

Modified Reptile Search Optimization Based Analysis of Inventory Framework With Interval-Valued Inventory Expenses

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Abstract: This study presents an inventory model that incorporates interval-valued inventory expenses and the effect of pre-payment (PP). The PP is a fixed proportion of the cycle's total procurement expense; this results in a discount on procurement expense but a loss of interest on the PP. It is assumed that the inventory expenses, including ordering, purchase, shortage, and carrying expenses, are interval-valued. We examine two scenarios: one with no shortages and the other allowing for partially backlogged deficiencies. Due to loyal customers and some customers transferring shops, the demand rate is projected to fall over a limited interval in the second case. Both cases use interval arithmetic to design mixed integer restricted optimization issues with interval targets. We proposed a Modified Reptile Search Optimization (MRSO) algorithm to tackle these issues. The suggested model is demonstrated numerically, and sensitivity analysis are carried out to determine the effects of various inventory factors on optimal profit. The inventory model with interval-valued (IV) expenses can be solved most effectively using the MRSO method. The results emphasize PP, IV expenses, and shortage cases in inventory management. The model and MRSO algorithms assist decision-makers in improving their inventory strategies and optimizing profitability under uncertain expense and demand conditions.

Keywords: Inventory model, advance payment, IV expenses, Modified Reptile Search Optimization (MRSO)

1. Introduction

Inventory management involves overseeing and controlling the acquisition, storage, and utilization of a business or organization's materials, goods, and products. An effective inventory model gives businesses insights and strategies to optimize inventory levels, streamline operations, and enhance overall performance [1]. An inventory model aims to determine the optimal inventory policies that minimize expenses while ensuring adequate stock availability. These policies include order quantities, reorder points, lead times, and replenishment strategies. By employing mathematical modeling and optimization techniques, businesses can analyze these factors and make informed decisions to achieve optimal inventory management [2].

Inventory models take into account several important considerations. These include demand patterns, such as seasonal fluctuations, trends, and variability, which affect

the quantity and timing of inventory replenishment. Additionally, expenses associated with inventory play a significant role in the model. These expenses encompass holding expenses, ordering expenses, stock-out expenses, and other relevant expenses that directly impact the overall inventory investment [3,4]. However, traditional inventory models often assume deterministic expenses, overlooking the inherent uncertainty associated with inventory expenses. Inventory expenses can fluctuate within certain intervals due to various factors such as market conditions, supplier pricing, and economic fluctuations.

This paper addresses these challenges by developing an inventory model that incorporates the effect of pre-payment (PP) and considers IV inventory expenses. PP is a proportion of the total purchasing expenses every cycle paid beforehand to receive a price reduction. While this discount is advantageous, the opportunity expenses of the interest on the advanced payment must also be considered. Additionally, the inventory expenses, including carrying, ordering, purchase, and shortage expenses, are considered to be interval-valued, acknowledging the uncertainty associated with these expenses.

Two cases are studied in this research. The first case focuses on inventory management without shortage, where the objective is to optimize inventory policies to minimize expenses and maximize profitability. In the second case,

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partially backlogged shortages are permitted, considering the impact on the demand rate during the shortage period. During a time of shortage, only loyal consumers would wait for subsequent purchases from the business, while a portion of customers may switch to other shops.

To tackle these inventory optimization problems, mixed integer-constrained optimization models with interval objectives (IO) are formulated using interval arithmetic. To solve these models, a Modified Reptile Search Optimization (MRSO) algorithm is developed, inspired by the search behavior of reptiles. The MRSO algorithm is designed to handle IV expenses and uncertainties in inventory optimization.

2. Related Works

In a study [5], a novel mathematical framework for multi-site ordering issues with an “all-unit discount policy and multi-mode RCPSP (MRCPSP)” was suggested. The model reduces project expenses and duration. A hybrid multi-objective uncertainty strategy for IV fuzzy mathematical model was also offered. Study [6] examines the benefits of simultaneously investing in “greening innovation (GI) and emissions-reducing technology (ERTs)” in a greenery production inventory model with carbon emission parameters and IV inventory expenses components. ERT investment determines the model. Both approaches consider defective manufacturing and reworking. The quantum-behaved particle swarm optimisation method achieves the IV optimal profit.

The study [7] has three phases. In the first step, a novel “interval-valued fuzzy (IVF) determines provider and circular economy grade. In the second step, a “multi-objective mathematical model” reduces expenses, environmental consequences, and social sustainability. IVF-robust solution technique addresses mathematical model instabilities. In the third step, the solution is combined with the “AUGMECON2” approach to providing Pareto-optimal solutions that balance expenses, emissions, and social accountability. IV functional clustering using Wasserstein distance is proposed in [8]. This distance completely exploits the data patterns by considering the data distribution's core and spread. This study also suggests using the IV functional entropy technique to thoroughly explain phenomena with numerous variables. Finally, a stock market example demonstrates the suggested method's efficacy and superiority.

In research [9], neural networks estimate the shortest and highest daily prices and then develop a daily scalping trading system that purchases and sells in the anticipated quantities. The most effective mean-variance daily scalping trading portfolio reduces trading system risk and increases trading positions. Paper [10] develops an

incomplete production inventory model under various carbon emission regulation strategies using interval numbers for carbon emission features. Four inventory models are established based on four policies: basic tax policy, cap and buy policy, cap and incentive policy, and strictly within authorised cap policy. The manufacturer's optimal profit was found using the “Quantum behaved Particle Swarm Optimisation (QPSO)” method.

The study's major goal [11] was to examine how product warranties affect the production firm's optimal strategy. This paper also examines how carbon pricing legislation affects manufacturing income. Centre-radius optimisation solves interval optimisation problems. Green production in interval environments was investigated in [12]. Due to IV demand and faulty rates, the suggested model's differential equations of inventory levels have an interval form. Interval order relations and centre-radius optimisation are employed to organise the optimisation issue into crisp form, and various metaheuristic algorithms are utilised to solve it. Study [13] used intervals for inventory parameters. They created a two-warehouse inventory model with advanced payment and partly backlogged shortages using this concept. Direct/indirect optimisation cannot handle this problem due to uncertainty. For this, several particle swarm optimisation approaches have been developed to tackle the suggested inventory model's problem, utilising interval arithmetic and interval order relations.

3. Method

3.1. Bounded intervals arithmetic

A closed interval that is specified by either its left and right bounds or its centre and radius is referred to as an interval-valued (IV) number.

$$B = [b_{LT}, b_{RG}] = \{y: b_{LT} \leq y \leq b_{RG}, y \in S\} \text{ or } = \langle b_d, b_x \rangle = \{y: b_d - b_x \leq y \leq b_d + b_x, y \in S\} \quad (1)$$

Here,

b_{LT}, b_{RG} – Right and left limits, respectively,

$b_d = (b_{LT} + b_{RG})/2$ and $b_x = (b_{RG} - b_{LT})/2$ – radius and centre of the interval,

S – set of the real number

For example, for any $y \in S$, x may be expressed as an interval $[y, y]$ with zero width. Each real number can be thought of specifically as an interval. Here, we will provide brief descriptions of the four interval arithmetical operations used in every standard interval analysis.

The definitions and concepts related to interval arithmetic are as follows:

Interval Arithmetic Operation: For a binary operation

(*) such as addition (+), subtraction (-), multiplication (\cdot), or division ($/$) on the set of real numbers, the operation can be extended to closed intervals B and C as follows:

$$B * C = \{b * c : b \in B \text{ and } c \in C\} \quad (2)$$

Interval Addition: For two interval numbers $B = [b_{LT}, b_{RG}]$ and $C = [c_{LT}, c_{RG}]$, the addition of intervals B and C is defined as $B + C = [b_{LT} + b_{RG}, c_{LT} + c_{RG}]$

Interval Subtraction: For two interval numbers $B = [b_{LT}, b_{RG}]$ and $C = [c_{LT}, c_{RG}]$, the subtraction of interval C from interval B is defined as $B - C = [b_{LT} - b_{RG}, c_{LT} - c_{RG}]$ (3)

Interval Scalar Multiplication: For an interval number $B = [b_{LT}, b_{RG}]$ and a scalar value λ , the scalar multiplication of interval B by λ is defined as:

$$\lambda B = [\lambda b_{LT}, \lambda b_{RG}] \text{ if } \lambda \geq 0 \quad (4)$$

$$\lambda B = [\lambda b_{RG}, \lambda b_{LT}] \text{ if } \lambda < 0 \quad (5)$$

Order Relations of Interval Numbers: In the context of optimization problems, when comparing two interval numbers $B = [b_{LT}, b_{RG}]$ and $C = [c_{LT}, c_{RG}]$, the following types of interval relationships can occur:

Type-1: Disjoint Intervals: B and C have no overlap.

Type-2: Partially Overlapping Intervals: B and C have some overlap but are not contained in each other.

Type-3: One Interval Contained in the Other: One interval is completely contained within the other interval.

These definitions and types of interval relationships are relevant for analyzing and solving optimization problems involving IV parameters, such as in the context of the inventory system discussed in the paper.

3.2. Optimistic decision-making

In the context of maximization problems and considering the order relation \geq_{omax} defined in Definition 2, the decision maker's approach is to choose the interval with the highest profit without taking uncertainty into account. The order relation \geq_{omax} is defined as follows:

$B \geq_{omax} C$: Interval B is considered superior to interval C if the right limit of B (b_{RG}) is greater than or equal to the right limit of C (c_{RG}). In other words, if the potential upper bound of B 's profit is greater than or equal to the potential upper bound of C 's profit.

$B >_{omax} C$: Interval B is strictly superior to interval C if $B \geq_{omax} C$ and B is not equal to C . This means that B has a higher potential upper bound of profit compared to C and is preferred by the optimistic decision maker.

It's important to note that the order relation \geq_{omax} is not symmetric, meaning that if B is considered superior to C , it does not necessarily mean that C is considered superior to

B . However, the relation is transitive.

In optimistic decision-making for maximization problems, the decision-maker focuses on the potential high-end outcomes and chooses the interval with the highest profit, ignoring the uncertainty associated with the intervals. This approach assumes that the optimal-case scenario will materialize and seeks to maximize the potential gains without considering the potential downside or risks.

3.3. Pessimistic decision-making

In the given context, the decision maker aims to maximize profit and follows the principle that "Less uncertainty is better than more uncertainty" or "More uncertainty is worse than less uncertainty." Based on this principle, a specific order relation is defined for maximization problems denoted as $>_{pmax}$, which allows the decision maker to compare and prioritize different intervals.

For type 1 and type 2 intervals, the order relation $>_{pmax}$ is determined by comparing the central values (b_D and c_D) of the intervals. If the central value of interval B is greater than the central value of interval C ($b_D > c_D$), then B is considered greater than C ($B >_{pmax} C$).

For type 3 intervals, the order relation $>_{pmax}$ is determined by considering both the central values (b_D and c_D) and the widths (b_x and c_x) of the intervals. If the central value of interval B is greater than or equal to the central value of interval C ($b_D \geq c_D$), and the width of interval B is strictly less than the width of interval C ($b_x < c_x$), then B is considered greater than C ($B >_{pmax} C$).

However, there is a scenario where the order relation cannot be determined using the pessimistic approach for type-III intervals. This occurs when both the central value and the width of interval A are greater than the corresponding values of interval C ($b_D > c_D \wedge b_x > c_x$). In this case, the pessimistic decision cannot be made based on the given principle. Instead, an optimistic decision may be considered, implying that a different approach or criteria may need to be employed to make the decision.

It's important to note that this approach and order relation is specific to the given context and principle stated in the question. Different decision-makers or decision contexts may have alternative principles or criteria for making decisions in maximization problems.

3.4. Assumptions

We are able to make additional improvements to the assumptions that are used in the numerical model of the suggested inventory system:

1. Single-item inventory: The system deals with only one type of item.

2. Single batch delivery: At the initial stage of each cycle, there is a single order that is placed, and the entirety of that order's quantity arrives in a single batch.

3. Finite replenishment size: The size of the order amount or replacement is limited and specified.

4. Fixed lead time: The lead time for replenishment remains constant and is known in advance.

5. Partial backlogging: Shortages are allowed in the system, and they are partially backlogged. This means that if demand exceeds the available inventory, some portion of the demand is backlogged while the rest is considered lost sales.

6. Finite planning horizon: The inventory planning horizon (PH) is finite and sufficiently longer than the lead time, indicating the time period over which the inventory system is analyzed and managed.

7. Advance payment and discounts: A specific portion of the overall purchase expenses every cycle is paid in advance, and this advance payment entitles the buyer to a certain portion of the unit purchase expense as a discount.

8. Known intervals for inventory expenses: The expenses associated with the inventory system, such as holding expenses, purchase expenses, ordering expenses, and shortage expenses, are known and defined within specific intervals.

9. Uniform demand rate with varying rates during stock-outs: Except for stock-out instances, when the customer demand rate may vary, the demand rate is expected to remain constant.

3.5. Model description in arithmetic

During the process of developing the model, it was assumed that the company had an initial inventory level of R units for the item. This was done so that the model could be properly validated. When the quantity of goods available meets R_a (at times $= t_1$), a fresh order is positioned for the subsequent cycle. During the lead time, the demand is equal to E_y . Two cases can arise based on the relationship between R_a and E_y :

Case 1: $R_a \geq E_y$

Case 2: $R_a < E_y$

Case 1

In the given scenario, Case-1 states that R_a (the reorder point) is greater than or equal to E_y (the demand during the lead time). This implies that there will be no shortage during the lead time. The total holding expenses $I_1(y)$ over the PH I is given by the following equation:

$$I_1(y) = D_i \left[\int_0^{t_1} r dt + \right.$$

$$\left. \sum_{k=1}^{n-1} \int_{t_k}^{t_{k+y}} r dt + \sum_{k=1}^{n-1} \int_{t_{k+y}}^{t_{k+1}} r dt + \int_{t_n}^I r dt \right] \quad (6)$$

$$= D_i \left[\int_{R_a}^{R_a} \frac{-r dt}{E} + (n-1) \int_{R_a}^{R_a+Ey} \frac{-r dt}{E} + (n-1) \int_{R_a+Ey}^{R_a} \frac{-r dt}{E} + \int_{R_a}^0 \frac{-r dt}{E} \right]$$

$$= \frac{D_i}{2D} [nR^2 + 2(n-1)\{R_a - Ey\}] \quad (7)$$

Here, R is calculated as EI/n , and P_p is defined as $J_d(1 - J_d)RD_q$.

The total profit over the PH I is given by the equation:

$$B_1 = nQR - nRD_q(1 - J_d) - (n-1)J_dRD_q(1 - J_d)yJ_c - nD_o - \frac{D_i}{2D} [nR^2 + 2(n-1)(R_a - Ey)] \quad (8)$$

The IV expenses parameters D_q , D_o , and D_i are considered, and the total profit B lies within the interval $[B_{LT}, B_{RG}]$.

$$B_{1LT} = nQR - nRD_{qRG}(1 - J_d) - (n-1)J_dRD_q(1 - J_d)yJ_cD_{qRG} - nD_{oRG} - \frac{D_{iRG}}{2D} [nR^2 + 2(n-1)(R_a - Ey)] \quad (9)$$

And

$$B_{1RG} = nQR - nRD_{qLT}(1 - J_d) - (n-1)J_dR(1 - J_d)yJ_cD_{qLT} - nD_{oLT} - \frac{D_{iLT}}{2D} [nR^2 + 2(n-1)(R_a - Ey)] \quad (10)$$

The lower bound of B_1 , B_{1LT} , is obtained by substituting the interval lower bounds of D_q , D_o , and D_i into the equation for B_1 . Similarly, the upper bound of B_1 , B_{1RG} , is obtained using the interval upper bounds of D_q , D_o , and D_i .

Thus, the optimization problem becomes:

Maximize $B_1(R_a, n)$ subject to $R_a \geq E_y$, where n is an integer.

This problem is a "non-linear maximization problem" with an IO. The objective is to maximize the total profit over the planning horizon, subject to the constraint that the reorder point R_a must be greater than or equal to the demand during the lead time E_y . The solution to this problem involves finding the optimal values of R_a and n that maximize the total profit within the given interval bounds.

Case 2

In Case-2, where R_a is less than E_y , shortages occur during the lead time. However, it is assumed that shortages are not allowed in the last cycle (n th cycle). The total holding expenses, $I_2(y)$, over the PH I is given by the following equation:

The function $I_2(y)$ represents the holding expenses in the inventory system. It is calculated as the sum of the holding expenses during different time periods: the holding

expenses in the time interval $(0, t_1)$, the holding expenses in the time interval $(t_k, t_k + 1)$ excluding the shortage time in the $(n - 1)$ cycle, and the holding expenses in the time interval (t_n, I) . This function quantifies the expenses associated with holding inventory at different stages of the inventory system, taking into account the specific time intervals and potential shortages.

$$\begin{aligned}
 &= D_i \left[\int_R^{R_a} \frac{-rdr}{E} + \sum_{k=1}^{n-1} \int_{R_a}^0 \frac{-rdr}{E} + \sum_{k=1}^{n-1} \int_{R-R_s}^{R_a} \frac{-rdr}{E} + \int_{R_a}^0 \frac{-rdr}{E} \right] \\
 &= D_i \left[\int_{R_a}^R \frac{rdr}{E} + (n-1) \int_0^{R_a} \frac{rdr}{E} + (n-1) \int_{R_a}^{R-R_s} \frac{rdr}{E} + \int_0^{R_s} \frac{rdr}{E} \right] \\
 &= \frac{D_i}{2D} [R^2 + (n-1)(R-R_s)^2] \quad (11)
 \end{aligned}$$

Here, R is calculated as $EI/n - (1 - \lambda)(n - 1)R_s/\lambda n$, and λ is a parameter. The total shortage expenses, $S_d(y)$, in the time horizon I is given by:

$$\begin{aligned}
 S_d(y) &= (n-1)D_s \int_0^{R_s} \frac{rdr}{\lambda E} \\
 &= (n-1) \frac{R_s}{2\lambda E} R_s^2 \quad (12)
 \end{aligned}$$

The total profit over the PH I is given by the equation:

$$\begin{aligned}
 B_2 &= nQR - nRD_q(1 - J_d) - (n-1)A_q y J_c - nD_o - \\
 &\frac{D_i}{2D} [R^2 + (n-1)(R-R_s)^2] - \frac{D_s(n-1)R_s^2}{2\lambda E} \quad (13)
 \end{aligned}$$

$$\begin{aligned}
 &= nQR - nRD_q(1 - J_d) - (n-1)J_e RD_q(1 - J_c) y J_c - \\
 &nD_o - \frac{D_i}{2D} [R^2 + (n-1)(R-R_s)^2] - \frac{D_s(n-1)R_s^2}{2\lambda E} \quad (14)
 \end{aligned}$$

For the IV expenses parameters D_q , D_o , and D_i :

$$\begin{aligned}
 B_2 \text{ lies within the interval } [B_{LT}, B_{RG}] \\
 B_{2LT} &= nQR - nRD_{qRG}(1 - J_d) - (n-1)J_d R(1 - \\
 &J_d) y J_c D_{qRG} - nD_{oRG} - \frac{D_{iRG}}{2D} [R^2 + (n-1)(R-R_s)^2] - \\
 &\frac{D_{sRG}(n-1)R_s^2}{2\lambda E} \quad (15)
 \end{aligned}$$

$$\begin{aligned}
 B_{2RG} &= nQR - nRD_{qLT}(1 - J_d) - (n-1)J_d R(1 - \\
 &J_d) y J_c D_{qLT} - nD_{oLT} - \frac{D_{iLT}}{2D} [R^2 + (n-1)(R-R_s)^2] - \\
 &\frac{D_{sRG}(n-1)R_s^2}{2\lambda E} \quad (16)
 \end{aligned}$$

Therefore, the problem becomes:

Maximize $B_2(R_a, n)$ Subject to $R_a > 0$, where n is an integer.

This is a “non-linear maximization problem” with an IO.

To obtain the optimal solution for the proposed inventory system, we need to choose the better solution between the two cases. The overall objective is to maximize the total profit, so the final solution is determined by comparing the

results from Case-1 and Case-2 and selecting the maximum value:

$$\begin{aligned}
 \text{Maximize } B &= \text{Maximize}(B_1, B_2) \\
 &(17)
 \end{aligned}$$

3.6. Solutions procedure

For this model solution we introduce MRSA to solve the issue.

3.6.1 . Reptile search optimization

Reptile search optimization is a metaheuristic algorithm inspired by the behavior of reptiles, specifically their movement and search patterns. The algorithm aims to explore the solution space and find optimal or near-optimal solutions for a given problem. It utilizes a population of solutions, referred to as reptiles, and simulates their behavior to guide the search process.

The key parameters in reptile search optimization include population size (p_size), maximum generation number (m_gen), crossover probability (p_cross), mutation probability ($p_mutation$), fitness function, selection mechanism, and search strategy. The population size determines the number of reptiles in the population, with larger populations enabling more comprehensive exploration but potentially increasing computational complexity. The maximum generation number sets an upper limit on the number of iterations or generations the algorithm will run, determining the termination condition.

Crossover probability (p_cross) determines the likelihood of performing crossover or recombination between two parent solutions within the population. Crossover allows the exchange of genetic information between solutions, combining beneficial traits from different individuals. Mutation probability (p_mute) determines the likelihood of introducing random variations or mutations into the solutions, enabling the exploration of new regions within the solution space.

The fitness function evaluates the quality or fitness of each solution in the population based on the problem's objectives. It measures how well a solution performs and guides the selection process. The selection mechanism determines how solutions are chosen for reproduction and survival. Various methods can be employed to balance exploration and exploitation.

The search strategy defines the movement and search behavior of reptiles within the solution space. It encapsulates the exploration-exploitation trade-off and guides how the algorithm explores new regions while exploiting promising areas. The specific search strategy can be designed to suit the problem domain and characteristics, incorporating domain knowledge or heuristics.

To achieve optimal performance with reptile search optimization, it is necessary to fine-tune these parameters based on problem complexity and domain expertise. It involves experimenting with different parameter settings, conducting sensitivity analysis, and evaluating the algorithm's performance using empirical studies. By iteratively adjusting and refining the parameters, the algorithm can effectively navigate the solution space and find high-quality solutions for the given problem.

3.6.2 Modified reptile search optimization

The Modified Reptile Search Algorithm (MRSA) solves high-dimensional nonconvex optimisation issues by improving the RSA. The original RSA may be inefficient, computationally demanding, and capture local minima. MRSA has been modified to address these issues. A sine operator while high walking is a key MRSA modification. The "Sine Cosine Algorithm (SCA)" dynamic exploration mechanism inspired this notion. MRSA option aspirants may widen the search space further by adding the sine operator. Global exploration assists the algorithm in minimising local minima and looking for the solution space.

Another adjustment in the MRSA involves utilizing Levy flights, which are random processes following the Levy distribution function. The parameter controlling the Levy flight, known as the levy flight parameter, determines the size of the random steps taken. By using a lower value for the levy flight parameter, the MRSA ensures that smaller random steps are taken, enabling solution candidates to search the areas closest to the obtained solutions. This improvement enhances the algorithm's exploitation capabilities, leading to improved global convergence.

These adjustments in the MRSA significantly reduce the complexity of the algorithm while simultaneously improving its performance. The improved global exploration and exploitation capabilities result in higher efficiency, faster convergence speed, and lower time complexity. As a result, the proposed modifications in the MRSA address the limitations of the original RSA and make it more suitable for solving high-dimensional nonconvex optimization problems.

4. Result and Discussion

4.1. Numerical demonstrations

The following five examples are used to demonstrate the suggested model as shown in Table 1. These numerical examples' decisions for the model parameters are all realistic, but they weren't chosen from any particular case study. The suggested MRSA has conducted five separate runs for each example, and for each run, the most optimal overall profit B value, represented by an interval, has been chosen in accordance with Definition 3 of interval order

relations. The optimal values $B, R, R_a, R_s, n,$ and P_p have been identified for each of the cases. The following MRSA parameter values are used in this calculation: $p_size = 105,$ $p_cross = 0.95,$ $p_mute = 0.3,$ and $m_gen = 550.$ Figs 1-4 illustrate the results of our graphic analysis of the effects of changes in MRSA parameters such as $p_size, m_gen, p_cross,$ and p_mute on the maximum total profit over the planning horizon $I,$ which is a different way we tested the performance of MRSA.

The results depicted in Fig. 1 indicate that the value of the optimal found profit reaches a stable state when the crossover probability (p_cross) exceeds 0.80. Beyond this threshold, further increases in p_cross do not significantly impact the stability of the solution.

Fig. 2 illustrates that the Modified Reptile Search Algorithm (MRSA) produces stable solutions when the mutation probability (p_mute) is within the range of 0.07 to 0.3. Within this range, the algorithm consistently converges to reliable solutions, and variations in p_mute do not significantly affect the stability of the solution.

Fig. 3 shows the ideal profit's stability when the maximum generation number (m_gen) surpasses 75. Once the algorithm reaches this threshold, additional iterations do not significantly impact the stability of the solution, and the optimal found profit remains relatively constant.

Finally, Fig. 4 shows that the MRSA produces stable solutions when the population size (p_size) is above 65. Beyond this value, increasing p_size does not notably affect the stability of the solution, and the algorithm consistently converges to reliable solutions.

Table 1. Instances' numerical solutions

Eg.	R	R_s	R_a	n	P_p	B	Remarks Solution
1	63,637	5.657	-	12	[79,85, 115.06]	[9080.57, 10550.73]	Case-2
2	58.34	19.668	-	14	[72,31, 104.30]	[8941.16, 10373.53]	Case-2
3	71.00	12.615	-	11	[87.51, 126.02]	[9080.57, 10541.16]	Case-2
4	116.668	-	44.273	07	[148.00, 211.00]	[9349.66, 10074.72]	Case-1
5	141.00	-	43.927	06	[176.41, 253.00]	[9405.58, 10866.26]	Case-1

It can be shown in Fig.1 that when p_cross is more than 0.76, the value of the optimal discovered profit begins

to stabilise. When the value of p_mute falls within the range of 0.07–0.2, it is clear that the MRSA provides a stable solution. This may be determined from Fig. 2 when m_gen is more than 75. The value of the optimal discovered profit is shown to remain stable in Fig. 3. Fig. 4 demonstrates that the MRSA is able to generate a stable solution when the p_size is greater than 65.

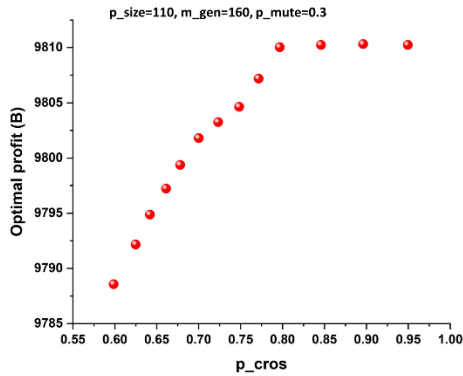


Fig. 1. Result of profit (B) vs. p_cross

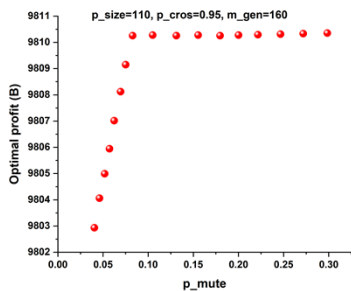


Fig. 2. Result of profit (B) vs. p_mute

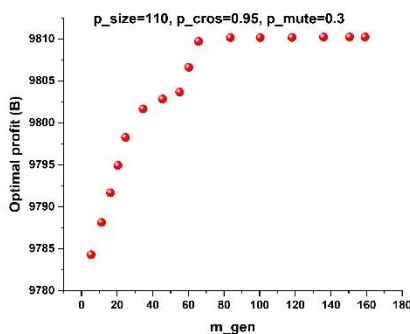


Fig. 3. Result of profit (B) vs. m_gen

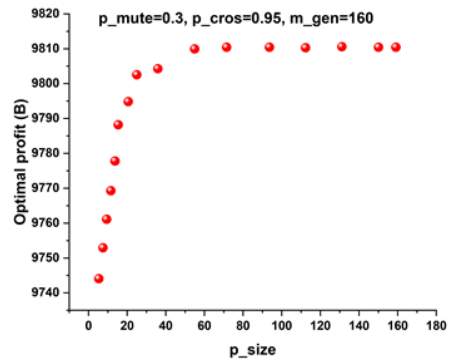


Fig. 4. Result of profit (B) vs. p_size

4.2. Analysis of sensitivity

Employing numerical demonstration, analyses were performed graphically to investigate the impact of under or overestimating parameters such as "demand rate, time horizon, the percentage of AP with respect to total purchase expenses, selling price, bank interest, and the percentage of discount on unit expenses on the centre value of the IV profit." These investigations were carried out by changing only one factor at a time while leaving the others (ranging from -15% to 15%) identical. Fifteen replicated iterations have been used to determine the optimal profit in each situation. These are depicted in the self-explanatory Fig. 5–10. Fig. 5 demonstrates that, as could be predicted, an increase in the rate of demand (E) has a corresponding increase in the optimal discovered profit. Additionally, the percentage changes in E are nearly identical to the percentage changes in the optimal discovered profit. The relationship between the optimal discovered profit and the time horizon (I) is evident in Fig. 6. In this instance, the optimal noticed profit increases as I grow, and vice versa.

Fig.7 shows how the "percentage of pre-payment with respect to total purchase expenses" affects the optimal observed profit. The plot suggests an inverse relationship between these two variables. This indicates that changes in the percentage of pre-payment have a limited impact on the percentage change in the optimal found profit. Other factors or parameters may have a more significant influence on the optimal profit.

Fig. 8, the impact of changes in expenses on the optimal found profit is examined. The plot indicates a direct effect, suggesting that an increase or decrease in expenses leads to a corresponding increase or decrease in the optimal found profit. However, price changes are slightly more than suitable found profit changes. This suggests that variations in price have a corresponding influence on the optimal discovered profit, although the profit change may be partially lower than the price change.

Fig. 9 shows an inverse relationship between changes in bank interest and changes in the optimal found profit.

However, the impact of changes in bank interest on the profit is relatively low compared to other factors.

Fig. 10 indicates a direct relationship between changes in the percentage of discount on unit expenses and changes in the optimal found profit. However, the effect of changes in interest on the profit is relatively low compared to changes in expenses or demand.

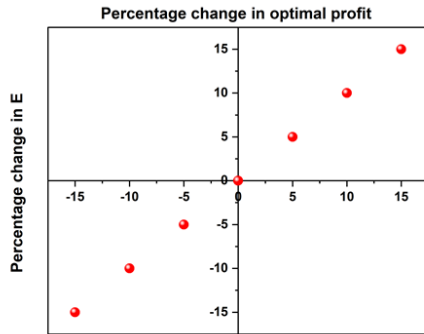


Fig. 5. Demand rate vs. profit (B).

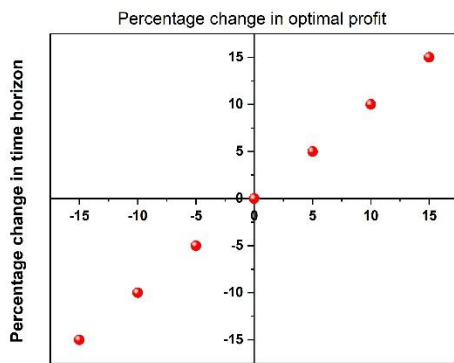


Fig. 6. Time horizon (I) vs profit (B).

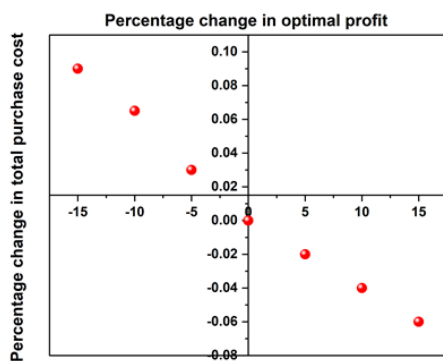


Fig. 7. The ratio of AP vs. profit (B).

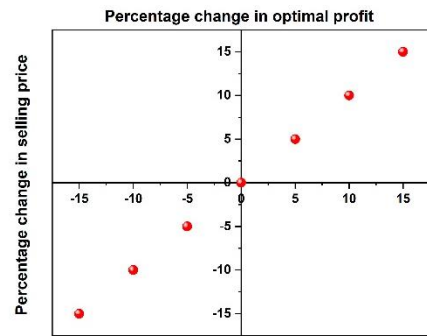


Fig. 8. Selling price (P) vs profit (B).

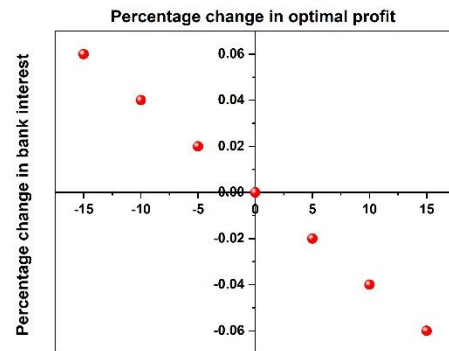


Fig. 9. Bank interest vs. profit (B).

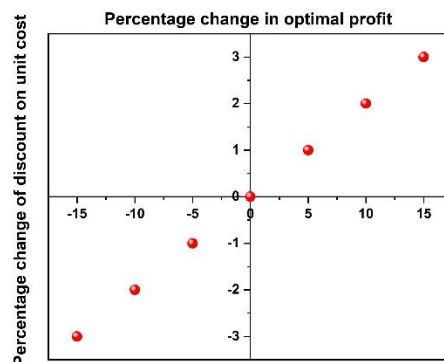


Fig. 10. Unit expenses discount vs. optimal profit (B).

5. Conclusion

In this study, we have presented an inventory model that incorporates IV inventory expenses and the impact of pre-payment (PP). The model considers IV expenses and examines two scenarios: one without shortages and another allowing for partially backlogged deficiencies. In the second scenario, the demand rate is expected to decrease over a limited interval due to factors such as loyal customers and customers transferring shops. To address the optimization problem associated with interval targets, we have proposed a Modified Reptile Search Optimization (MRSO) algorithm. This algorithm efficiently solves the inventory model with interval-valued expenses, allowing decision-makers to improve their inventory strategies and

optimize profitability under uncertain expense and demand conditions. Numerical demonstrations of the proposed model have been provided, and sensitivity analysis has been conducted to assess the effects of various inventory factors on optimal profit. The results highlight the significance of pre-payment, IV expenses, and different cases of shortages in inventory management. By utilizing the model and the MRSO algorithm, decision-makers can enhance their inventory management strategies, considering uncertain expenses and demands and ultimately maximizing profitability. The findings offer valuable insights for decision-makers to make informed choices regarding inventory strategies and achieve profitability in dynamic and uncertain environments.

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