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Infinite Capacity and Unbounded Spectral Efficiency in Massive MIMO Networks: Overcoming Pilot Contamination with MC-MMSE Precoding and Combining

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Abstract: Massive Multiple-Input Multiple-Output (MIMO) technology, hailed as a cornerstone for next-generation cellular networks, has been marred by the challenge of pilot contamination, which was widely believed to impose a finite limit on network capacity. This research reevaluates this notion, employing Multi-Cell Minimum Mean Square Error (MC-MMSE) precoding and combining techniques. Unlike previous studies relying on rudimentary models and suboptimal strategies, we demonstrate that capacity in Massive MIMO networks grows infinitely as the number of antennas, M, increases. By ensuring asymptotically linearly independent covariance matrices and employing MC-MMSE, both uplink and downlink spectral efficiencies were observed to rise without bounds. Our findings, supported by extensive simulations, challenge prior assumptions, paving the way for harnessing the full potential of Massive MIMO technology, even in the face of pilot contamination. This research fundamentally alters our understanding of the capacity limits in Massive MIMO networks, opening new avenues for higher spectral efficiency and unbounded network performance.

Keywords: *Channel Estimation, Coherent Interference, Massive MIMO, MC-MMSE Precoding/Combining, Pilot Contamination, Spatial Multiplexing, Spectral Efficiency, Uplink Data Transmission.*

1. Introduction

The Shannon capacity of a channel serves as an indicator of the spectral efficiency (SE) it can support. The utilisation of Massive MIMO (multiple-input multiple-output) technology results in an improvement of the overall spectral efficiency of cellular networks by employing spatial multiplexing techniques to accommodate a significant number of user equipment (UEs) within each cell [2], [27]. Consequently, it is widely recognised as a crucial time division duplex (TDD) technology that holds significant utility for the next generation cellular networks [1],[3],[4],[25]. The fundamental difference between the Massive MIMO and standard multi-user MIMO is each base station (BS) has the large number of antennas M, radio-frequency chains to process the signals independently. When channel estimations are used for coherent receive combining, the uplink signal strength of a specified User Equipment (UE) is amplified by a factor designated as M. Simultaneously, this approach leads to a reduction in the power of both noise and independent interference. The user has provided references in the form of numerical citations [6],[19]. The aforementioned concept is also applicable to downlink transmit precoding. The utilization of uplink pilot signalling is necessary for obtaining channel estimations, and due to the limited availability of pilot resources dictated by the channel coherence time, it becomes necessary to reuse the same pilots across multiple cells [14]. The channel estimates utilization in for coherent receive combining results in an enhancement of the uplink signal strength of a selected User Equipment (UE) by a factor M, concurrently diminishing the noise strength and independent interference [1] [19]. The aforementioned concept is also applicable to downlink transmit precoding. The phenomenon of pilot contamination gives rise to two significant outcomes. Firstly, the quality of channel estimation is compromised due to the interference caused by the presence of pilots [33]. Secondly, the channel estimate of a desired user equipment (UE) is influenced by the channels of other UEs that are causing interference, as they share the same pilot. In the context of maximum ratio (MR) combining/precoding, considering identically distributed independent (i.i.d.) Rayleigh fading channels, it has been shown in references [1] and [2] that the interference caused by these user equipment’s (UEs) while transmitting data is amplified by a factor M. This suggests that as the number of users, denoted as M, approaches infinity, the presence of pilot contamination leads to a limited spectral efficiency. The limit for a greater number of antennas has also been explored in other approaches of combining/precoding, such as the least mean squared error (MMSE) [8]. The authors in references [5], [6], and [7] examined the concept of Single-Cell Minimum Mean Square Error (SC-MMSE), while the concept of Multi-Cell Minimum Mean Square Error (MC-MMSE) was explored in references [8] and [9]. The key differentiation lies in the utilization of channel estimates by

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the base station (BS) in MMSE of a multi-cell scenario (MC-MMSE) as well as for MMSE of a single-cell (SC-MMSE) scenario. Specifically, in MC-MMSE, the BS incorporates channel estimates obtained from all user equipment's (UEs) within the network, while in SC-MMSE, the BS solely relies on channel estimates acquired from the UEs present within its own cell. In the context of independent and identically distributed (i.i.d.) Rayleigh fading channels, which do not exhibit spatial correlation, it has been demonstrated that the spectral efficiency (SE) converges to a finite value as the number of antennas (M) approaches infinity. Certain specific instances of spatially correlated fading can result in the formation of covariance matrices that are rank deficient, as indicated by references [10] and [12]. However, in the case where the user equipment's (UEs) sharing the pilot with orthogonal support exhibit covariance matrices with rank deficiency, the phenomenon of pilot contamination gradually diminishes as the spectral efficiency (SE) increases without bounds [11]. By considering that every element of arbitrary covariance matrices $A_1 A_2$ must be zero, as:

$$A_1 = \begin{bmatrix} a & c \\ c^* & b \end{bmatrix} \qquad A_2 = \begin{bmatrix} d & f \\ f^* & b \end{bmatrix} \tag{1}$$

Representing the real covariance matrices of two arbitrarily positioned User Equipment's (UEs) as instances of a random variable having a continuous distribution, then the occurrence of the first element in it $ad + cf^* = 0$ happens with a probability of zero [35]. Consequently, the occurrence of orthogonal support is highly unlikely in practical scenarios, albeit it may be observed in exceptional instances. According to references [10] and [12], the uniform linear arrays (ULAs) single-ring model suggests that if the angular support in channels is non-overlapping, orthogonal support is expected. However, reference [13] presents ULA microwave calculations that indicate practical channels do not exhibit orthogonal support and instead exhibit a high degree of irregularity. Practical covariance matrices, particularly at microwave frequencies, are observed to lack orthogonal support [11]

In the existing literature, a range of methodologies have been identified and referred to as pilot decontamination, which aim to address the issue of pilot contamination. In the context of covariance matrices, category one involves the allocation of pilots to user equipment's (UEs) with the objective of identifying pairings that exhibit significant differences in support [10],[11],[12],[14]. The aforementioned approach has the potential to effectively mitigate pilot contamination. However, it is important to note that the finite limit can only be eliminated under exceptional circumstances, specifically while the covariance matrices hold a support that is orthogonal in nature. The category number two of techniques makes use of semi-blind estimation to differentiate the channel subspace of the targeted UE from the channel subspace of the interfering signals [15], [16], [17], [19]. In the scenario where both M and the channel coherence block size approach infinity, it has been observed that this technique has the potential to eliminate pilot contamination entirely [14],[16],[18]. In practice, the channel coherence is limited and cannot accommodate unique pilots for each cell, posing challenges in approaching this limit. The third type utilizes more than one pilot phases with distinct sequences of pilot in order to progressively mitigate pilot contamination, as evidenced by references [20], [21], and [33], without relying on statistical data. Nevertheless, the total duration of the pilot exceeds or equals the integer value of the count of User equipment's (UEs), enabling the allocation of mutually orthogonal pilots to all UEs. Consequently, this approach effectively resolves the issue of pilot contamination. The current approach lacks scalability when applied to expansive cell networks. Pilot contamination precoding, the fourth category of precoding, is a technique that effectively mitigates interference by enabling coherent shared transmission and reception throughout the whole network [22], [23]. The proposed approach appears to offer the potential for generating an unlimited spectral efficiency (SE). However, it should be noted that this claim has not been conclusively demonstrated and relies on the assumption that complete user equipment (UE) data is accessible at each base station (BS). It is important to acknowledge that this requirement may not be feasible or practical in real-world scenarios.

In summary, pilot contamination is a fundamental concern that imposes a constraint on the achievable spectral efficiency, unless under exceptional circumstances of extreme nature. This study illustrates that the observed phenomenon can be attributed primarily to the prevalence of investigating suboptimal combining/precoding techniques, such as Maximum Ratio Combining (MRC) and Successive Cancellation Minimum Mean Square Error (SC-MMSE), while focusing on idealized independent and identically distributed (i.i.d.) Rayleigh fading channels, as previously explored in the literature [8] & [9] on MC-MMSE. In the context of pilot contamination, it has been exhibited that the utilization of MC-MMSE combining/precoding leads to an unbounded increase in spectral efficiency (SE) when the pilot-sharing user equipment (UEs) possess asymptotically-linearly independent covariance matrices [22], [35]. It is important to highlight that the vectors [a b c] T and [d e f] T in (1) are linearly independent if they are non-parallel, which is typically the situation for covariance matrices that are randomly generated. Consequently, our findings are contingent upon a condition that is highly likely to be fulfilled in practical scenarios, representing the general case. In contrast, prior studies on the asymptotic aspects of Massive MIMO have predominantly investigated special circumstances that are improbable in practical settings. In contrast to prior research, this study does not employ multicell cooperation and does not necessitate support of orthogonality for covariance matrices.

We can refer paper [24] which presents a demonstration of a scenario involving a two-user uplink, which yields similar findings. This study aims to validate the findings within a broader scope, encompassing both uplink and downlink scenarios. Section II presents a generalization of the findings to a configuration involving multiple cells. Section III of the document presents the numerical data, whereas Section IV provides a concise summary of the significant conclusions.

In this academic discourse, it is pertinent to discuss the topic of notations. In the context of matrix Z, it is conventionally understood that the lowercase letter z represents a vector. The superscripts Z^T, Z^{-1}, and Z^H denote the transpose, inverse, and Hermitian (conjugate-transpose) of the matrix Z, respectively. The Frobenius norm of matrix Z is denoted as $||Z||^F$. The identity matrix is represented as I. The complex Gaussian random vector with mean m and covariance R is denoted as $\mathcal{N}$ (m, R).

2. Spectral Efficiency of a Multi-User System

This analysis shall help us to examine a Massive MIMO network consisting of L cells, and each cell composes of a Base Station (BS) equipped with a total of M antennas, serving K User Equipments (UEs) [13], [29]. The received signal at base station j is given as: $\boldsymbol{y_j}(\boldsymbol{y_j} \in \mathbb{C}^{\boldsymbol{M}})$

$$y_j = \sum_{l=1}^{L}\sum_{i=1}^{K} \sqrt{\rho}\, \mathrm{h}_{jli}x_{li} + \mathrm{n}_j \quad (2)$$

where ρ indicates the normalized transmit power, for the i^{th} UE in lth cell, x_{li} stands for the unit power. $\mathrm{h}_{jli} \sim \mathcal{N}_{\mathbb{C}}(0, \mathrm{A}_{jli})$ is the channel, n_j is the noise at the base station. Using the uplink pilot power ρt of every user along with estimation methods as in [7], the estimate of j^{th} base station can be given as:

$$\hat{h}_{jli} = A_{jli}Q_{jli}^{-1}\left(\sum_{j'}^{L} h_{jl'i} + \frac{1}{\sqrt{\rho^t}}n_{ji}\right) \sim \boldsymbol{\mathcal{N}}_{\mathbb{C}}(\boldsymbol{0}, \boldsymbol{\Phi_{jli}}) \quad (3)$$

where n_{ji} is the noise, $\mathrm{Q}_{jli}^{-1} = \frac{1}{\sum_{l'=1}^{L}\mathrm{A}_{jl'i}+\frac{1}{\rho t}\mathrm{I}_M}$ and $\Phi_{jli} = \mathrm{A}_{jli}(\mathrm{Q}_{jli}^{-1}\,\mathrm{A}_{jli})$, the error of estimation is independent of $\hat{\mathrm{h}}_{jli}$ It can be estimated as, $\tilde{\mathrm{h}}_{jli} = \mathrm{h}_{jl\,i} - \hat{\mathrm{h}}_{jli} \sim \mathcal{N}_{\mathbb{C}}(0, \mathrm{A}_{jli} - \Phi_{jli})$, all the UE estimates can be correlated $\mathbb{E}\left\{\hat{\mathrm{h}}_{jni}\mathrm{h}_{jmi}^{\mathrm{H}}\right\} = \mathrm{A}_{\mathrm{jni}}(\mathrm{Q}_{\mathrm{ji}}^{-1}\,\mathrm{A}_{\mathrm{jmi}})$.

2.1. Calculating Uplink Data Transmission

The lower bound capacity in the uplink, as described interferences [5] and [26], can be expressed as:

$$SE_{jk}^{ul} = \left(1 - \frac{\tau_p}{\tau_c}\right)\mathbb{E}\{log_2(1 + \gamma_{jk}^{ul})\} \quad (4)$$

The metric denoted as bits per hertz (bits/Hz) is utilized to quantify the spectral efficiency, while the effective signal-to-interference-plus-noise ratio (SINR) is employed to assess the quality of the received signal.

$$\gamma_{jk}^{\mathrm{ul}} = \frac{|\mathrm{v}_{jk}^{\mathrm{H}}\hat{\mathrm{h}}_{jjk}|^2}{\mathbb{E}\left\{\sum_{(l,i)\neq(j,k)}|\mathrm{v}_{jk}^{\mathrm{H}}\mathrm{h}_{jli}|^2 + |\mathrm{v}_{jk}^{\mathrm{H}}\tilde{\mathrm{h}}_{jjk}|^2 + \left|\frac{\mathrm{v}_{jk}^{\mathrm{H}}\mathrm{v}_{jk}}{\rho^{\mathrm{ul}}}\right|\hat{\mathrm{h}}_j\right\}} \quad (5)$$

$$= \frac{|\mathrm{v}_{jk}^{\mathrm{H}}\hat{\mathrm{h}}_{jjk}|^2}{\mathrm{v}_{jk}^{\mathrm{H}}\left(\sum_{(l,i)\neq(j,k)}\hat{\mathrm{h}}_{jli}\hat{\mathrm{h}}_{jli}^{\mathrm{H}} + \mathrm{Z}_j\right)\mathrm{v}_{jk}} \quad (6)$$

Where $\mathbb{E}\{|\hat{\mathrm{h}}_j|\}$ is the conditional expected channel estimate value for base station j. The SINR (Signal-to-Interference-plus-Noise Ratio) expressed in equation (6) can be optimized to achieve maximum effectiveness for user k within cell j.

$$\mathrm{v}_{jk}^{\mathrm{H}} = \left(\sum_{l=1}^{L}\sum_{i=1}^{K}\hat{\mathrm{h}}_{jli}\,\hat{\mathrm{h}}_{jli}^{\mathrm{H}} + \mathrm{Z}_j\right)^{-1}\hat{\mathrm{h}}_{jjk} \quad (7)$$

The proposed approach can be referred to as the optimization of the receive combine scheme for MC-MMSE. With minor modifications, it can also be adapted for the SC-MMSE combiner.

$$\mathrm{v}_{jk}^{\mathrm{H}} = \left(\sum_{i=1}^{K}\hat{\mathrm{h}}_{jli}\;\hat{\mathrm{h}}_{jli}^{\mathrm{H}} + \bar{\mathrm{Z}}_j\right)^{-1}\hat{\mathrm{h}}_{jjk} \quad (8)$$

The algorithmic complexity for a MC-MMSE method is more compared to the SC-MMSE method, as stated in reference [9]. Apart from this distinction, both schemes rely on channel estimates that are computed at the base station. Additionally, the pilot overheads are similar in nature, as the pilots serve the purpose of estimating both intra and inter cell channels.

By substituting the value of (7) into equation (6), we obtain the following result:

$$\gamma_{jk}^{\mathrm{ul}} = \hat{\mathrm{h}}_{jjk}\left(\sum_{(l,i)\neq(j,k)}\hat{\mathrm{h}}_{jli}\,\hat{\mathrm{h}}_{jli}^{\mathrm{H}} + \mathrm{Z}_j\right)^{-1}\hat{\mathrm{h}}_{jjk} \quad (9)$$

Under the assumption that, as $M \to \infty$
$j, l, i, \lim\inf_M \frac{1}{M}\mathrm{tr}(\mathrm{A}_{jli}) > 0$ and $\lim\sup_M ||\mathrm{A}_{jli}||2<\infty$.

It can be postulated that the signal-to-interference-plus-noise ratio (SINR) exhibits an unbounded increase as the number of base station antennas (M) approaches infinity, regardless of the existing contamination of pilots. There is a distinct disparity to the finite limit discussed in MR [6], as well as other single-cell combining techniques [1], [5], [7]. This distinction arises from the undeniable ability of M-MMSE to effectively attenuate the interference, which is coherent and results from contamination of pilots, given that certain assumptions are met. The covariance matrices $\{\mathrm{A}_{jlk}$:

l=1, 2, …, L} generated by the channels associated with the User Equipment (UE) devices that utilize the same pilots and transmit to the base station j are linearly independent. This condition can also be extended to the roughly calculated channels $\{\hat{h}_{jkl}: l = 1, \ldots, L\}$. In a more comprehensive framework, where hjjk exhibits asymptotic linear independence from the estimated channels of all pilot-interfering UEs, it is worth noting that certain estimates of interfering channels can be represented as linear and straight combinations of additional interfering channels [23].

Let Sjk be defined as a subset of the interfering channel estimates, where Sjk is a subset of the set $S_{jk} \subseteq \{\hat{h}_{jkl}: \forall l \neq j\}$. The combining vector calculation in equation (7), only the estimates within this subset are considered. Another noteworthy observation is that the received combined vector has become orthogonal to the range of values it achieved through Sjk with multi-cell zero forcing (MC-ZF) [28]. The aforementioned procedure demonstrates unlimited spectral efficiency when condition M approaches infinity. The system maintains its array gain as M increases and effectively mitigates interference caused by all user equipment (UE), including those originating from Sjk. The arrangement can be represented as a:

$$v_{jk} = \hat{H}_{jk}(\hat{H}_{jk}^{H}\hat{H}_{jk})^{-1}e_1 \quad (10)$$

In this context, e1 refers to the first column, while $\hat{H}_{jk}$ represents the matrix with $\hat{h}_{jjk}$ as the first column and the remaining columns containing channel estimates. The utilization of M-MMSE combining has been identified as the most effective technique, thereby implying that it possesses an unlimited spectral efficiency when employed in conjunction with MC-ZF, as stated in reference [28].

2.2. Calculating Downlink Data Transmission

Let us now consider the downlink data transmission scenario, in which BS from cell n transmits the signal $x_n = \sqrt{\rho^{dn}}\sum_{n=1}^{K} w_{ni}\varsigma_{ni}$, $\varsigma_{ni} \sim \mathcal{N}_{\mathbb{C}}(0, A_{jli})$ is the data signal considered for UE i, and the standardized power is given as ρ^{dn} This signal serves the actual purpose of transmitting the precoding vector w_{ni} where it has to meet the condition where it has to meet the condition $\mathbb{E}\left\{\left|\left|w_{ni}\right|\right|^2\right\} = 1$, so that it equals the standardized power allocated to that particular UE, resulting in $\mathbb{E}\left\{\left|\left|w_{ni}\right|\right|^2\right\} = \rho^{dn}$. The methods employed in [5],[26] for the channel capacity at downlink of UE k of cell j can be lower bounded by $SE_{jk}^{dl} = \left(1 - \frac{\tau_p}{\tau_c}\right)\log_2(1 + \gamma_{jk}^{dl})$

with bits/Hz as units.

$$\gamma_{jk}^{dl} = \frac{|\mathbb{E}\{h_{jjk}^{H}w_{jk}\}|^2}{\sum_{n=1}^{L}\sum_{i=1}^{K}\mathbb{E}\{|h_{njk}^{H}w_{ni}|^2\} - \mathbb{E}\{|h_{jjk}^{H}w_{jk}|^2\} + \frac{1}{\rho^{dn}}} \quad (11)$$

γ_{jk}^{dl} has the dependency on all precoding vectors $\{w_{ni}\}$. As there is a need of selective precoding, it can ideally be done by collaboratively selecting across cells, making precoding optimization problematic in practice. Using uplink-downlink duality as in [9], It is fair to choose $\{w_{ni}\}$depending on the MC-MMSE combining vectors depending on the MC-MMSE combining vectors $\{v_{jk}\}$ as in [32]. With this MC-MMSE precoding can be given as:

$$w_{jk} = \sqrt{v_{jk}}v_{jk} = \sqrt{v_{jk}}\left(\sum_{n=1}^{L}\sum_{i=1}^{K}\hat{h}_{jni}\hat{h}_{jni}^{H} + Z_j\right)^{-1}\hat{h}_{jjk} \quad (12)$$

Under the assumption that,

as $M \to \infty \quad \forall j, n, i, \lim\inf_M \frac{1}{M}\mathrm{tr}(A_{jni}) > 0$ and lim $\sup_M \left|\left|A_{jni}\right|\right|_2 < \infty$, if MC-MMSE precoding is used then the SINR γ_{jk}^{dl} increases without any bound. Regardless of the inferior assumptions made in precoding in M-MMSE, uniform power allotment, and not estimating the immediate understanding of the precoded channels, all UEs in the network achieve an asymptotically no bound downlink SE [22]. The sole condition to satisfy is the estimates of channel for the intended user equipments (UEs) must not have any linear dependency from the pilot contaminating UEs channel estimates that are located in neighboring cells. The numerical illustration that the downlink SE rises with no bound as M→∞ is done in this section.

2.3. Achievable SE Calculation with MC-MMSE Combining and Precoding

If the base station (BS) lacks familiarity in matrices of covariance, a different approach for estimation of channel involves independently estimating every entry of h_k, disregarding the inter relation the elements have. The element or component-wise minimum mean square error (EW-MMSE) estimator, as described in reference [30], is obtained by utilizing solely the principal diagonals of matrices A1 and A2. Efficient diagonal calculation can be achieved using a limited quantity of samples, without the necessity for the sample size to increase with M [30], [32]. The MC-MMSE scheme, when employing the covariance matrix diagonals, exhibits unbounded growth as the value of M approaches infinity. To ensure simplicity and accommodate spatial limitations, we will focus on the uplink scenario for detecting the kth user equipment (UE) of cell j. This detection can be achieved by utilizing the approximate combining vector of minimum mean square error (MC-MMSE).

$$v_{jk} = \left(\sum_{l=1}^{L}\sum_{i=1}^{K}\hat{h}_{jli}\hat{h}_{jli}^{H} + S_j\right)^{-1}\hat{h}_{jjk} \quad (13)$$

$SE_{jk}^{ul} = \left(1 - \frac{\tau_p}{\tau_c}\right)\log_2(1 + \gamma_{jk}^{ul})$where, $S_j = \sum_{l=1}^{L}\sum_{i=1}^{K}(D_{jli} - D_{jli}\Lambda_{ji}^{-1}D_{jli}) + \frac{1}{\rho^{ul}}I_M$ is the diagonal matrix, the diagonal elements D_{jli} and Λ_{ji} allow EW-MMSE estimate to have a far lower computing complexity than MMSE estimation. The combining strategy described in equation (13) can be implemented without requiring knowledge of the complete channel covariance matrices. This is possible because the strategy relies solely on the diagonal elements of A_{jli} and the separately estimated $\hat{h}_{jli}$ elements, without considering spatial channel correlation [34]. The lower bound SE for kth UE of cell j can be expressed as: [bits/Hz] where,

$$\gamma_{jk}^{ul} = \frac{|\mathbb{E}\{v_{jk}^{H} h_{jjk}\}|^2}{\sum_{n=1}^{L}\sum_{i=1}^{K}\mathbb{E}\{|v_{jk}^{H} h_{jli}|^2\} - \mathbb{E}\{|v_{jk}^{H} h_{jjk}|^2\} + \frac{1}{\rho^{ul}}\mathbb{E}\left\{\left\|v_{jk}\right\|^2\right\}}$$

The underlying premise in this context is to solely utilize the diagonal elements of covariance matrices for the purposes of channel-estimation and receive-combining. This assumption can be substantiated by employing MC-MMSE combining. Based on the previously conducted calculations, it has been observed that the signal to interference plus noise-ratio (SINR) value γ_{jk}^{ul} experiences an unbounded growth as the quantity of antennas (M) advances towards infinity. In order to attain an unrestricted uplink spectral efficiency (SE) and consequently an infinite capacity, it is necessary for the diagonals of the covariance matrices to be asymptotically linearly independent and for them to be known at the base station (BS) [27]. The aforementioned criterion is commonly satisfied due to the fact that even small stochastic changes in the components of the covariance matrix are adequate for establishing the property of asymptotic linear independence.

3. Results and Discussion

Let us commence by contrasting three techniques employe d for the generation of channel covariance matrices and the resulting spatial correlation.

1) A Uniform Linear Array (ULA) having antenna spacing of half of the wavelength and average valued large-scale fading β is represented by a single-ring model. θ being the angle of arrival (AOA) and uniformly distributed scatters with an angular interval of $[\theta - \Delta, \theta + \Delta]$, the (x, y) th element of A is $[A]_{x,y} = \frac{\beta}{2\Delta}\int_{-\Delta}^{\Delta} e^{\pi i(x-y)\sin(\theta+\delta)} d\delta$.
2) The ULA model of exponential correlation factor $r \in [0,1]$ among neighboring antennas, β, the average large-scale fading and θ being the angle of arrival (AOA), leading to $[A]_{x,y} = \beta r^{|x-y|} e^{i(x-y\theta)}$.
3) Over the array, uncorrelated Rayleigh fading with average large-scale fading and independent log-normal large-scale fading fluctuations.

3.1 Uplink

We analyze the difficult symmetric arrangement with L = 2 cells, each cell having K = 2 UEs, pilots with duration $\tau_p = K$, and coherence slabs/blocks of $\tau_c = 200$channel uses. The base stations (BSs) are strategically positioned at the four corners of the region, while the user equipment (UE) is in close proximity to the cell borders. The AoAs (Angles of Arrival) and distances between the UEs and BSs are comparable, although not identical. Consequently, the level of pilot contamination is significantly elevated in this configuration.

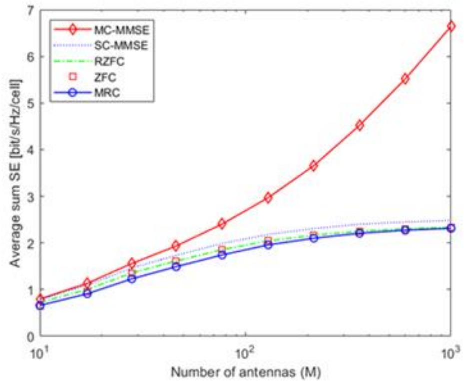

Fig. 1. Uplink SE as a function of M

The graph showcases (Figure 1) the spectral efficiency per cell for various transmission schemes, namely MC-MMSE, SC-MMSE, MRC, and MC-ZF. It is observed that these schemes yield similar spectral efficiency values when the number of antennas (M) is 10. However, a significant disparity becomes apparent as M increases. The findings presented in this study indicate that there is a positive correlation between the slope of the curve for MC-MMSE and the number of M per UE. The remaining combining strategies are not capable to reduce the coherent interference resulting from of User Equipments (UEs) from neighbouring cells, as they approach their finite asymptotic boundaries. Figure 2 illustrates the logarithmic representation of the uplink spectral efficiency per user equipment (UE) as a relation of the quantity of antennas, considering the exponential correlation model with a correlation coefficient of 0.5.

the asymptotic behaviour of the spectral efficiency. In the pilot and data transmission, the SNR average value recorded near the BS antenna is put to be equal: $\frac{\rho^{ul}\mathrm{tr}(A_{jli})}{M} = \frac{\rho^{t}\mathrm{tr}(A_{jni})}{M}$.

In case of intracell UE's the average valve of SNR is observed around -6.8 dB and it lies between -7.5 dB and -12 dB for interfering UE's in adjacent cells. MC-MMSE gives a spectral efficiency that increases without bound. The SE exhibits a linear growth pattern when plotted on a logarithmic scale along the horizontal axis. The immediate effective Signal to Interference plus Noise-Ratio (SINR) exhibits a linear relationship with the parameter M.

Due to the similarity of the channel estimations, comprehensive interference suppression results in significant degradation of the desired signal. The MC-MMSE technique, however, reveals a significant balance between reducing interference and effectively combining the desired signal, leading to enhanced spectral efficiency.

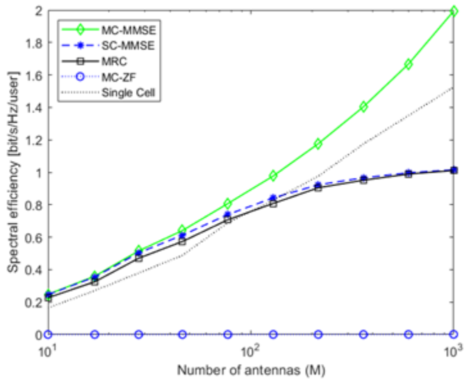

Fig. 2 Standard error of uplink as a function of M, for exponential correlation model for covariance matrices.

In order to mitigate the issue of pilot contamination, the reference curve known as "Single Cell" is designed to specifically cater to a scenario where the four cells are actively operating within separate coherence blocks. The utilisation of MMSE combining is implemented as a result of the inclusion of an additional pre-log factor of 1/4. Consequently, the spectral efficiency (SE) exhibits unbounded growth, albeit at a slower pace compared to MC-MMSE. Consequently, even in the case of a small system with a parameter L equal to 4, the avoidance of pilot contamination through single cell approaches can be deemed inefficient. Next, we inspect the non-related Rayleigh-fading-type, which assumes that there are non-dependent large-range fading variations across the array. Figure 3 illustrates the up-link spectral efficiency (SE) using M = 200 antennas, with standard deviations varying from 0 to 5. In the event that σ = 0, representing the exceptional scenario where the covariance matrices exhibit linear dependence, it can be observed that the MC-MMSE technique does not offer any distinct benefits when compared to the SC-MMSE or MRC methods. This particular occurrence has garnered significant attention within scholarly communities, predominantly due to its capacity to streamline mathematical examination.

Nevertheless, when moving away from the scaled identity type by incorporating minute changes in the large-range fading across the array, resulting in linearly independent covariance matrices, the MC-MMSE method demonstrates notable performance benefits compared to SC-MMSE and MRC techniques. As the variances increase, the standard error (SE) with MMSE channel assessment improves at a faster rate compared to the SE with zero forcing (ZF) channel estimation, eventually converging to a similar level. The MC-ZF scheme is unlikely to achieve superiority due to the fact that the MC-MMSE scheme is considered maximum. The inspiration for our simulation is derived from the calculated data provided in reference [29], which presents significant difference of approximately 4 dB across a massive MIMO-array with a standard deviation of approximately 4.

3.2 Downlink

The achieve same SNR the setup for downlink is similar to that of uplink. The configuration includes spatial channel interrelation and large-scale changes throughout the arrangements made as array. We assume a combination of the exponential correlation model: $[A]_{x,y} = \beta r^{|y-x|} e^{i(y-x)\theta} 10^{(f_x+f_y)/20}$ the correlation factor includes r = 0.5 and AOA(θ).

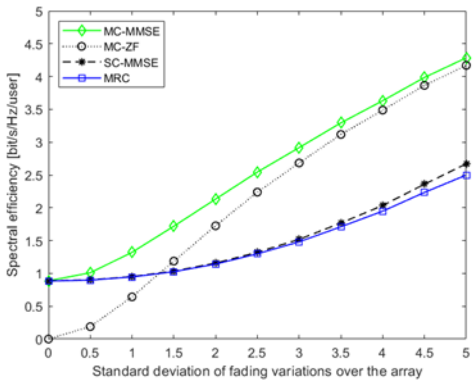

Fig. 3 Standard deviation of the uplink SE, considering large-scale fading variations.

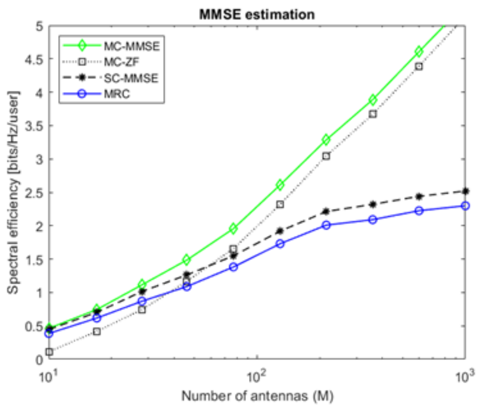

Fig. 4 Standard error of downlink with M as the function using MMSE estimator with full covariance information.

The values of the function f exhibit significant independent variations across the entire array, with a standard deviation of σ = 4. Figure 4 illustrates the connection among the downlink spectral efficiency and the quantity of antennas, denoted as M, when employing the minimum mean square error (MMSE) estimator with complete channel covariance matrices.

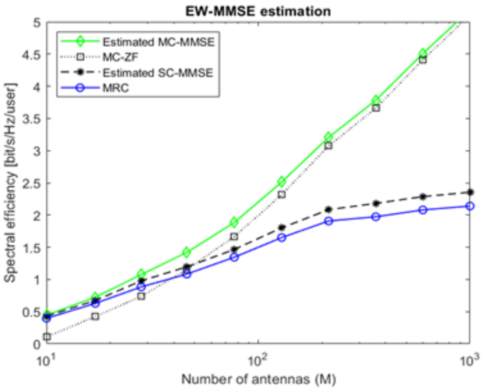

Fig. 5 EW-MMSE calculator with knowledge of covariance matrices diagonals.

The findings depicted in Figure 4 exhibit a resemblance to those illustrated in Figure 3, which pertains to the uplink scenario. The spectral efficiencies (SEs) achieved by multi-carrier minimum mean square error (MC-MMSE) and multi-carrier zero-forcing (MC-ZF) techniques exhibit unbounded growth, while the SEs attained by single-carrier minimum mean square error (SC-MMSE) and maximum ratio combining (MRC) methods range to finite boundaries. In disparity to the up-link, the downlink implementation of both MC-MMSE and MC-ZF precoding techniques is considered suboptimal. However, it is worth noting that these two methods exhibit asymptotic equivalence. The aforementioned behavior is also evident in the case of EW-MMSE, as depicted in Figure 5. As a suboptimal estimator, it fails to account for the off-diagonal elements present in covariance matrices.

4. Conclusion

In contrast to previous research findings suggesting the presence of a finite limit, our study provides evidence of such capacity in massive-MIMO systems exhibits unrestricted growth as the quantity of antennas (M) approaches infinity, particularly in the context of contamination of pilots. The study successfully demonstrated that the implementation of MC-MMSE precoding/combining results in an unbounded increase in the typical lower limits on capacity. These schemes exploit the linear non-depending covariance matrices, as well as the linear independence of the MMSE-channel estimations of UEs which make use of the same pilot. The auto-correlation matrices may possess full rank, and minimal fluctuations in eigenvalues are adequate for our conclusions to remain valid. In specific scenarios, it has been observed that the channel covariance matrices exhibit linear correlation. However, it has been noted that these matrices are not robust against minor perturbations in their covariance structure [34]. Consequently, these anomalies are considered highly improbable occurrences that are unlikely to manifest in real-world scenarios or be selected from a random sample, despite their extensive scrutiny in scholarly publications. The finite limit of the SE of MRC, also referred to as conjugate/combined beamforming or matched filtering, suggests that this approach is generally not asymptotically appropriate in the context of Massive MIMO [31]. It is imperative to acknowledge that our research findings do not imply the absence of the pilot contamination effect. While there remains a decline in performance attributable to estimation errors and interference rejection, the fundamental capacity constraint is no longer observed. Due to the simplifications inherent in the capacity lower bounds examined, it is deemed that the linear MC-MMSE technique is adequate in practical scenarios, wherein interference could be treated like noise at the receivers.

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