robot's three links effectively. In [21], was proposed to present a model for designing a multi-fingered robotic hand using a DC motor to perform grasping tasks.

In similar work [22, 1], a bee algorithm (BA) used to tune the parameters of input control to swing the robot gymnast. Gray Wolf Optimization (GWO) Algorithm becoming one of the most important optimization algorithms that used to find the optimal solutions, being introduced by [23, 24, 25] to solve various optimization problems by applying a hierarchy similar to that seen in grey wolves. This is thus applied in the current paper.

The rest of the paper is arranged as follows, discussion the dynamic and mathematical modelling of the system in Section 2. Examines the robot swing control problem in Section 3. Introduces the Gray Wolf Optimization technique to solve the problem of controlling the swing in Section 4. Tuning the coefficients of swing of the system using the GWO algorithm in Section 5. Section 6 discusses the simulation results. Finally, Conclusion and Further Work in Section 7.

2. Robot System and Dynamics

The schematic diagram shown in Figure.1, depicts a three-joint robot that resembles gymnastics human that swing

on a bar high that freely rotates from 0 to 360° in complete revolutions; this features two powered joints and one un-powered free-moving joint, where the upper joints is represented by arms, minus the elbow and wrist joints. The middle section is the trunk, head, and neck. The lower joints is represented by legs, neglecting the knee and ankle joints. A potentiometer is installed in the first joint, while not installed to the second and third joints that represented by shoulder and hip joints in a gymnastics human, a DC motor and sensor (potentiometer) is installed in each, measuring the angles between adjacent links. Table 1 defines the Vocabulary of System Symbols used in this work.

Table 1. Vocabulary of System Symbol

Symbols	Vocabulary
l_n	length of link n
a_n	the centre of gravity of link n
m_n	mass of link n
i_n	moment of inertia of link n
θ_i	measured angle between link n and the vertical line

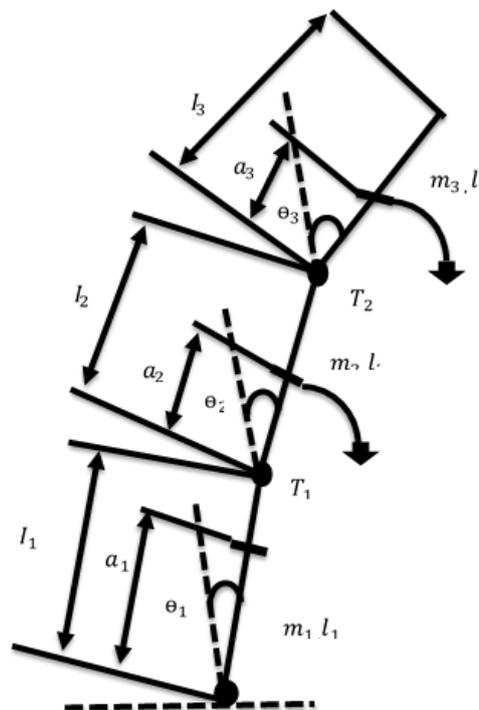


Fig. 1 Schematic Diagram of a Robot Gymnast

The kinematic equations for the three-jointed nonlinear mathematical model seen in figure 1 can be derived using Lagrange equations [1-2-3] to describe the necessary differential equations. The equations related to the angles of the robot are then solved to create the system dynamics

$[\theta_1 \ \theta_2 \ \theta_3]$. A linear continuous time model is achieved by considering the system in the vertical position ($\theta_1 = \theta_2 = \theta_3=0$). The state space representation of the system [1], thus takes the form:

$$\dot{x}=Ax + Bu \quad (1)$$

$$y= Cx +Du \quad (2)$$

Where

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -36.42 & -0.35 & 0.21 & -0.20 & 88.38 & 9.17 \\ 13.10 & -22.06 & -2.23 & 0.20 & -168.29 & 7.70 \\ 2.14 & -1.50 & -5.68 & 0.02 & 7.69 & -201.45 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -15.19 & -0.74 \\ 28.92 & -0.62 \\ -1.32 & 16.21 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad D=0, \quad x = \begin{bmatrix} \theta_1 \\ \theta_2 - \theta_1 \\ \theta_3 - \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 - \dot{\theta}_1 \\ \dot{\theta}_3 - \dot{\theta}_2 \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad y = \begin{bmatrix} \theta_1 \\ \theta_2 - \theta_1 \\ \theta_3 - \theta_2 \end{bmatrix}$$

3. Control Problem

The main challenge in controlling this type of robot gymnast lies in determining how to make it swinging from the lower stable to an upper unstable position, which requires determining the appropriate swing time to make the angle of the first joint reach the vertical position ($\theta_1 = \pm 180$) without risking damage to the components of the robot structure. The solution for this type of movement determines the motor control signals in joints 2 and 3, and this must be achieved based on the value of the first angle ($p_1 = (\theta_1)$), that is, from a steady state $p_1 = 0$ to vertical position, whether that is $p_1 = \pi$ or $p_1 = -\pi$. The input control signals [1], are given in the equations as

$$u_1 = A_1 \alpha \sin(\phi_1) \quad (3)$$

$$u_2 = A_2 \alpha \sin(\phi_2) \quad (4)$$

Where A_1, A_2 are constants used to create movement between position 1 and 2. Were initially $A_1 = 3, A_2 = 2.5, \alpha = 1, \eta = 0.3142$ and $\delta = 1$ (E. E. Eldukhri and H. G. Kamil, 2015). The control signals and frequencies must be equal ($\phi_1 = \phi_2$), and both ϕ_1 and ϕ_2 are thus dependent on δ , such that

$$\phi_1(K) = \phi_2(K) = \phi_1(K-1) + \pi / \delta \quad (5)$$

Continuous time model at the selected sample time ($T_s = 0.025$) allows δ and α to be calculated using two equations:

$$\delta(k) = \delta(k-1) + \Delta\delta \quad (6)$$

$$\alpha(k) = \alpha(k-1) + \Delta\alpha \quad (7)$$

In this study, the most critical aspects are how to reduce the saturated control signal in DC motors to allow the robot to reach the vertical position in the least amount of time without the robot structure being damaged and with the appropriate values of frequencies and amplitudes achieved at each sample time. The manual calculation required to achieve this would be a very tedious task; In this work, the Gray Wolves Optimization (GWO) algorithm applied to find the ideal values for the necessary increments in $\Delta\delta$ and $\Delta\alpha$.

4. Gray Wolf Optimisation Algorithm

The GWO algorithm is intelligence swarm mechanism that simulates wolf hunting mechanisms seen in nature; it has become a widely used search technique for computing near-perfect solutions to optimisation problems [23, 24], as its features include a minimal need for tuning. Four groups are used, identified as alphas (leaders), betas (subordinate), deltas, and omegas (lower orders), which are used to simulate a hierarchy of command [26, 27, 28, 29]. This type of algorithm have also a special hunting strategy. They track the prey and approaching them. Then they encircle, harass and pursue the prey until it stops moving. Step finally they attack the prey. Detailed information of the mathematical model of the GWO algorithm can be found in [30]. The GWO algorithm required number of parameters to be set namely: number of population size, number of iteration, knowledge the search space, and $a = 0$ to 2. The algorithm was used to optimization the increments $\Delta\delta, \Delta\alpha$ and attaining reasonably swing-up of gymnastics robot smoothly and that is discussed in the following section. The main steps of this algorithm are shown in Figure 2 (Pseudo code for GWO Algorithm [30]).

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Initialize the grey wolf population  $X_i$  ( $i=1, 2, \dots, n$ )
Initialized values of a, A, C
Calculate the fitness of each search agent
-  $X_\alpha$  = the best solution in the search agent of algorithm
-  $X_\beta$  = the second best solution in the search agent of algorithm
-  $X_\delta$  = the third best solution in the search agent of algorithm
While ( $t < \text{Max number of iterations}$ )
  For each search agent
    Update the position of the current search agent
  End for
  Update a, A, C
  Calculate the fitness of all search agents
  Update  $X_\alpha, X_\beta,$  and  $X_\delta$ 
   $t=t+1$ 
end while
return  $X_\alpha$ 

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Fig. 2 Pseudo code for GWO Algorithm [30].

5. Controlling the Tuning of Swing Coefficients Using the GWO algorithm

In this paper, the determining the optimal tuning of the control unit by finding the optimal values for the increment in α ($\Delta\alpha$), and increment in δ ($\Delta\delta$) for use in equations (1) and (2), as calculated using the GWO algorithm. This was intended to achieve a smooth swing

of a three-link gymnastic robot in a reasonable time, with the control signal used to maintain the amplitudes and frequencies in equations (3) and (4). The range of the parameters values of the GWO algorithm were show in Table 2. Several averages of the resulting $\Delta\alpha$ and $\Delta\delta$ emerged where 30 random values were used initially and the conditions $0.3 < \Delta\alpha < 0.6$ and $3 < \Delta\delta < 6$ were applied.

Table 2. GWO algorithm parameters.

N	D	a
30	2	2

The select random values of $\Delta\delta$ and $\Delta\alpha$ ensure that at every change in time point, the GWO algorithm code is re-executed. For each chosen $\Delta\delta$ and $\Delta\alpha$, Table 3 show how the gymnast robot took swing long time to the upright

position ($\theta_1 = -180$) and how to determine the category of each solution according to the criteria margin of error and duration time (it means the time it reaches the vertical position).

Table 3. An initial population of randomly solution to find the fitness of the sites.

$\Delta\alpha$	$\Delta\delta$	The angle of the first link position (θ_1 Deg).	Time (sec) take to reach an upward position
0.3507	4.9473	-180.2031	125.1500
0.3352	3.8900	-180.2345	121.1250
0.3786	4.0061	-180.1055	123.3500
0.3567	5.0603	-180.4408	127.6500
0.3514	5.1181	-180.0462	129.0250
0.3488	3.3570	-180.2130	124.4000
0.3390	4.7065	-180.0617	120.2000

0.3458	4.0234	-180.1785	124.4500
0.3323	5.7189	-180.1826	143.1000
0.3536	4.2687	-180.4227	131.0500
0.3101	3.2064	-180.1198	139.4500
0.3290	5.4544	-180.0870	137.1750
0.3274	4.2159	-180.0061	130.1250
0.3573	4.3276	-180.0019	111.4250
0.3095	3.8308	-180.1458	140.8750
0.3103	4.3162	-180.0898	133.4000
0.3251	3.6869	-180.3926	135.7750
0.3149	5.7081	-180.0345	143.2500
0.3088	5.7866	-180.0273	145.2000
0.3086	4.4697	-180.0445	137.6000
0.3334	5.3408	-180.3839	134.4750
0.3228	3.7197	-180.2824	136.9250
0.4187	4.1944	-180.4263	106.9000
0.3143	4.0464	-180.4056	147.8750
0.3162	4.5924	-180.3021	140.7750
0.3107	5.5474	-180.2210	139.8750
0.4178	5.3073	-180.0092	108.3500
0.3056	5.0243	-180.1022	153.0000
0.3323	5.7189	-180.1826	143.1000
0.3536	4.2687	-180.4227	131.0500

The Flowchart in Figure. 3, explains several of steps of using the Gray Wolves Algorithm (GWO) to optimality the parameters of ($\Delta\alpha$, $\Delta\delta$) for attainable reasonably smoothly control swing up of Gymnastics Robot.

6. Results and Discussion

The results displayed in Table 4, were used by the GWO algorithm to tune the values of the $\Delta\delta$ and $\Delta\alpha$ to achieve the ideal values, defined as those where $\theta_1 = -180$ and the margin of error rate is < 0.01 . Duration times of between 120 and 130 seconds caused the algorithm to stop the search.

Table 5 shows the results implemented to affect robot's behavior during the swing process: three selected values for the $\Delta\delta$ and $\Delta\alpha$ are thus identified.

According to comparing the simulation results with the results reported in [1]. The system succeeded in obtaining a satisfactory response to reach the vertical position in a

The control input signals u_1 and u_2 were recalculated as per equations (3) and (4) for each sampling period, and the calculated values then applied in equations (1) and (2).

This shows that the values of 0.4187 and 4.1944 for the time it takes for the robot to swing vertically can cause damage to the robot's chassis; however, when $\Delta\alpha$ equal to 0.3056 and $\Delta\delta$ equal to 5.0243, the robot moves smoothly but takes a very long time to reach the vertical position (153 seconds) as shown in Fig. 5. When a value of $\Delta\alpha$ equal to 0.3280 and value of $\Delta\delta$ equal to 3.9221 offer the most satisfactory response in terms of reasonable duration (122.2750 seconds) and minimal damage, as shown in Figure. 6. To examine the behaviour of a gymnastic robot during the phase of swinging up, three values of $\Delta\alpha$ and $\Delta\delta$ selected from Table 5 that show in Figure 4-6.

reasonable duration time (122.2750 seconds) as explain in Figure.6 and with minimum margin of error when use the GWO algorithm.

Table 5. GWO Results.

$\Delta\alpha$	$\Delta\delta$	The angle of the first link position (θ_1 Deg).	Time (sec) take to reach an upward position
0.3283	4.7956	-180.0581	122.6000
0.3450	4.9788	-180.0690	126.0250
0.3280	3.9221	-180.0004	122.2750
0.3352	4.9222	-180.0044	124.9750
0.3672	5.0035	-180.0359	126.1250
0.3871	4.8513	-180.1883	122.3750
0.3358	5.8195	-180.0055	120.1250
0.3782	4.7831	-180.4173	120.9250
0.3323	5.7189	-180.1826	143.1000
0.3536	4.2687	-180.4227	131.0500
0.3101	3.2064	-180.1198	139.4500
0.3290	5.4544	-180.0870	137.1750
0.3274	4.2159	-180.0061	130.1250
0.3573	4.3276	-180.0019	111.4250
0.3095	3.8308	-180.1458	140.8750
0.3103	4.3162	-180.0898	133.4000
0.3251	3.6869	-180.3926	135.7750
0.3149	5.7081	-180.0345	143.2500
0.3088	5.7866	-180.0273	145.2000
0.3086	4.4697	-180.0445	137.6000
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0.3228	3.7197	-180.2824	136.9250
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0.3143	4.0464	-180.4056	147.8750
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0.4178	5.3073	-180.0092	108.3500
0.3056	5.0243	-180.1022	153.0000
0.3323	5.7189	-180.1826	143.1000
0.3536	4.2687	-180.4227	131.0500

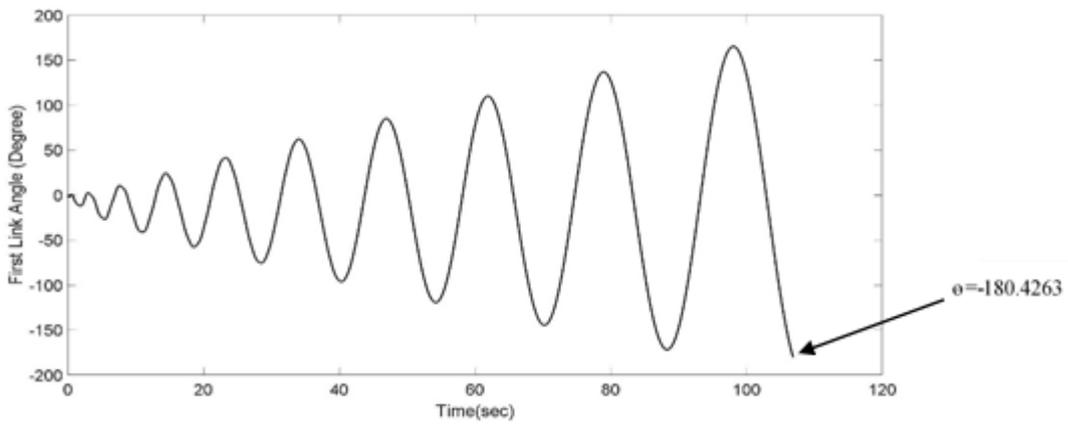


Fig. 4 Simulated angular position θ_1 at $\Delta\alpha=0.4187$ and $\Delta\delta= 4.1944$

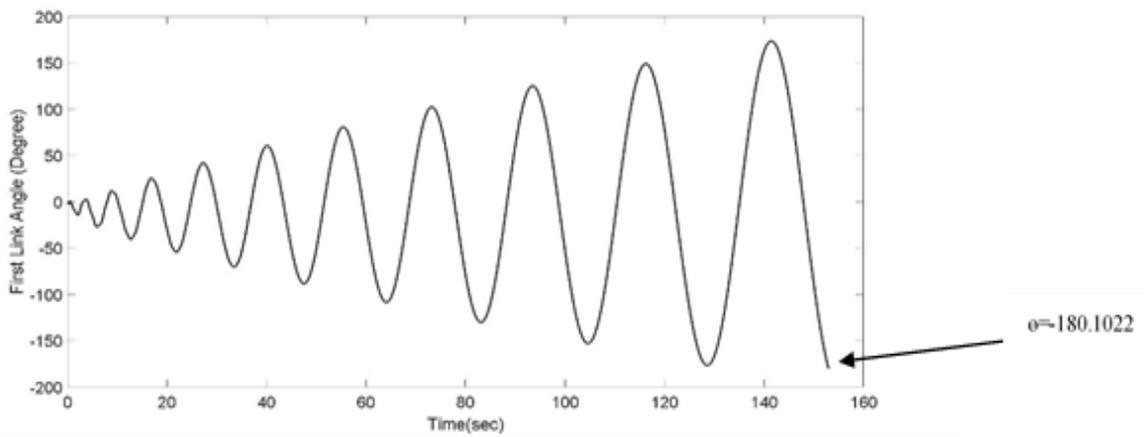


Fig. 5 Simulated angular position θ_1 at $\Delta\alpha=0.3056$ and $\Delta\delta= 5.0243$

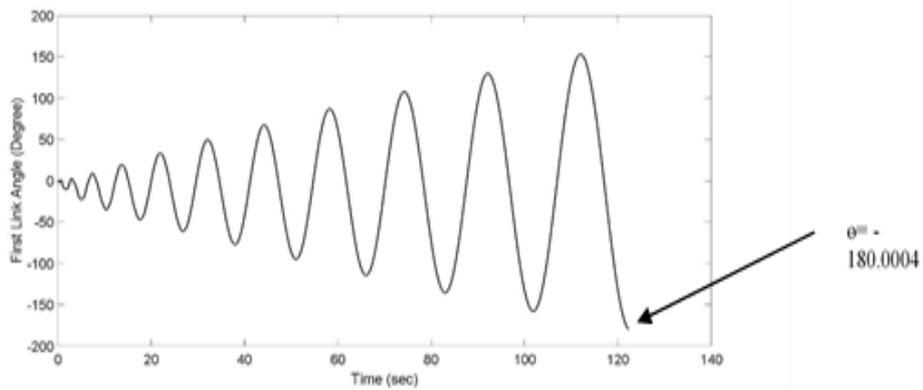


Fig. 6 Simulated angular position θ_1 at $\Delta\alpha=0.3280$ and $\Delta\delta= 3.9221$

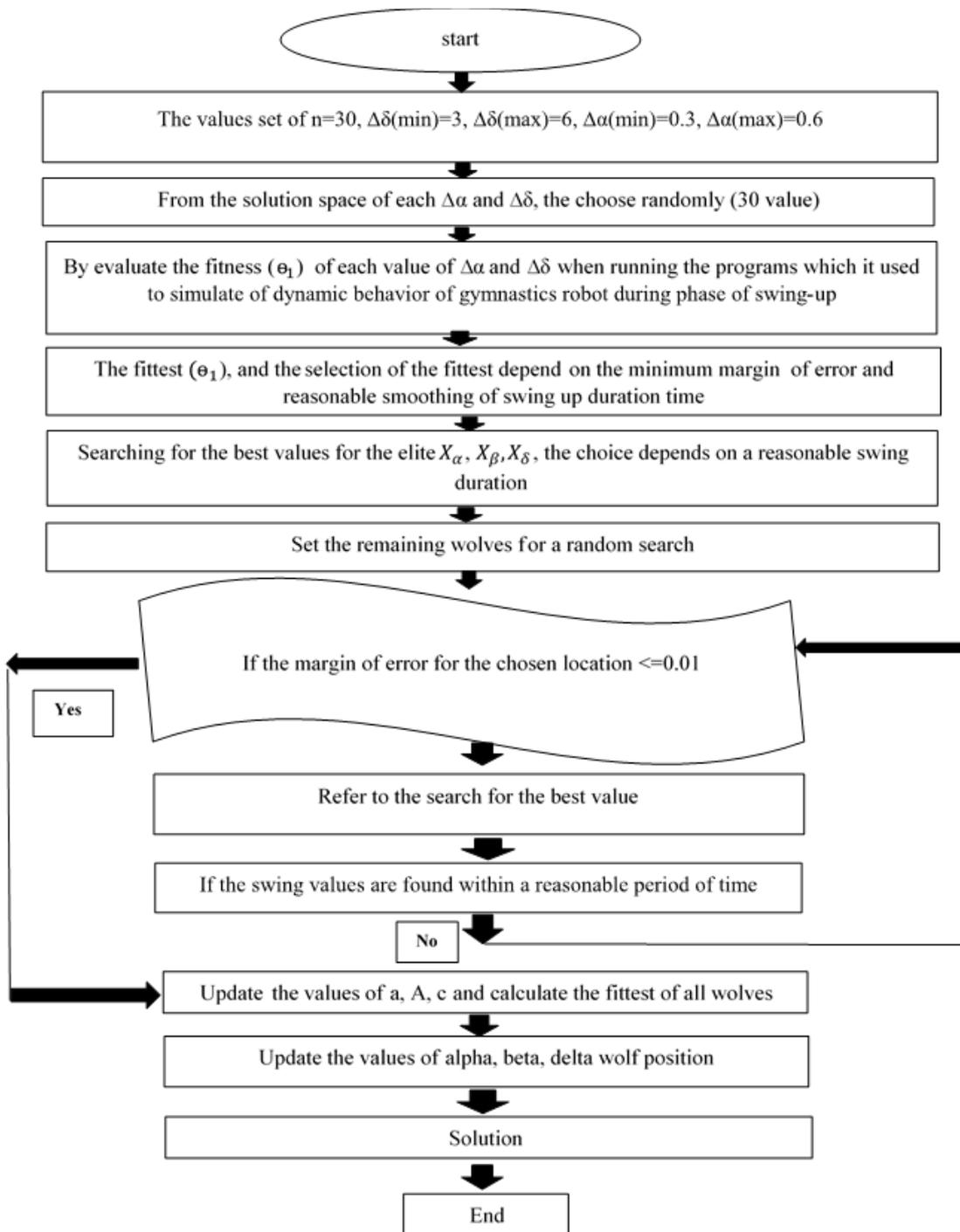


Fig. 3: Flowchart of the swing up control parameter optimization using the Gray Wolves Algorithm.

7. Conclusion and Further Work

The paper aimed to find a way to optimally control the inputs that allow a three-jointed robot to oscillate smoothly to the vertical plane in the least possible time. A modern algorithm was used to achieve this. In contrast to the previous optimization methods, the Gray Wolves Algorithm allowed good flexible selection and randomizer of values for parameters that affect the frequencies applied to the two DC motors to move the robot. Experimental and simulations results showed improvements in the response of the gymnastic robot with significantly reduced swing up time comparing to the

results that reported in [1]. Future work may, however, need to be undertaken using a different algorithm, to ensure that the results can be implemented across the entire system.

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