International Journal of
Intelligent Systems and Applications in Engineering

ISSN:2147-6799 www.ijisae.org Original Research Paper

Preferences, Utility and Prescriptive Decision Control in Complex Systems

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Received 28th June 2013, Accepted 18th September 2013

Abstract: *The evaluation of the preferences based utility function is a goal of the human cantered control (management) design. The achievement of this goal depends on the determination and on the presentation of the requirements, characteristics and preferences of the human behaviour in the appropriate environment (management, control or administration of complex processes). The decision making theory, the utility and the probability theory are a possible approach under consideration. This paper presents an approach to evaluation of human's preferences and their utilization in complex problems. The stochastic approximation is a possible resolution to the problem under consideration. The stochastic evaluation bases on mathematically formulated axiomatic principles and stochastic procedures. The uncertainty of the human preferences is eliminated as typically for the stochastic programming. The evaluation is preferences-oriented machine learning with restriction of the "certainty effect and probability distortion" of the utility assessment.The mathematical formulations presented here serve as basis of tools development. The utility and value evaluation leads to the development of preferences-based decision support in machine learning environments and iterative control design in complex problems.*

Keywords: *Preferences, Utility, Stochastic Approximation, Complex systems, Edgeworth box.*

1. Introduction

The aspiration for quantity measurements, estimations and prognosis at all phases of the decision making and problem-solving is natural. But this task is carried out with very scarce initial information, especially in the initial development phase in complex problems and situations. In the initial stage of a decision process the heuristic of the investigator is very important, because in most of the cases there is a lack of measurements or even clear scales under which to implement these measurements and computations. This stage is often outside of the strict logic and mathematics and is close to the art, in the widest sense of the word, to choose the right decision among great number of circumstances and often without associative examples of similar activity. The correct assessment of the degree of informativity and usability of these types of knowledge requires careful analysis of the terms measurement, formalization, and admissible mathematical operations under the respective scale, which do not distort the initial empirical information.

In the paper we describe approaches and methods for measurement and analytical presentations of empirical and scientific knowledge expressed as preferences. Due to multidisciplinary nature of the cognitive process and to multidisciplinary nature of the fields of applications our choice of scientific methods is oriented toward the utilization of the stochastic programming, the theory of measurement and utility theory [1], [7], [11], [13], [19]. In this manner we can pose the decision making problem as a problem of constructing value and utility functions based on stochastic recurrent procedures as machine learning, which can later be used in decision support, in intelligent information systems and human-adapted design process of optimization problems in complex systems with human participation. Validate mathematical evaluation of the human preferences as utility (value) is the first step in realization of a human-adapted design process and decision making [3], [11], [20].

The analytical description of the expert's preferences as value or utility function will allow mathematically the inclusion of the decision maker (DM) in the model description of the complex system "Technologist-process" [18]. Value based design is a systems engineering strategy which enables multidisciplinary design optimization. Value-driven design creates an environment that enables optimization by providing designers with an objective function [5]. The objective (value/utility) function inputs the important attributes of the system being designed, and outputs a score. In this way we introduce the Model-driven decision making. Model-driven decision making and control emphasizes access to and manipulation of a statistical, financial, optimization, or simulation models and uses data and parameters provided by users to assist decision process in analyzing a complex situation. The American psychologists Griffiths and Tenenbaum by analyzing intuitive evaluations in the conditions of repetitive life situations have proved the statistical optimality of human assessments [8]. The idea of this study is that humans process the new data about the surrounding world by interpreting them in the framework of a built in their consciousness probability model. That means that the Bayesian approach was a natural basis on which human beings form their decisions, using their previous empirical experience expressed as preferences [7], [11], [20]. In such case the utility theory and its prescription to make decision based on the optimal mathematical expectation of

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the utility has another scientific validation as methodology in the decision making.

We will demonstrate this system engineering, value driven approach within two examples, determination of the equilibrium points in competitive trading, modeled by the Edgworth box and the control design based on the human evaluation of the best growth rate of a biotechnological process.

2. Measurement, Scales, Preferences, Value and Utility Evaluation

The objective of value based decision making is to develop a mathematical framework (econometric) for management and modeling of complex systems. The aspiration for measurements, quantity estimations and prognosis is natural but the correct assessment requires careful analysis of the terms *measurement*, formalization and admissible mathematical operations. In complex processes, there is a lack of measurements or even clearly identifiable scales for the basic heuristic information. Internal human expectations and heuristic are generally expressed by qualitative preferences. The common sources of information in such a basic level are the human preferences. According to social-cognitive theories, people's strategies are guided both by internal expectations about their own capabilities of getting results, and by external feedback [3]. Probability theory and expected utility theory address decision making under these conditions [11].

The mathematical description on such a fundamental level requires basic mathematical terms like sets, relations and operations over them, and their gradual elaboration to more complex and specific terms like functions, operators on mathematically structured sets as well, and equivalency of these descriptions with respect to a given real object. In the last aspect of equivalency of the mathematical descriptions we enter the theory of *measurements and scaling* [13,19]**.**

People's preferences contain uncertainty of probabilistic nature due to the qualitative type of both the empirical expert information and human notions. A possible approach for solution of these problems is the *stochastic programming* [1, 15, 18]. The uncertainty of the subjective preferences could be considered as an additive noise that could be eliminated, as is typical in the stochastic approximation procedures. The main objective is the productive merger of the mathematical exactness with the empirical uncertainty in the human notions.

We start by a brief introduction in the measurement theory. *System with relations* (SR) is called the set $\boldsymbol{A}$ in conjunction with a set of relations $\boldsymbol{R_i}$, $\boldsymbol{i}\in\boldsymbol{I}$, $\boldsymbol{I}=\{1,2,3,...,n\}$ defined over it and we denote it by $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$. In this manner we introduce an algebraic structure in the set $\boldsymbol{A}$. Relation of *congruency* is called a relation of equivalency ($\approx$) (reflexive, symmetric and transitive relation) defined over the basic set $\boldsymbol{A}$, if the property of *substitution* is satisfied, i.e. from the fulfillment of relations $(x_1, x_2, x_3, ...,x_{hi})\in\boldsymbol{A}^{hi}$ and $(x_j\approx y_j)$ for every j=1, 2, 3 4,...,h_i it follows that $\boldsymbol{R_i}(x_1, x_2, x_3, ...,x_{hi})=\boldsymbol{R_i}(y_1, y_2, y_3, ...,y_{hi})$ for $\forall$ $\boldsymbol{i}, \boldsymbol{i}\in\boldsymbol{I}$. We say that the relation of equivalency ($\approx_2$) is coarser than the equivalency ($\approx_1$), if the inclusion $(\approx_1) \subseteq (\approx_2)$ is satisfied. It is known that there always exists a coarsest relation ($\approx_A$) over the SR $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$. This means that if two elements are in congruency ($x\approx_A y$), then they are undistinguishable with respect to the properties in the set $\boldsymbol{A}$ (the real object under investigation), described with the set of relations $((\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$. If we factorize the set $\boldsymbol{A}$ by the coarsest congruency ($\approx_A$), then in the factor set $\boldsymbol{A}/\approx_A$ the congruency ($\approx_A$) is in fact equality (=). A SR $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$, in which the congruency ($\approx_A$) is coarsest is called *irreducible*. In this case SR $(\boldsymbol{A}/\approx_A,(\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$ is irreducible.

A homomorphism is an image *f, f*: $\boldsymbol{A}\rightarrow\boldsymbol{B}$ between two systems with relations SR $(\boldsymbol{A},(\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$ and SR $(\boldsymbol{B}, (\boldsymbol{S_i}), \boldsymbol{i}\in\boldsymbol{I})$ from the same type. The systems with relations SR $(\boldsymbol{A},(\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$ and SR $(\boldsymbol{B}, (\boldsymbol{S_i}), \boldsymbol{i}\in\boldsymbol{I})$ are from the same type if for which $\forall$ $\boldsymbol{i}, \boldsymbol{i}\in\boldsymbol{I}$ and $(x_1, x_2, x_3, ...,x_{hi})\in\boldsymbol{R_i}$ is satisfied $\boldsymbol{R_i}(x_1, x_2, x_3, ...,x_{hi}) \Leftrightarrow \boldsymbol{S_i}(f(x_1), f(x_2), f(x_3), ..., f(x_{hi}))$.

DEFINITION: We call k-dimensional scale every homomorphism from irreducible empirical system into a number system SR $(\boldsymbol{A}, (\boldsymbol{Q_i}), \boldsymbol{i}\in\boldsymbol{I})$.

The empirical system of relations SR $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$ is an object from the reality with the properties described by the relations $((\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$, while the numbered system of relations SR $(\boldsymbol{B}, (\boldsymbol{S_i}), \boldsymbol{i}\in\boldsymbol{I})$ is a mathematical object which reflects the properties of the real object. For example the set $\boldsymbol{B}$ could be the k-ary Cartesian product of the set of the real numbers $\boldsymbol{R}^K$.

In the scale definition the correspondence f_0: $\boldsymbol{A}\rightarrow \boldsymbol{R}^K$ is not simply defined. In general sense, there exists entire class of scales converting the irreducible empirical system of relations SR $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$ into the number system SR $(\boldsymbol{R}^K, (\boldsymbol{S_i}), \boldsymbol{i}\in\boldsymbol{I})$. We denote this class of homomorphisms by $\aleph(\boldsymbol{A}, \boldsymbol{R}^K)$. Every homomorphism of $\aleph(\boldsymbol{A}, \boldsymbol{R}^K)$ is injective because the empirical system is irreducible and surjective with regard to $f(\boldsymbol{A})$).

Let $\boldsymbol{A_0}$ be a subset of $\boldsymbol{A}$. We denote by $G_A(A_0)$ all injective inclusion (*partial endomorphism*) from SR $(\boldsymbol{A_0}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$ in SR $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$. If a scale $f_0\in\aleph(\boldsymbol{A}, \boldsymbol{R}^K)$ is given, then we can characterize the whole class of scales $\aleph(\boldsymbol{A}, \boldsymbol{R}^K)$ in the following way: $\aleph(\boldsymbol{A}, \boldsymbol{R}^K)=\{\gamma\circ f_0/ \text{ where } \gamma\in G_{\boldsymbol{R}^K}(f_0(\boldsymbol{A}))\}$. In other words two scales are equivalent with precision up to a partial endomorphism $\gamma\in G_{\boldsymbol{R}^K}(f_0(\boldsymbol{A}))$. The elements of $G_{\boldsymbol{R}^K}(f_0(\boldsymbol{A}))$ are called *admissible manipulations* of the scale f_0 [19]. An example is the measurement of the temperature. If the scale $f_0(.)$ is the temperature in Celsius, then every partial endomorphism is an affine correspondence of the type $\gamma(x)=\boldsymbol{a}x+\boldsymbol{b}$, $\boldsymbol{a}\in\boldsymbol{R}$, $\boldsymbol{b}\in\boldsymbol{R}$ and $\boldsymbol{a>0}$. The temperature in Kelvin is determined by shifting the zero point by $\boldsymbol{b}$, $\boldsymbol{b}\in\boldsymbol{R}$ and by changing the magnitude by multiplying by $\boldsymbol{a}$, $\boldsymbol{a>0}$.

From the definition of the measurement and scale it follows that there are infinitely many types of scales. In informal terms measurement is an operation in which a given state of the observed object is mapped to a given denotation. An example is the so-called *nominal scale* which is an expression of the equivalence of two phenomena only. Let $\mathbf{X}$ be the set of alternatives ($\mathbf{X}\subseteq\mathbf{R}^m$). Let x and y be two alternatives ($(x,y)\in\mathbf{X}^2$). For this weakest scale the following axioms are valid:

1. $((x\approx y \vee \neg x\approx y)\equiv 1) \wedge ((x\approx y \wedge \neg x\approx y)\equiv 0) \wedge x\approx x$;
2. $(x\approx y \Rightarrow y\approx x)$;
3. $((x\approx y \wedge x\approx z) \Rightarrow y\approx z)$.

Here ($\approx$) denotes equivalence and $\neg(\approx)$ is the opposite (non-equivalence). The above three properties define the relation *equivalence*, which splits the set $\mathbf{X}$ into non overlapping subsets (classes of equivalence). In this scale only the Kronecker symbol may be used as a measure.

When the observed phenomenon allows to distinguish the differences between states and to compare them by preference a stronger scale needs to be used – the *ordering scale*. The preference in the ordering scale is denoted by $(x\succ y)$. In accordance with a long-standing tradition, $x\succ y$ is taken to represent "x is better than y". In this scale together with the above three axioms two more are satisfied:

4. $\neg(x \succ x)$ for $\forall$ $x\in X$, $((x \succ y) \Rightarrow \neg(y \succ x))$;
5. $(x\succ y \wedge y\succ z) \Rightarrow x\succ z$.

If incomparable alternatives exist, then the scale is called *partial ordering*. Under these five axioms an analytical preferences representation by value function u(.) is searched for. A *value function* is a function u(.) for which it is fulfilled $((x, y)\in X^2, x \succ y) \Leftrightarrow (u(x)>u(y))$ [11]. In this definition, in addition to axioms (4, 5), weak connectedness is also assumed $\neg(x \approx y) \Rightarrow ((y \succ x) \vee (x \succ y))$. Depending on the type of the function – continuous, partially continuous or discrete – there exist different types of scales, measuring the above relations. A transformation with an arbitrary monotonous function leads to another ordinal scale (*admissible manipulations* γ, $\gamma \in G_B$ $(f_0(\boldsymbol{A}))$. When using those scales, apart from comparison by magnitude, we can search the minimum and maximum of the function as feasible mathematical operations. Under this scale it is impossible to talk about distance between the different alternatives.

If together with the ordering of the alternatives, the distance between them can be evaluated, we can talk about *interval scale***.** For these scales the distances between the alternatives have the meaning of real numbers. For these scales the central moments and the variance are sensible evaluations and have physical meaning, whereas the mathematical expectation depends on the origin of the scale and thus is unfeasible. The transition from one interval scale to another is achieved with affine transformation $\boldsymbol{x = ay + b, (x, y) \in X^2, a > 0, b \in R}$. Among these type of scales is also the measurement of the utility function by the so called "gambling approach". We emphasize that the calculations are done with numbers related to the distances between the alternatives, and not with the numbers relating to the alternatives themselves. For instance, if we say that a body is twice as warm as another in Celsius, this will not be true if the measurements were done in Kelvin.

A stronger scale is the *ratio scale***.** This is an interval scale with fixed origin $\boldsymbol{x = ay, (x, y) \in X^2, a > 0}$. For example the weight measurement is in the ration scale. For these scales in addition to the previous 5 axioms the following additivity axioms are satisfied:

6. $(x=y \wedge z>0) \Rightarrow ((x+z)>y)$;
7. x+y=y+x;
8. $(x=y \wedge z=q) \Rightarrow (x+z=y+q)$;
9. q+(x+y)=(q+y)+x.

The *absolute scale* is the most powerful. For it the zero and one are absolute and it is a one of a kind and unique scale.

2.1. Value Function and Measurement Scale

From practical point of view the empirical system of human preferences relations is a SR $(\boldsymbol{X}, (\approx), (\succ))$, where $(\approx)$ can be considered as the relation "indifferent or equivalent", and $(\succ)$ is the relation "prefer". We look for equivalency of the empirical system with the numbered system of relations SR (***R***-real numbers**,** (=)**,** (>)). The *"indifference"* relation $(\approx)$ is based on $(\succ)$ and is defined by $((x \approx y) \Leftrightarrow \neg((x \succ y) \vee (x \prec y)))$. Let **X** be the set of alternatives ($\mathbf{X} \subseteq \mathbf{R}^m$). A *Value function* is a function (u^*: $\mathbf{X} \rightarrow \mathbf{R}$) for which it is fulfilled [11]:

$((x, y)\in \mathbf{X}^2, x \succ y) \Leftrightarrow (u^*(x)>u^*(y))$.

It is proved that for a finite set of alternatives and partial ordering (axioms 4, and 5) there always exists such a function with precision up to monotonous transformation [7]. In this manner we can move from the language of binary relations and preferences to the language of control criteria as objective value function. The assumption of existence of a value function *u*(.) leads to the "*negatively transitive*" and "*asymmetric*" relation $(\succ)$, "*weak order*". A "*strong order*" is a "*weak order*" for which is fulfilled $(\neg(x \approx y) \Rightarrow ((x \succ y) \vee (x \prec y))$. The existence of a "*weak order*" $(\succ)$ over ***X*** leads to the existence of a "*strong order*" over $\boldsymbol{X}/\approx$ [7]. Consequently the assumption of existence of a value function u(.) leads to the existence of: asymmetry $((x \prec y) \Rightarrow (\neg(x \succ y))$, axiom 4), transitivity $((x \succ y) \wedge (y \succ z) \Rightarrow (x \succ z)$, axiom 5) and transitivity of the "indifference" relation $(\approx)$ (axiom 3).

The ordering scale was defined via homomorphisms, monotone functions. But if we are looking for the equivalency between SR $(\boldsymbol{X}, (\approx), (\succ))$ and SR (***R***-real numbers**,** (=)**,** (>)) practically, the homomorphisms have to be not only monotonic but continuous as well. In this case the ordering in the real numbers ***R*** will be reflected in the empirical set ***X*** with the properties of the interval topology generated by the relation (>) in ***R*.** Then the term for convergence in the measurements coincides with the standard generally accepted term for convergence [19].

2.2. Utility Function and Measurement Scale

According to the *Utility theory* let ***X*** be the set of alternatives and ***P*** is a set of probability distributions over ***X*** and $\boldsymbol{X \subseteq P}$. A utility function *u*(.) will be any function for which the following is fulfilled:

$(p \succ q, (p,q)\in \mathbf{P}^2) \Leftrightarrow (\int u(.)dp > \int u(.)dq)$.

To every decision choice and action corresponds a probability distribution of appearance of final alternatives (results). The notation $(\succ)$ expresses the preferences of DM over ***P*** including those over ***X*** ($\boldsymbol{X \subseteq P}$). The interpretation is that the integral of the utility function *u*(.) is a measure concerning the comparison of the probability distributions *p* and *q* defined over ***X*** (figure 1).

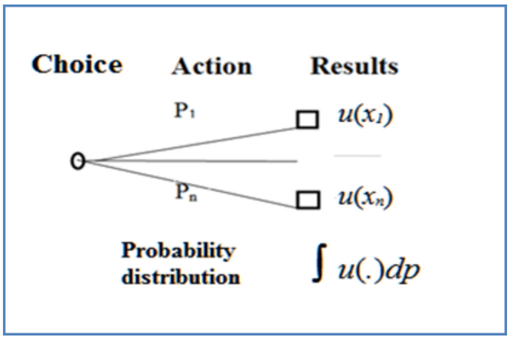

Fig.1. Probability distribution and utility function

There are different systems of mathematical axioms that give satisfactory conditions of a utility function existence. The most famous of them is the system of Von Neumann and Morgenstern's axioms [7]:

- (A.1) The *preferences* relations $(\succ)$ and $(\approx)$ are transitive, i.e. the binary preference relation $(\succ)$ is weak order**;**
- (A.2) *Archimedean Axiom*: for all $p,q,r \in \boldsymbol{P}$ such that $(p \succ q \succ r)$, there is an $\alpha,\beta \in (0,1)$ such that $((\alpha p + (1-\alpha)r) \succ q)$ and $(q \succ (\beta p + (1-\beta)r))$;
- (A.3) *Independence Axiom*: for all $p,q,r \in \boldsymbol{P}$ and any $\alpha \in (0, 1]$, then $(p \succ q)$ if and only if $((\alpha p + (1-\alpha)r) \succ (\alpha q + (1-\alpha)r))$.

Axioms (A1) and (A3) cannot give solution. Axioms (A1), (A2) and (A3) give solution in the interval scale (precision up to an affine transformation):

$((p \succ q) \Leftrightarrow (\int v(x)dp \succ \int v(x)dq) \Leftrightarrow (v(x)= \boldsymbol{a}u(x)+\boldsymbol{b}$,
$\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{R}, \boldsymbol{a}>0, x \in \boldsymbol{X}))$.

It is known that the assumption of existence of a utility (value) function *u*(.) leads to the "*negatively transitive*" and "*asymmetric*" relation $(\succ)$ and to transitivity of the relation $(\approx)$. So far we are in the preference scale, the *ordering scale*. The assumption of equivalence with precision up to affine transformation has not been included. In other words we have only a value function. For value, however, the mathematical expectation is unfeasible, but we underline that the mathematical expectation is included in the definition of the utility function. For this reason it is accepted that ($\mathbf{X}\subseteq\mathbf{P}$) and that $\mathbf{P}$ is a convex set: $((q, p)\in\mathbf{P}^2\Rightarrow(\alpha q+(1-\alpha)p)\in\mathbf{P}$, for $\forall\alpha\in[0,1])$.

Then utility $u(.)$ is determined in the interval scale [7]:

Proposition 1. If $((x\in X \wedge p(x)=1)\Rightarrow p\in\mathbf{P})$ and $(((q, p)\in\mathbf{P}^2) \Rightarrow ((\alpha p+(1-\alpha)q)\in\mathbf{P}, \alpha\in[0,1]))$ are realized, then the utility function $u(.)$ is defined with precision up to an affine transformation: $(u_1(.)\approx u_2(.))\Leftrightarrow(u_1(.)=au_2(.)+b, a>0)$.

Following from this proposition, the measurement of the preferences is in the *interval scale*. That is to say, this is a utility function. Now it is obvious why in practice the gambling approach is used to construct the utility function in the sense of von Neumann. The reason is that to be in the interval scale the set of the discrete probability distributions $\mathbf{P}$ have to be convex. The same holds true in respect of the set $\mathbf{X}$. The utility function is evaluated by the "gambling approach". This approach consists within the comparisons between lotteries. A "lottery" is called every discrete probability distribution over X. We denote as $<x, y,\alpha>$ the simplest lottery: α is the probability of the appearance of the alternative x and $(1-\alpha)$ - the probability of the alternative y. In the practice, the utility measurement is based on the comparisons between lotteries as is shown in figure 2 [11].

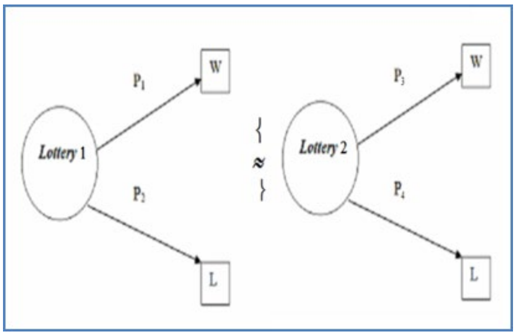

Fig.2. Gambling approach, comparisons of lotteries

The weak points of the gambling approach are the violations of the transitivity of the preferences and the so called "certainty effect" and "probability distortion" identified by the Nobel prizeman Kahneman and Tversky. The violations of the transitivity of the relation equivalence (≈) also lead to declinations in the utility assessment. All these difficulties explain the DM behavior observed in the Allais Paradox [2].

Following the research of Kahneman and Tversky and the debates about the well known Allais paradox, extensions and further developments of von Neumann's theory were sought [4], [10], [21]. Among these theories the rank dependent utility (RDU) and its derivative cumulative Prospect theory are currently the most popular. In the RDU the decision weight of an outcome is not just the probability associated with this outcome. It is a function of both the probability and the rank the alternative. Based on empirical researches several authors have argued that the probability weighting function has an inverse S-shaped form, which starts on concave and then becomes convex.

3. Utility And Value Stochastic Approximation Evaluation

Starting from the properties of the preference relation (≻) and indifference relation (≈), we propose the next stochastic approximation procedure for evaluation of the utility function $u(.)$. In correspondence with Proposition 1, it is assumed that $(\mathbf{X}\subseteq\mathbf{P})$, $((q,p)\in\mathbf{P}^2 \Rightarrow (\alpha q + (1-\alpha)p)\in\mathbf{P}$, for $\forall\alpha\in[0,1])$ and that utility function $u(.)$ exists. We define two sets:

$\mathbf{A}_{u*}=\{(\alpha,x,y,z)/(\alpha u^*(x)+(1-\alpha)u^*(y))>u^*(z)\}$,

$\mathbf{B}_{u*}=\{(\alpha,x,y,z)/(\alpha u^*(x)+(1-\alpha)u^*(y))>u^*(z)\}$,

where $u^*(.)$ is DM's empirical utility. The next proposition is in the foundation of the used stochastic approximation procedures [18]:

Proposition 2. We denote $\mathbf{A}_u=\{(\alpha,x,y,z)/(\alpha u(x)+(1-\alpha)u(y))>u(z)\}$. If $\mathbf{A}_{u1}=\mathbf{A}_{u2}$, then $u_1(.)=au_2(.)+b$, $a>0$.

The approximation of the utility function is constructed by recognition of the set $\mathbf{A}_u$ [15], [18]. The proposed assessment is machine learning based on DM's preferences. The machine learning is a probabilistic pattern recognition ($\mathbf{A}_{u*}\cap\mathbf{B}_{u*}\neq\emptyset$) and the utility evaluation is a stochastic programming pattern recognition with noise (uncertainty) elimination**.** Key element in this solution is Proposition 2.

The evaluation procedure is presented as follows. The DM compares the "lottery" $<x,y,\alpha>$ with the simple alternative z, $z\in\mathbf{Z}$ **("better-≻, f(x,y,z,α)=(1)", "worse-≺, f(x,y,z,α)=(-1)", or "can't answer or equivalent- ~ , f(x,y,z,α)=0", f(.) denotes the qualitative DM answer).** This determines a learning point **((x,y,z,α), f(x,y,z,α)).** The following recurrent stochastic algorithm constructs the polynomial utility approximation:

$$u(x)=\sum_i c_i\Phi_i(x),$$

$$c_i^{n+1}=c_i^n+\gamma_n\left[f(t^{n+1})-\overline{(c^n,\Psi(t^{n+1}))}\right]\Psi_i(t^{n+1})$$

$$\sum_{\mathbf{n}}\gamma_{\mathbf{n}}=+\infty,\ \sum_{\mathbf{n}}\gamma_{\mathbf{n}}^2<+\infty,\ \forall\mathbf{n},\gamma_{\mathbf{n}}>0.$$

In the formula are used the following notations (based on $\mathbf{A}_u$): $t=(x,y,z,\alpha)$, $\psi_i(t)=\psi_i(x,y,z,\alpha)=\alpha\Phi_i(x)+(1-\alpha)\Phi_i(y)-\Phi_i(z)$, where $(\Phi_i(x))$ is a family of polynomials. The line above the scalar product $v=\overline{(c^n,\Psi(t))}$ means: ($\overline{v}=1$), if ($v>1$), ($\overline{v}=-1$) if ($v<-1$) and ($\overline{v}=v$) if ($-1<v<1$). The notation $(c^n,\Psi(t))=\alpha g^n(x)+(1-\alpha)g^n(y)-g^n(z)=G^n(x,y,z,\alpha)$ is a scalar product. The coefficients c_i^n take part in the polynomial presentation $g^n(x)=\sum_{i=1} c_i^n\Phi_i(x)$. The learning points are set with a pseudo random sequence. Practically the assessment process is the following. The expert (DM) relates intuitively the "learning point" (x,y,z,α)) to the set $\mathbf{A}_{u*}$ with probability $D_1(x,y,z,\alpha)$ or to the set $\mathbf{B}_{u*}$ with probability $D_2(x,y,z,\alpha)$. The probabilities $D_1(x,y,z,\alpha)$ and $D_2(x,y,z,\alpha)$ are mathematical expectation of $f(.)$ over $\mathbf{A}_{u*}$ and $\mathbf{B}_{u*}$ respectively,

$(D_1(x,y,z,\alpha)=M(f/x,y,z,\alpha))$ if $(M(f/x,y,z,\alpha)>0)$,

$(D_2(x,y,z,\alpha)=-M(f/x,y,z,\alpha))$ if $(M(f/x,y,z,\alpha)<0)$.

Let $D'(x,y,z,\alpha)$ is the random value:

$D'(x,y,z,\alpha)=D_1(x,y,z,\alpha)$ if $(M(f/x,y,z,\alpha)>0)$;

$D'(x,y,z,\alpha)=(-D_2(x,y,z,\alpha))$ if $(M(f/x,y,z,\alpha)<0)$;

$D'(x,y,z,\alpha)=0$ if $(M(f/x,y,z,\alpha)=0)$.

We approximate $D'(x,y,z,\alpha)$ by a function of the type $G(x,y,z,\alpha)=(\alpha g(x)+(1-\alpha)g(y)-g(z))$, where $g(x)=\sum_i c_i\Phi_i(x)$. The coefficients c_i^n take part in the polynomial approximation of $G(x,y,z,\alpha)$:

$$G^n(x,y,z,\alpha)=(c^n,\Psi(t))=\alpha g^n(x)+(1-\alpha)g^n(y)-g^n(z)$$

$$g^n(x)=\sum_{i=1}^{N} c_i^n\Phi_i(x)$$

The function $G^n(x,y,z,\alpha)$ is positive over $\mathbf{A}_{u*}$ and negative over $\mathbf{B}_{u*}$ depending on the degree of approximation of $D'(x,y,z,\alpha)$. The function $g^n(x)$ is the approximation of the utility function $u(.)$. In another notation the stochastic procedure has the following form:

$$c_i^{n+1}=c_i^n+\gamma_n\left[D'(t^{n+1})+\xi^{n+1}-\overline{(c^n,\Psi(t^{n+1}))}\right]\Psi_i(t^{n+1}),$$

$$\sum_n \gamma_n = +\infty, \sum_n \gamma_n^2 < +\infty, \forall n, \gamma_n > 0,$$

$$f(t^{n+1}) = \left[D'(t^{n+1}) + \xi^{n+1} \right].$$

We follow the evaluation approach described in the well known books [11], [20]. DM compares the "lottery" $<x,y,\alpha>=(\alpha x+(1-\alpha)y)$ with the separate elements (alternative) z, $z \in \mathbf{X}$. This lottery is of the simplest possible type and is sufficient for the utility evaluation. The expressed preferences, the answers of DM and comparisons are of cardinal (qualitative) nature and contain the inherent DM's uncertainty and errors. The stochastic convergence of the Potential function method (Kernel method) is analyzed in [1], [15], [18].

The same approach is used for of value evaluation. The difference is only within the form of the sets $\mathbf{A}_{u^*}$ and $\mathbf{B}_{u^*}$. Let $\mathbf{A}_{u^*}$ and $\mathbf{B}_{u^*}$ be the sets:

$A_{u^*}=\{(x,y)\in R^{2m}/\ (u^*(x))>u^*(y)\}$,

$B_{u^*}=\{(x, y)\in R^{2m}/\ (u^*(x))<u^*(y)\}$.

If there is a function F(x,y) of the form F(x,y)=f(x)-f(y), positive over A_{u^*} and negative over B_{u^*}, then the function f(x) is a value function, equivalent to the empirical value function u*(.). Such approach permits the use of stochastic "pattern recognition" for solving the problem. In the deterministic case it is true that $A_{u^*}\cap B_{u^*}=\emptyset$. In the probabilistic case it is true that $A_{u^*}\cap B_{u^*}\neq\emptyset$ and here have to be used the probabilistic pattern recognition [1, 12, 18].

4. Value Driven Decision Making and Equilibrium Analysis in Edgeworth Economic: Edgeworth Box and Competitive Trade

Competitive trade is a setting in which there are prices for two goods in question and many people who take these prices as given. Hence, the situation is as in the competitive market, except for the fact that we now consider two markets simultaneously. A useful tool for description the competitive trade is the Edgeworth Box. Essentially, it merges the indifference map between the parties in the trade by inverting one of the agents (individuals, consumers, markets and so on) diagram. Given two consumers $\boldsymbol{O}_1$ and $\boldsymbol{O}_2$, two goods, and no production, all non-wasteful allocations can be drawn in the box shown in figure 3.

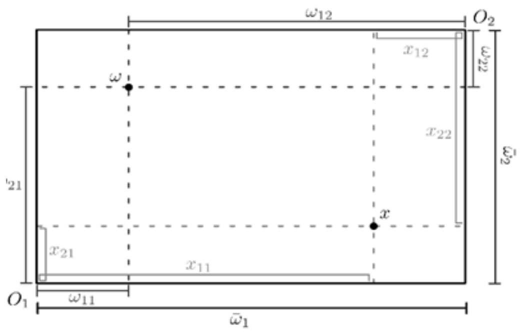

Fig.3. Edgeworth Box, initial endowment and allocations

Every point in the box represents a complete allocation of the two goods to the two consumers. Each of the two individuals maximizes his utility according to his preferences [11], [6]. The demand functions or the utility functions which represent consumers' preferences are convex and continuous, because in accordance with the equilibrium theory the preferences in are continuous, monotone and convex as is shown in figure 4 [6].

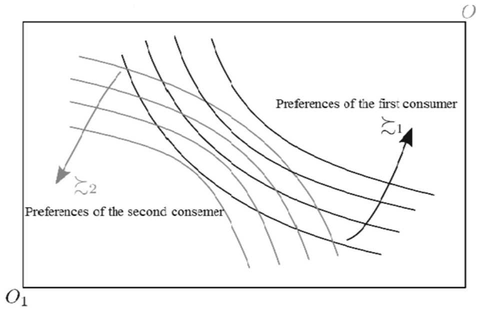

Fig.4. Convex indifference curves

Each consumer is characterized by an endowment vector, a consumption set, and regular and continuous preferences [6]. The two consumers are each endowed (born with) a certain quantity of goods. They have locally non-satiated preferences and initial endowments:

$(\boldsymbol{w}_1, \boldsymbol{w}_2) = ((w_{11}, w_{21}), (w_{12}, w_{22}))$.

In the box the vector $w = (w_1, w_2)$ is the total quantities of the two goods:

$$w_1 = w_{11} + w_{12}, w_2 = w_{21} + w_{22}.$$

An allocation $\boldsymbol{x}=(\boldsymbol{x}_1, \boldsymbol{x}_2) = ((x_{11}, x_{21}), (x_{12}, x_{22}))$ represents the amounts of each good that are allocated to each consumer. A no wasteful allocation $\boldsymbol{x}=(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is one for which is fulfilled:

$$w_1 = x_{11} + x_{12}, w_2 = x_{21} + x_{22}.$$

In terms of aggregate amounts of the two agents, the total amounts needs to be equal to the total endowment of the two goods. The consumers take prices of the two goods $\boldsymbol{p} = (p_1, p_2)$ as given and maximize their utilities. The budget (income) set $B_i(\boldsymbol{p})$ of each consumer is given by: $B_i(\boldsymbol{p}) = \{\mathbf{x}_i \in \mathbf{R}^2_+ \ / \ \boldsymbol{p}\boldsymbol{x}_i \le \boldsymbol{p}\boldsymbol{w}_i\}, (\mathbf{i}=1,2)$, where $(\boldsymbol{p}\boldsymbol{x}_i)$ and $(\boldsymbol{p}\boldsymbol{w}_i)$ mean scalar products. For every level of prices, consumers will face a different budget set. The locus of preferred allocations for every level of prices is the consumer's offer curve.

An allocation is said to be Pareto efficient, or Pareto optimal, if there is no other feasible allocation in the Edgworth economy for which both are at least as well off and one is strictly better off. The locus of points that are Pareto optimal given preferences and endowments is the Pareto set, noted as $\boldsymbol{P}$ in figure 5. The part of the Pareto set in which both consumers do at least as well as their initial endowments is the Contract curve shown in figure 5 and noted as $\boldsymbol{N}$ (kernel of market game).

We are interested in the equilibrium point(s) of the process of exchange where is fulfilled the Walrasian equilibrium [6]. Walrasian equilibrium is a price vector $\boldsymbol{p}$ and an allocation $\boldsymbol{x}$ such that, for every consumer the prices (i.e. the terms of trade) are such that what one consumer (group of consumers) wants to buy is exactly equal to what the other consumer (group of consumers) wants to sell. In other words, consumers' demands are compatible with each other. We note the locus of points that are in Walrasian equilibrium as $\boldsymbol{W}$ (two points in figure 5).

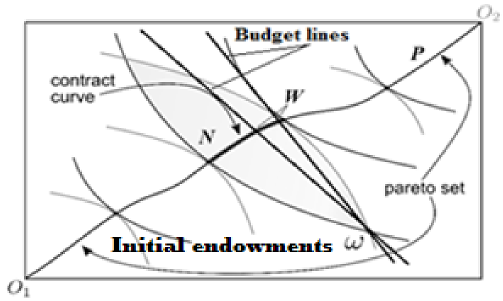

Fig.5. Pareto set and contract curve

In still other words, the quantity each consumer wants to buy at the given market prices is equal to what is available on the market. The following inclusion is true in the Edgworth economy [6]: $P \supset N \supset W$. In that sense a contract curve in the Edgeworth Box shows an exchange market in equilibrium and this is a particular representation of the Walrasian equilibrium theorem.

We had evaluated the consumer's preferences as value functions. In figure 6 are shown the indifference curves, calculations of the Pareto set ***P*** and the determination of the contract curve ***N***.

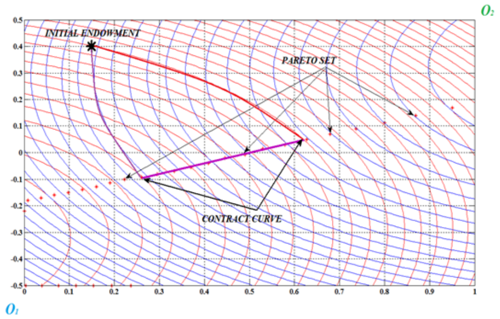

Fig.6. Real experiment-Pareto set and contract curve

The indifference curves in figure 6 are determined based on values functions evaluated by direct comparisons of couples of allocations $\boldsymbol{x}=(\boldsymbol{x}_1, \boldsymbol{x}_2) = ((x_{11}, x_{21}), (x_{12}, x_{22}))$. This is made through the discussed in the paper approach and algorithms for exact value function evaluation ($\mathbf{A}_{\mathbf{u}^*}\cap\mathbf{B}_{\mathbf{u}^*}=\varnothing$) [18]. After that we made quadratic approximation of the constructed value function. The little divergence from the theoretical convex requirements is due to the finite number of learning points and to the uncertainty in the expressed consumer's preferences. In the experiment for determination of the set $\mathbf{A}_{\mathbf{u}^*}$ and $\mathbf{B}_{\mathbf{u}^*}$we used a finite number of preferences expressed for couples of allocations ($\boldsymbol{x}=(\boldsymbol{x}_1, \boldsymbol{x}_2)$, $\boldsymbol{y}=(\boldsymbol{y}_1, \boldsymbol{y}_2)$):

$\mathbf{A}_{\mathbf{u}^*}=\{(x,y)\in\mathbf{R}^{2m} / (\mathbf{u}^*(x))>\mathbf{u}^*(y)\}$,

$\mathbf{B}_{\mathbf{u}^*}=\{(x, y)\in\mathbf{R}^{2m} / (\mathbf{u}^*(x))<\mathbf{u}^*(y)\}$.

The indifference curves could be determined by utility function evaluation also. The discussed previously in the paper stochastic procedures could be used for this purpose. In this case the learning points have to be defined as lotteries with Edgworth box allocations and consumers preferences in reference to learning triples of allocations. The described methodology and procedures allow for the design of individually oriented information systems [9]. Our experience is that the human estimation contains uncertainty at the rate of [10, 30] %. Such systems allow for exact evaluation of the Pareto set ***P***, a reasonable determination of the contract curve ***N*** and calculation of the Walrasian set ***W*** and may be autonomous or parts of larger decision support system [5, 6, 9]. The demands functions could be evaluated by direct comparisons or by the gambling approach. In that manner the incomplete information is compensated with the participation of qualitative human estimations.

In that manner we can state and solve the market-clearing equilibrium in principle and we can determine the contract curve and the Walrasian set in the Edgeworth box. The set of the Walrasian equilibriums ***W*** and the appropriate prices $\boldsymbol{p} = (p_1, p_2)$ are calculated based on the determined demand utility (value) functions and this is a meaningful prognosis of the market equilibrium. In that way can be forecast the competitive market equilibrium allocations $\boldsymbol{x}=(\boldsymbol{x}_1, \boldsymbol{x}_2) = ((x_{11}, x_{21}), (x_{12}, x_{22}))$ and the appropriate prices $\boldsymbol{p} = (p_1, p_2)$. The contract curves are specified on the individual consumers' preferences and show that there are possibilities to be made mutually advantageous trades. This means that one could unilaterally negotiate a better arrangement for everyone.

5. Utility Evaluation of The Best Growth Rate and Control Design

The complexity of the biotechnological systems and their singularities make them difficult objects for control. They are difficult to control because it is difficult to determine their optimal technological parameters [14], [17]. These parameters can depend on very complicated technological, ecological or economical market factors. Because of this in practice expert estimates are used. From outside the estimates are expressed by qualitative preferences of the technologist. The preferences themselves are in rank scale and bring the internal indetermination, the uncertainty of the qualitative expression. Our experience is that the human estimation of the process parameters of a cultivation process contains uncertainty at the rate of [10, 30] %. Because of this reason mathematical methods and models from the Utility theory and stochastic programming could be used in biotechnology. These stochastic methods, because of their essence, eliminate the uncertainty and could neutralize the wrong answers if one uses the gambling evaluation approach. Thus we achieve analytical math description of the complex system "Technologist-biotechnological process".

The approach used in the paper permits exact mathematical evaluation of the optimal specific growth rate of the fed-batch cultivation process according to the DM point of view. Let **Z** be the set of alternatives (**Z**={specific growth rates of the biotechnological process-μ}, **Z**≡[0, 0.6]) and **P** be a convex subset of discrete probability distributions over **Z**. The expert "preference" relation over **P** is expressed through ($\succ$) and this is also true for those over **Z** (**Z** $\subseteq$ **P**). The utility growth-rate function $U(.)$ is stochastically approximated by a polynomial [18].

$$U(\mu) = \sum_{i=1}^{n} c_i \mu^i$$

This polynomial representation permits analytical determination of the derivative of the utility function and easy implementation in the optimal control theory [16]-[18]. Following the approach we are looking for pattern recognition of the sets of positive preferences $\boldsymbol{A}_{u^*}$ and negative preferences $\boldsymbol{B}_{u^*}$:

$\boldsymbol{A}_{u^*} = \{(x,y,z,\alpha) / (\alpha u^*(x)+(1-\alpha)u^*(y))>u^*(z)\}$,

$\boldsymbol{B}_{u^*} = \{(x,y,z,\alpha) / (\alpha u^*(x)+(1-\alpha)u^*(y))<u^*(z)\}$.

The star in the notations means an empirical estimate of the utility of the technologist. The utility function $U(\mu)$ itself is built as a recurrent procedure for the recognition of the set $\boldsymbol{A}_{u^*}$. The DM compares "lotteries" ($\alpha x+(1-\alpha)y$, $x,y\in\mathbf{Z}$, $\alpha\in[0,1]$) with simple alternatives $z\in\mathbf{Z}$ and the answer is determined from him (" better", "worse" or "indifference, equivalency or impossibility for explicit delimitation"). The Biotechnologist (DM) determines his answer (for every comparison): $f(x,y,z,\alpha)=1$ for ($\succ$), $f(x,y,z,\alpha)=-1$ for ($\prec$) and $f(x,y,z,\alpha)=0$ for ($\approx$). The function $f(x,y,z,\alpha)$ is a probability function, subjective characteristic of the DM depicturing intuition and empirical knowledge and also including subjective and probability uncertainty of the answers. In the recurrent procedure "the training point" $(x,y,z,\alpha, f(x,y,z,\alpha))$ is treated as point from the set $\boldsymbol{A}_u$ with probability $D_1(x,y,z,\alpha)$ or a point from $\boldsymbol{B}_u$ with probability $D_2(x,y,z,\alpha)$). We suppose that (x,y,z,α) are given by probability distribution $F(x,y,z,\alpha)$. In fact this is a pseudo-random Lp_τ sequence of Sobol. Then probabilities $D_1(x,y,z,\alpha)$ and $D_2(x,y,z,\alpha)$ are the conditional mathematical expectations of $f(.)$ over the sets $\boldsymbol{A}_u$ and $\boldsymbol{B}_u$, respectively. With $D'(x,y,z,\alpha)$ we denote the conditional random value:

$$D'(x,y,z,\alpha) = \begin{cases} D_1(x,y,z,\alpha), & \text{when } M(f/x,y,z,\alpha)>0, \\ -D_2(x,y,z,\alpha), & \text{when } M(f/x,y,z,\alpha)<0, \\ 0, & \text{when } M(f/x,y,z,\alpha)=0. \end{cases}$$

The measurable function $D'(x,y,z,\alpha)$ is approximated by function of the type $G(x,y,z,\alpha)=(\alpha g(x)+(1-\alpha)g(y)-g(z))$. The function $g(x)$ is an approximation of the utility $U(.)$. The coefficients c_i^n take part in:

$$g^n(x)=\sum_{i=1}^{N} c_i^n \Phi_i(x)$$

$$(c^n,\Psi(t))=\alpha g^n(x)+(1-\alpha)g^n(y)-g^n(z)=G^n(x,y,z,\alpha)$$

The function $G^n(x,y,z,\alpha)$ is positive over $\boldsymbol{A_u}$ and negative over $\boldsymbol{B_u}$ depending on the degree of approximation of $D'(x,y,z,\alpha)$. In fact total recognition is impossible, because of the wrong preferences of the technologist caused by the uncertainty within his preferences ($A_{u^*}\cap B_{u^*}\neq\emptyset$). The process of the recognition of the sets A_{u^*} и B_{u^*} is shown on the figure (7). The polynomial approximation of the DM utility $U(\mu)$ is the smooth line in figure (7). The maximum of the utility function determines the „best" growth rate of the fed-batch process after the technologist. A session with 128 questions learning points (x,y,z,α, $f(x,y,z,\alpha)$) takes no more than 45 minutes.

The Value based control design is determined by the solution of the next optimal control problem: $\boldsymbol{max}(U(\mu))$, where the variable μ is the specific growth rate, ($\mu\in[0, \mu_{\max}]$, $D\in[0, D_{\max}]$).

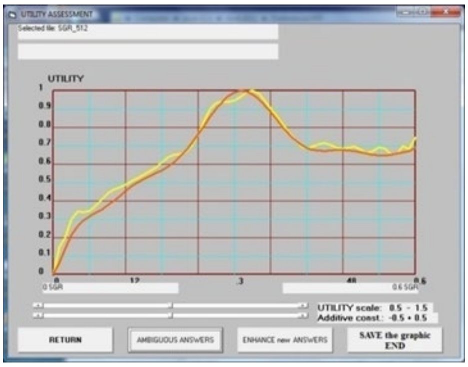

Fig.7. Growth rate utility

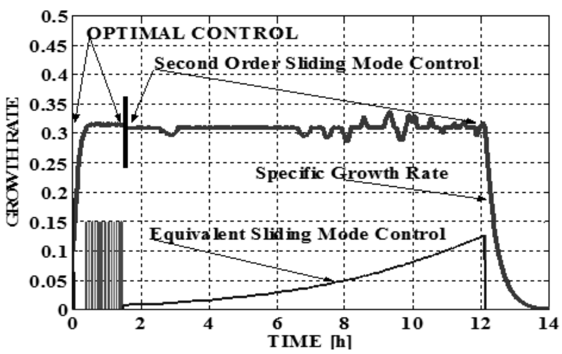

Fig.8. Stabilization of the fed-batch process

Here $U(\mu)$ is an aggregation objective function (the utility function) and D is the control input (the dilution rate):

$$\max(U(\mu)),\ \mu\in[0,\mu_{\max}],\ t\in[0,T_{\text{int}}],\ D\in[0,D_{\max}]$$

$$\dot{X}=\mu X-DX$$

$$\dot{S}=-k\mu X+(So-S)D$$

$$\dot{\mu}=m(\mu_m\frac{S}{(K_S+S)}-\mu)$$

The differential equation describes a *continuous biotechnological process*. The Monod-Wang model permits exact linearization to Brunovsky normal form following the procedures in papers [16], [17]. The optimal solution is determined with the use of the equivalent Brunovsky normal form of the differential equation above:

$$\dot{Y}_1=Y_2$$

$$\dot{Y}_2=Y_3$$

$$\dot{Y}_3=W.$$

In the formula, W denotes the control input of the Brunovsky model. The two differential equations above are equivalent as objects for control. The vector (Y_1, Y_2, Y_3) is the new state vector:

$$Y_1=u_1$$

$$Y_2=u_3(u_1-ku_1^2)$$

$$Y_3=u_3^2(u_1-3ku_1^2+2k^2u_1^3)+m(\mu_m\frac{u_2}{(K_S+u_2)}-u_3)(u_1-ku_1^2)$$

$$\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=\Phi(X,S,\mu)=\begin{pmatrix}\frac{X}{S_0-S}\\S\\\mu\end{pmatrix}$$

The derivative of the function Y_3 determines the interconnection between W and D, the inputs of the equivalent models. The control design is a design based on the Brunovsky normal form and the application of the Pontrjagin's maximum principle step by step for sufficiently small time periods T. The optimal control law has the analytical form [16], [17]:

$$D_{opt}=\text{sign}\left(\left(\sum_{i=1}^{6} ic_i\mu^{(i-1)}\right)(T-t)\left[\frac{(T-t)\mu(1-2kY_1)}{2}-1\right]\right)D_{\max},$$

$$where:\ \text{sign}\ (r)=1, r>0, \text{sign}\ (r)=0, r\le 0.$$

The sum is the derivative of the utility function. It is clear that the optimal "time-minimization" control is determined from the sign of the utility function derivative. The control input is $D=D_{\max}$ or D = 0. The solution is in fact a ***"time-minimization"*** control (if the time period T_{int} is sufficiently small). The control brings the system back to the set point for minimal time in any case of specific growth rate deviations.

The control law *of the fed-batch process* has the same form because $D(t)$ is replaced with $F(t)/V(t)$ in Monod-Wang model [16], [17]:

$$\dot{X}=\mu X-\frac{F}{V}X,$$

$$\dot{S}=-k\mu X+(So-S)\frac{F}{V},$$

$$\dot{\mu}=m(\mu_m\frac{S}{(K_S+S)}-\mu),$$

$$\dot{V}=F,$$

$$\dot{E}=k_2\mu E-\frac{F}{V}E$$

Thus, the feeding rate $F(t)$ takes $F(t)=F_{\max}$ or $F(t)=0$, depending on $D(t)$ which takes $D=D_{\max}$ or $D=0$. We conclude that the control law brings the system to the set point (optimal growth rate) with time minimization control, starting from any deviation of the specific growth rate as is shown in figure 8. We use this control law as a main part in a more complex chattering control law for stabilization of the system in the "best" growth rate [14], [16]-[18]. The deviation of the fed-batch process with this chattering control is shown on figure (8). After the stabilization of the system in equivalent sliding mode control position the system can be maintained around the optimal parameters with sliding mode control.

6. Conclusions

Human values (utilities) are integral part of the decision making process of the individual. They are the internal motivation for determining the main objective in the goal-oriented systems. Unfortunately, in most scientific investigations and developments, the subjective values and probabilistic expectations are not explicitly related and directly oriented towards the considered problem. In this aspect, especially important is the task of connecting the two contradicting tendencies: the requirement of ordinal information from mathematical and computational point of view and the cardinal nature of the empirical knowledge.

One of the possible scientific approaches in regards to these problems is that of multiattribute utility. In this manner in difficult for formalization and even verbally expressed weakly structurized problems and complex events we introduce the strict analytical approach, as analysis and analytically based synthesis, which allows for logically sound and mathematically precise decision formation. We achieve analytic model description of complex process with human participation. Such models ensure exact mathematical descriptions of problems in various areas for which the quantitative modeling is difficult: economics, biotechnology, ecology, and so on. These models guarantee that the powerful optimal control theory could be applied for exact mathematical solutions in such complex areas.

By the Edgworth box and the growth rate control examples we saw that the utility approach permits exact mathematical evaluation according to the consumers' point of view even though the human thinking is qualitative and pierced by uncertainty. Measurement, Expected utility theory and stochastic programming are some of the approaches for attainment of these purposes. These examples show that the presented methodology and mathematical procedures allow for the design of individually oriented decision support systems. Such systems may be autonomous or parts of larger intelligent information or decision support systems and can permit reasonable optimal solutions and prognoses.

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