International Journal of
INTELLIGENT SYSTEMS AND APPLICATIONS IN ENGINEERING
ISSN:2147-6799 www.ijisae.org Original Research Paper

Optimization of Multi-Objective Fixed Charge Bulk Transportation Problem

Sangita Sheoran [1] and Kuldeep Tanwar[2]

Submitted: 15/03/2024 **Revised**: 30/04/2024 **Accepted**: 07/05/2024

Abstract: In this work, a multi-objective fixed charge bulk transportation problem (MOFCBTP) is considered that involves optimizing the transportation of bulk commodities from multiple sources to multiple destinations. The considered problem involves reducing the delivery cost and time simultaneously while satisfying supply and demand constraints and considering fixed charges associated with each transportation route. This problem is commonly encountered in logistics and supply chain management. Prior to this research, no work has been found on MOFCBTP in the literature. A method has been proposed in this work to provide acceptable optimal solutions of the problem. Finally, a numerical example is solved by using the proposed method to show the practical application of the method.

Keywords: *bulk transportation problem; fixed charge; multi objective optimization; optimal solution; zero suffix method; solution pairs.*

1.Introduction

Bulk transportation refers to the movement of large quantities of goods or materials, such as coal, grains, oil, chemicals, etc., from one location to another. This type of transportation typically involves the use of specialized equipment, such as trucks, trains, or ships, that can handle large volumes of cargo efficiently. Bulk transportation presents unique challenges that must be carefully managed to ensure that goods are delivered on time and in good condition. Effective management of bulk transportation requires careful planning, coordination, and optimization of resources, including transportation routes, modes of transport, and storage facilities. This involves the use of sophisticated algorithms and models to identify the most efficient and cost-effective solutions to transport goods, taking into account factors such as cost, time, distance, supply, and demand variability.

Bulk transportation problem (BTP) refers to the challenge of designing and implementing transportation strategies that can efficiently and effectively move large volumes of goods or materials from one location to another. This problem has important implications for the competitiveness of industries, the cost and availability of goods, and the sustainability of transportation systems. In this problem, the cost of transportation does not depend upon the quantity of the material to be delivered. Initially, Maio and Roveda [1] presented the concept of BTP with a least cost objective. Srinivasan and Thompson [2] proposed an approach comprising two stages for solving the problem solved by Maio and Roveda [1].

The single-objective BTP is extended into the multi-objective BTP (MOBTP), which has two objectives: minimizing the cost and time of transportation. Here, as there are two objectives, cost and time, which are to be minimized, the optimum solution can only be a trade-off between the two.

Prakash and Ram [3] used the branch and bound technique to reduce the cost and duration of transportation simultaneously. They converted the MOBTP into a single-objective BTP using priority factors. Prakash et al. [4] discussed Pareto optimal solutions to MOBTP. Prakash et al. [5] presented an efficient heuristic to get pareto optimal solutions of MOBTP. Prakash et al. [6] presented a method based on the lexicographic minimum to attain cost-time trade-offs of MOBTP by considering a sequence of prioritized MOBTPs whose solution gives the set of all cost-time trade-offs of MOBTP. A lexicographic minimum is used to solve the prioritized MOBTPs.

[1]*Research Scholar, MVN University, Palwal, Haryana, India*
[2]*Department of Mathematics, MVN University, Palwal, Haryana, India*

A problem in which a fixed charge is associated between each delivery point and the receiving point is called a fixed-charge transportation problem (FCTP). This charge is added to the transportation cost for moving a specific quantity of a commodity. This charge may be the cost of renting a vehicle, toll charges, arrival charges at airplane terminal, set-up charges needed for assembling the item, and so forth.

Balinski [7] presented an approximate method for the solution of FCTP. Cooper and Drebes [8] presented two heuristic methods for solving moderate-sized linear programming problems with fixed charges and demonstrated that the heuristic methods provide an optimal solution in more than 90% of the cases and a very near optimal solution in the remaining cases. Hirsch and Dantzig [9] discussed FCTP and demonstrated that FCTP has a concave objective function and a bounded convex feasible region. Steinberg [10] presented exact methods on the basis of the branch-and-bound method, but the presented methods cannot be used to manually solve an FCTP. It requires a lot of distributions to find a solution to a problem of size 4 by 4.

Walker [11] presented a simple heuristic to provide a solution to small problems. There is an advantage of solving small problems manually, as it gives a comprehensive knowledge of how to get an optimal solution. Sadagopan and Ravindran [12] presented a dual approach for solving the FCTP. Sandrock [13] presented a simple method for solving a small FCTP wherein the fixed charge is connected with the supply points rather than routes. Sun et al. [14] presented a tabu search approach for the FCTP. Adlakha and Kowalski [15] proposed a simple heuristic to solve small FCTP. The first part of the heuristic can be applied to solve the large FCTP. Kowalski and Lev [16] considered the cost as a step function in a FCTP and presented a heuristic method for the solution. Raj and Rajendran [17] introduced fast heuristic approaches for FCTP and showed the efficiency of the proposed heuristic approaches over some existing methods.

Adlakha et al. [18] presented an analytical branching approach based on the computation of an upper bound and a lower bound. In the presented method, the number of branch stages is independent of the size of the problem. Farag [19] presented a modified Vogel's method to obtain a solution that can be used as a lower bound for the optimal solution of FCTP. Singh and Singh [20] presented a modified particle swarm approach for FCTP. Kaushal and Arora [21] are the first one who considered the fixed charge BTP (FCBTP) and presented a method for solving the problem. Later on, Kaushal and Arora [22] discussed an extension of FCBTP and presented a Lexi-Search algorithm for solving the problem.

Multiple objectives in FCTP that conflict with one another make up the multi-objective FCTP (MOFCTP). These objectives may include not only minimizing transportation costs but also minimizing the number of vehicles used, minimizing time, minimizing the carbon footprint, maximizing customer satisfaction, and so on. Therefore, MOFCTP is a challenging problem that requires sophisticated methods to generate and evaluate solutions that satisfy the multiple conflicting objectives.

Roy et al. [23] studied a MOFCTP having parameters as random rough variables, and three different approaches have been proposed to provide an optimal solution. Roy and Midya [24] proposed a mathematical model of a multi-objective fixed-charge solid transportation problem (MOFCSTP) in which parameters are triangular intuitionistic fuzzy numbers. Haque et al. [25] formulated a MOFCSTP with parameters that are closed intervals and devised a method based on the interval's parametric perception.

MOFCBTP is an extension of MOFCTP when transportation cost does not depend upon the quantity of the commodity being delivered. MOFCBTP has significant practical relevance, as it provides decision-makers with a more comprehensive understanding of the trade-offs between different objectives and helps them make decisions. It is to be noted that several researchers have worked on MOFCTP, but no work is available in the literature on MOFCBTP prior to this work. The practical significance of this problem motivates us to do work on it and develop some efficient and effective algorithm for solving the MOFCBTP.

The following are the goals of this research work:

- To formulate a MOFCBTP.
- To propose a solution approach for solving the MOFCBTP.
- To validate the proposed solution approach by applying on numerical example.

2.Notations and assumptions

The problem is developed using the notations and presumptions listed below:

m The number of supply points,

n The number of demand point,

$S_1, S_2, S_3, \ldots, S_m$ The set of supply points,

$D_1, D_2, D_3, \ldots, D_n$ The set of demand points,

a_i (i=1, 2, 3,….., m) The quantity of product available at the i^{th} supply point,

b_j (j=1, 2, 3, ….., n) The demand of product at the j^{th} demand point,

c_{ij} The cost of bulk transportation for i^{th} supply point to j^{th} demand point,

t_{ij} The time of transportation from i^{th} supply point to j^{th} demand point,

f_{ij} The fixed charge associated with the route from i^{th} supply point to j^{th} demand point,

x_{ij} The decision variable taking value 1 if i^{th} supply point is used to meet the demand of j^{th} demand point and 0 otherwise,

Z Objective function

Assumptions

$$a_i > 0, b_j > 0, c_{ij} > 0, \; t_{ij} > 0 \text{ and } f_{ij} > 0 \;\; \forall\, i, j.$$

3.Mathematical Model of MOFCBTP

The objective of the presented model of MOFCBTP is to minimize both the cost and time of bulk transportation while satisfying the constraints. There are m supply points and n demand points, and any of the supply points can transport to any of the demand points at a transportation cost of c_{ij} along with a fixed charge f_{ij}.

The proposed model for MOFCBTP is as follows:

Minimize

$$Z = \left(C = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} x_{ij} + f_{ij} y_{ij}),\, T = max\{t_{ij} : x_{ij} = 1\} \right)$$

subject to the constraints

$$\sum_{j=1}^{n} b_j x_{ij} \le a_i (i = 1,2,3, \ldots \ldots .., m) \quad (2)$$

$$\sum_{i=1}^{m} x_{ij} = 1 (j = 1,2,3, \ldots \ldots .., n) \quad (3)$$

$$x_{ij} = 1 \; or \; 0 \quad (4)$$

$$y_{ij} = \begin{cases} 1, \; x_{ij} \neq 0 \\ 0, x_{ij} = 0 \end{cases} \quad (5)$$

and $a_i,\; b_j, c_{ij}, t_{ij},$ and $f_{ij} \ge 0,\; \forall\, i, j$

4.Efficient Solution Pairs: Let the given MOFCBTP be optimized by keeping cost at first priority and time at second priority, and let (C_1, T_1) be the first cost-time solution pair. Then, the (C_2, T_2) solution pair of MOFCBTP is said to be the 2nd efficient cost-time solution pair if there does not exist any solution pair (C, T) of MOFCBTP such that C_1<C<C_2 and T_2<T<T_1.

5.Solution Procedure for MOFCBTP

We will solve the problem by keeping minimization of cost including fixed charge at first priority and time at second priority. For solving MOFCBTP, we have proposed the modified version of Sudhakar et al. [26]:

The steps listed below generate the solution pairs of MOFCBTP:

Step 1 Put the given MOFCBTP in tabular form.

Step 2 Convert the cost and fixed charge of transportation into a single Balinski [7] cost matrix using $C_{ij} = c_{ij} + \frac{f_{ij}}{m_{ij}}$ where $m_{ij} = \min\,(a_i, b_j)$.

Step 3 Construct the table by leaving all the time entries.

Step 4 Remove all the cell-entries from the table for which $a_i < b_j$ as the transportation is bulk.

Step 5 Subtract the row minimum entry from each of the entries of the concerned row to get the row reduced form. Do the same for each column also to get column reduced form.

Step 6 Determine the suffix value for each zero by dividing the sum of adjacent elements of zero by the number of adjacent elements that are sum up and write the suffix values in brackets in the concerned zero-cells.

Step 7 (a) Make allocation at the zero-cell that has the largest suffix value. If there is a tie between the largest suffix values, then choose that zero-cell (from the zero-cells for which tie exists) for allocation where transportation cost including fixed charge is minimum. If again there is a tie, then choose that zero-cell (from the zero-cells for which tie exists) for which the maximum can be allocated. If again there is a tie, then choose any zero-cell from the zero-cells for which tie exists.

(b) Remove the j^{th} demand point whose demand has been fulfilled and update the availability of the i^{th} supply point.

Step 8 Repeat Steps 4-7 until all demand points meet their demands.

Step 9 Let (C_1, T_1) be the 1st cost-time solution pair.

Step 10 Delete entries of the cells from the Table obtained in Step 2 for which $t_{ij} \geq T_1$.

Step 11 Repeat Steps 3-7 until all demand points meet their demands.

Step 12 Let (C_2, T_2) be the 2nd cost-time solution pair.

We can obtain subsequent cost-time solution pairs by replicating the exact procedure used to generate the 2nd cost-time solution pair.

6.Application

To validate the proposed methods, a numerical example is provided in this section.

6.1 Numerical Example

Example 1. In the numerical example, the cost of bulk transportation and the fixed charge in rupees and the time of transportation in hours are considered. It is desired to minimize the total transportation charge and time of transportation simultaneously.

Table 1 Transportation cost, fixed charge and time of transportation $\left(\mathrm{c_{ij}}, \mathrm{f_{ij}}, \mathrm{t_{ij}}\right)$

	D_1	**D_2**	**D_3**	**D_4**	a_i
S_1	(24,4,5)	(25,6,3)	(32,8,2)	(23,8,4)	15
S_2	(25,7,4)	(31,13,2)	(27,6,3)	(28,10,3)	14
S_3	(29,9,2)	(24,8,4)	(32,11,1)	(31,13,1)	21
b_j	10	9	12	7	

Applying Steps 2,3,4, and 5 successively, we obtain the Tables 2,3,4 and 5 respectively shown below.

Table 2 Balinski [7] cost matrix along with time of transportation $\left(C_{ij} = c_{ij} + \frac{f_{ij}}{m_{ij}}, t_{ij}\right)$

	D_1	**D_2**	**D_3**	**D_4**	a_i
S_1	(24.4,5)	(25.7,3)	(32.7,2)	(24.1,4)	15
S_2	(25.7,4)	(32.4,2)	(27.5,3)	(29.4,3)	14
S_3	(29.9,2)	(24.9,4)	(32.9,1)	(32.9,1)	21
b_j	10	9	12	7	

Table 3 Balinski [7] cost matrix

	D_1	**D_2**	**D_3**	**D_4**	a_i
S_1	24.4	25.7	32.7	24.1	15
S_2	25.7	32.4	27.5	29.4	14
S_3	29.9	24.9	32.9	32.9	21
b_j	10	9	12	7	

Table 4 Row reduced cost matrix

	D_1	**D_2**	**D_3**	**D_4**	a_i
S_1	0.3	1.6	8.6	0	15
S_2	0	6.7	1.8	3.7	14
S_3	5	0	8.	8	21
b_j	10	9	12	7	

Table 5 Column reduced cost matrix

	D_1	**D_2**	**D_3**	**D_4**	a_i
S_1	0.3	1.6	6.8	0	15
S_2	0	6.7	0	3.7	14
S_3	5	0	6.2	8	21
b_j	10	9	12	7	

For each zero-cell, the suffix value is written in brackets in the concerned cell, as shown in Table 6.

Table 6 Reduced cost matrix along with suffix values of zero-cells

	D_1	D_2	D_3	D_4	a_i
S_1	0.3	1.6	6.8	0(5.25)	15
S_2	0(4)	6.7	0(5.85)	3.7	14
S_3	5	0(5.97)	6.2	8	21
b_j	10	9	12	7	

Make allocation at the zero cell (3, 2) as this cell has the largest suffix value, removing the demand point D_2, and updating the table. The updated Table 7 is shown below.

Table 7 Reduced cost matrix after 1st allocation

	D_1	D_2	D_3	a_i
S_1	0.3	6.8	0	15
S_2	0	0	3.7	14
S_3	5	6.2	8	12
b_j	10	12	7	

Applying Steps 5 and 6 successively to get the reduced cost matrix along with suffix values for each zero-cell shown in Table 8.

Table 8 Reduced cost matrix with suffix values for zero-cells

	D_1	D_3	D_4	a_i
S_1	0.3	6.8	0(5.25)	15
S_2	0(0.1)	0(2.93)	3.7	14
S_3	0(0.6)	1.2	3	12
b_j	10	12	7	

Making allocation at the zero-cell (1, 4) as this cell has the largest suffix value 5.25, removing the demand point D_4, and updating the table. The updated Table 9 is shown below.

Table 9 Reduced cost matrix after 2nd allocation

	D_1	D_3	a_i
S_1	-	-	8
S_2	0	0	14
S_3	0	1.2	12
b_j	10	12	

Continuing in the same way, finally we get the decision variables x_{31}, x_{32}, x_{23} and x_{14}.

The overall minimum transportation charge including fixed charge and time of transportation when time is kept at 2nd priority for minimization are $C_1 = 134$ and $T_1 = 4$, respectively.

Thus, the 1st solution of MOFCBTP is $(C_1, T_1) = (134, 4)$.

To obtain the subsequent solution of MOFCBTP, delete the entries of the cells for which $t_{ij} \geq T_1 = 4$ from the Table 2. We get the cost-time matrix shown in Table 10.

Table 10 Balinski [7] cost matrix along with time of transportation $\left(C_{ij} = c_{ij} + \frac{f_{ij}}{m_{ij}}, t_{ij}\right)$

	D_1	D_2	D_3	D_4	a_i
S_1	-	(25.7,3)	(32.7,2)	-	15
S_2	-	(32.4,2)	(27.5,3)	(29.4,3)	14
S_3	(29.9,2)	-	(32.9,1)	(32.9,1)	21
b_j	10	9	12	7	

Repeat Steps 3 to 7 successively until all demand points meet their demands. The decision variables are as follows: x_{31}, x_{12}, x_{23} and x_{34} and the overall minimum transportation charge including fixed charge and time of transportation when time is kept at 2nd priority for minimization are $C_2 = 146$ and $T_2 = 3$, respectively. Thus, the 2nd solution of MOFCBTP is $(C_2, T_2) = (146, 3)$.

Continuing in the same way, the 3rd and last solution of MOFCBTP is $(C_3, T_3) = (166, 2)$, and the decision variables are x_{31}, x_{22}, x_{13} and x_{34}.

7.Conclusion and future work

Research on MOFCBTP has been carried out for the first time in literature. In such problems, there are two types of costs, namely, transportation costs and fixed charges associated with each route. The objectives are of a conflicting nature, as when someone wants to reduce the cost of transportation, time increases, and vice versa. To handle this type of problem, it is better to provide more flexibility in solution pairs to the decision maker so that he/she can opt for the solution that suits him/her best. The proposed method is simple to understand and apply on numerical problems of different sizes without any complex calculations. This study may be carried forward by including the cases when there are more than one indices.

References

[1] Maio AD, Roveda, C. An all zero-one algorithm for a certain class of transportation problems. Operation Research. 1971; 19:1406-1418.

[2] Srinivasan V, Thompsons GL. An algorithm for assigning users to sources in special classes of transportation problem. Operations Research. 1973; 21: 284-295.

[3] Prakash S, Ram PP. A bulk transportation problem with objectives to minimize total cost and duration of transportation. The Mathematics Student. 1995; 64: 206-214.

[4] Prakash S, Kumar P, Prasad BVNS, Gupta A. Pareto optimal solutions of a cost-time trade –off bulk transportation problem. European Journal of Operational Research. 2008; 188: 85-100.

[5] Prakash S, Sharma MK, Singh A. An efficient heuristic for multi-objective bulk transportation problem. In proceedings of the 39th International Conference on Computers & Industrial Engineering 2009(pp. 1005-1009). IEEE.

[6] Prakash S, Saluja RK, Singh P. Pareto optimal solutions to the cost-time trade-off bulk transportation problem through a newly devised efficacious novel algorithm. Journal of Data and Information Processing. 2014; 2:13-25.

[7] Balinski ML. Fixed cost transportation problems. Naval Research Logistics Quarterly. 1961; 8: 41–54.

[8] Cooper L, Drebes C. An approximate algorithm for the fixed charge problem. Naval Research Logistics Quarterly. 1967; 14: 101–13.

[9] Hirsch WM, Dantzig GB. The fixed charge problem. Naval Research Logistics. 1968; 15: 413-424.

[10] Steinberg DI. The fixed charge problem. Naval Research Logistics Quarterly. 1970; 17: 217–35.

[11] Walker WE. A heuristic adjacent extreme point algorithm for the fixed charge problem. Management Science. 1976; 22: 587 –96.

[12] Sadagopan S, Ravindran A. A vertex ranking algorithm for the fixed-charge transportation problem. Journal of Optimization Theory and application. 1982; 37: 221–230.

[13] Sandrock K. A simple algorithm for solving small fixed-charge transportation problems. Journal of Operational Research Society. 1988; 39: 467-475.

[14] Sun M, Aronson JE, Mckeown PG, Drinka D. A tabu search heuristic procedure for the fixed charge transportation problem. European Journal of Operations Research. 1998; 106: 441-456.

[15] Adlakha V, Kowalski K. A simple heuristic for solving small fixed charge transportation problems. OMEGA: The International Journal of Management Science. 2003; 31: 205–211.

[16] Kowalski K, Lev B. On step fixed-charge transportation problem. Omega. 2008; 36: 913-917.

[17] Raj K, Rajendran C. Fast heuristic algorithms to solve a single-stage fixed-charge transportation problem. International Journal of Operational Research. 2009; 6: 304-329.

[18] Adlakha V, Kowalski K, Lev B. A branching method for the fixed charge transportation problem. Omega. 2010; 38: 93-397.

[19] Farag HH. An approach for solving the fixed charge transportation problem. Journal of University of Shanghai for Science and Technology. 2021; 23: 583-590.

[20] Singh G, Singh A. Solving fixed-charge transportation problem using a modified particle swarm optimization algorithm. International Journal of System Assurance Engineering and Management. 2021; 12: 1073-1086.

[21] Kaushal B, Arora S. Fixed charge bulk transportation problem. Operations Research and Optimization. 2017; 45:11-29.

[22] Kaushal B, Arora S. Extension of fixed charge bulk transportation problem. Advanced Modelling and Optimization. 2018; 19: 517-525.

[23] Roy SK, Midya S, Yu VF. Multi-objective fixed-charge transportation problem with random rough variables. International Journal of uncertainty, Fuzziness and Knowledge-Based Systems. 2018; 26: 971–996.

[24] Roy SK, Midya S. Multi-objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment. Applied Intelligence. 2019; 49: 3524–3538.

[25] Haque S, Bhurjee AK, Kumar P. Multi-objective non-linear solid transportation problem with fixed charge, budget constraints under uncertain environments. System Science and Control Engineering. 2022; 10: 899-909.

[26] Sudhakar VJ, Arunasankar N, Karpagam T. A new approach for finding an optimal solution for transportation problems. European Journal of Scientific Research, 2012; 68: 254-257.