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Event Triggered Super Twisting Sliding Mode Control for Networked Nonlinear Systems

Ajay Kulkarni[1]*, Nitesh Kumar Soni[2], Sachin Puntambekar [3]

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Abstract: This paper presents an event-triggered super twisting sliding mode control scheme to achieve effective tracking performance of nonlinear systems configured in closed loop through a communication channel. Super twisting sliding mode scheme which has been proved to be a reliable control scheme to alleviate chattering is utilized to design a baseline tracking control policy. Control policy so framed is transformed into an event triggered version so as to extend the applicability of the control policy to the networked systems with limited resources. This policy also reduces the computational complexity of the system. Proof of the stability of the closed loop system is established using Lyapunov stability theory and Zeno-free behaviour of networked control system is also ensured. Finally, a simulation study is performed to illustrate the performance of the suggested control plan

Keywords: *Super twisting sliding mode control, Networked systems, Event triggering, Zeno behaviour*

1. Introduction

Sliding mode control has been proved to a highly efficient control scheme due to its characteristics of robustness and fast response. However, due to switching terms like sign function this control scheme is associated with chattering phenomenon leading to performance degradation of the system due to runaway of components like actuator. To deal with the problem of chattering different variants of sliding mode control are cited in the literature. Among these variants, super twisting sliding mode control has been proved to be an efficient control scheme to supress chattering and preserve robustness [1-4].

Event triggered control has emerged as an optimal scheme for the closed loop systems connected though a digital channel. In event control schemes, control term is updated only when some predefined triggering mechanism is violated.

This mechanism thus requires the information transmission only at some discrete instants thereby reducing the computational and communication burden. [5-9].

Event triggering mechanisms are required to exhibit Zeno free behaviour. Zeno behaviour indicates infinite triggering in finite duration. This phenomenon degenerates the system performance and may even lead to instability. Therefore, a Zeno free control algorithm is required to be designed [5-14].

Major contribution of this article are as follows

a. Super twisting sliding mode control scheme is framed to reduce chattering and preserve the robustness.

b. Event triggered super twisting sliding mode controller is designed with nonexistence of Zeno behaviour.

c. Event triggering condition is designed to ensure system stability and confinement of sliding variable to sliding manifold during the sliding phase.

d. Lyapunov stability analysis is carried out to snow the convergence of closed loop signals to a residual set containing origin.

e. Robustness and control quality are analysed under the condition of external disturbance.

The rest of the paper is structured as follows: System preliminaries are presented in section II, event triggered control methodology, allied aspects and control term formulated are detailed in section III. Validation of stability using Lyapunov analysis is carried out in section IV. Section V presents the proof of nonexistence of Zeno behaviour whereas section VI illustrates the simulation study and section VII concludes the paper.

2. System Preliminaries

2.1 System Model

A strict feedback nonlinear system model is considered [14]

[1] *Professor, Department of Electronics Engineering Medi-Caps University Indore – 453331, India*
Email: ajay.kulkarni@medicaps.ac.in*(Corresponding Author)
ORCID ID : 0000-0003-1514-8289
[2] *Assistant Professor, Department of Electronics Engineering Medi-Caps University Indore – 453331, India*
Email:nitesh@medicaps.ac.in
ORCID ID : 0000-0002-9946-0212
[3] *Associate Professor, Department of Electronics Engineering Medi-Caps University Indore – 453331, India*
ORCID ID : 0000-0003-1311-1482
Email:sachin.puntambekar@medicaps.ac.in

$$\begin{cases} \dot{x}_i = x_{i+1} \ \ ; i = 1, n-1; \\ \dot{x}_n = f(x) + u + d \,; x(0) = x(t_0) \end{cases} (1)$$

where $x = [x_1 \ \cdots \ x_n]^T \in R^n$ represents state vector, $u \in R$ denotes the control variable, $f(x): R^n \to R$ represents the nonlinear term and d is the external disturbance.

Assumption 1: Nonlinear term $f(x)$ satisfies the Lipschitz continuity condition on a compact set $S_x \subset R^n$ such that

$$|f(x(t)) - f(x(t_k))| \leq L_1 \{\sum_{i=1}^{n} |x_i(t) - x_i(t_k)|\} (2)$$

where $L_1 > 0$ is the Lipschitz constant.

Assumption 2: Disturbance term $d(t)$ satisfies the condition of boundedness, there exist some positive constant δ_d such that

$$|d(t)| \leq \delta_d < \infty \,; \ \forall t \leq 0 \ (3)$$

This paper now presents an event triggered control scheme designed to solve the tracking problem of the nonlinear dynamics (1) with stated assumptions.

3. Event Triggered Super Twisting Sliding Mode Controller Design

Procedure for designing event triggered super twisting mode control is presented in this section [2].

Considering a desired trajectory $y_d(t)$ with given assumption. State variable $x_1(t)$ is required to follow $y_d(t)$ with a prescribed degree of accuracy.

Assumption 3: Desired trajectory $y_d(t) \in R$ and its derivatives up to nth order $\{y_d(t), \dot{y}_d(t), \ddot{y}_d(t), \ldots\}$ are bounded and known.

Defining a sliding manifold as

$$S = \{x \in R^n; \ \|s\| = \|\lambda e\| \leq \rho \} \ (4)$$

where s is the sliding variable and e is the tracking error vector defined as

$$e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} = \begin{bmatrix} x_1 - y_d \\ x_2 - \dot{y}_d \\ \vdots \\ x_n - \overset{n-1}{y}_d \end{bmatrix} \qquad (5)$$

Additionally, ρ is a positive constant and λ is gain vector $\lambda = [\lambda_1 \ \ \lambda_2 \ \ \ldots \ \ \lambda_n]^T$ with $\lambda_i < 0$ and $\lambda_n = 1$.

For the nonlinear dynamics governed by (4) and (5) super twisting sliding mode controller is formulated as [10]

$$u(t) = \left\{-\lambda_1 e_2 - \lambda_2 e_3 - \cdots - \lambda_{n-1} e_n + \overset{n}{y}_d - f(x) - k_1 s - k_1 |s|^{\frac{1}{2}} sign(s) + v\right\} \qquad (6)$$

where k_1 is a positive constant and the input component v is defined using the following dynamics

$$\dot{v} = -(1 + \tfrac{1}{2}|s|^{-\frac{1}{2}})(s + |s|^{\frac{1}{2}} sign(s)) \qquad (7)$$

Lemma 1 Under the action of control law (5) and (6), system (1) displays finite time convergence. There exist a positive definite Lyapunov function $V(e)$ and positive definite matrices P and Q such that $\dot{V}(e) \leq -\frac{\lambda_{min}(Q)}{\lambda_{max}(P)} V(e) - \frac{1}{2} \frac{\lambda_{min}(Q)}{\lambda_{max}^{\frac{1}{2}}(P)} V(e)^{1/2}$ and all the closed loop signals settle down in the time bounded by $T \leq \frac{\ln\left\{\frac{V^{\frac{1}{2}}(t_0) + \lambda_{max}^{\frac{1}{2}}(P)}{\lambda_{max}^{\frac{1}{2}}(P)}\right\}}{1 / \frac{\lambda_{min}(Q)}{2\lambda_{max}(P)}}$ with $V(t_0)$ as the initial value of $V(e)$.

In event triggering scheme, control terms are updated only at the discrete triggering instants $\{t_k\}_{k=0}^{\infty}; k \in Z^+$. These triggering instants are controlled by certain system performance-based triggering rules and the updation occurs only when the defined rule is violated. This strategy often results in reduction in computational complexity and optimum use of network resources and thus has been proved highly efficacious for networked control systems

At some instant t such that $\forall t \in [t_k t_{k+1})$,event triggered version of the control law (6) can be defined as

$$u(t) = u(t_k) = \left\{-\lambda_1 e_2(t_k) - \lambda_2 e_3(t_k) - \cdots - \lambda_{n-1} e_n(t_k) + \overset{n}{y}_d(t_k) - f(x(t_k)) - k_1 s(t_k) - k_1 |s(t_k)|^{\frac{1}{2}} sign(s(t_k)) + v(t_k)\right\} (8)$$

From (7), dynamics of $v(t)$ can be defined as

$$\dot{v}(t) = -\left(1 + \frac{1}{2}|s(t)|^{-\frac{1}{2}}\right)\left(s(t) + |s(t)|^{\frac{1}{2}} sign(s(t))\right)\Big|_{t=t_k} = -\left(1 + \frac{1}{2}|s(t_k)|^{-\frac{1}{2}}\right)\left(s(t_k) + |s(t_k)|^{\frac{1}{2}} sign(s(t_k))\right) (9)$$

Triggering rule designed to control the updates of the control term (8) is

$$t_{k+1} = inf\{t \geq t_k | \sum_{i=1}^{n} |e_i(t) - e_i(t_k)| \ \geq \varphi\} \qquad (10)$$

where

$$\varphi = \frac{\rho}{2} + \frac{\rho}{2} e^{-\beta t} + \vartheta e^{-\alpha t} + \sigma(k_1 |\zeta_1(t_k)| + |\zeta_2(t_k)|) + \theta |\zeta_1(t_k)|$$

Here, the design parameters ϑ, α, β are intended to ensure that $\varphi > 0, \forall t$. Parameter σ and θ will be designed later and the variable $\zeta_1(t_k)$ and $\zeta_2(t_k)$ will be introduced in next section.

Event triggering mechanism does not allow the sliding variable $s(t)$ (3) to cross the bounds imposed by $\varphi(t)$ (10) and so by appropriately selecting the constants sliding variable can be confined to sliding manifold during the sliding phase.

During the reaching phase i.e. $sign(s(t)) = sign(s(t_k))$, system trajectories originate from outside the sliding manifold and under the action of control law (8) converges to the sliding manifold. Size of the bound imposed by φ(10) is greater than ρ(4) and gradually reduces as sliding variable converges. During the sliding phase i.e. when $sign(s(t)) \neq sign(s(t_k))$, sliding variable is inside the sliding manifold and by optimally designing the triggering rule (10) bound size can be made to approach sliding manifold.

Let at $t = t_s \in [t_k t_{k+1})$ sliding variable $s(t)$ enters the sliding phase with $sign(s(t)) \neq sign(s(t_k))$ then maximum bound during the reaching phase is

$$\Delta_s = \frac{\rho}{2} + \frac{\rho}{2}e^{-\beta t_s} + \vartheta e^{-\alpha t_s} + |\sigma|(k_1|\zeta_1(t_k)| + |\zeta_2(t_k)|) + |\theta(t_s)|(|\zeta_1(t_k)|) \quad (11)$$

Design parameters ϑ and α can now be designed as

$$\vartheta \geq sup(|\sigma|(k_1|\zeta_1(t_k)| + |\zeta_2(t_k)|) + |\theta|(|\zeta_1(t_k)|)$$

$$\alpha = \gamma_1 + \gamma_2\left(sign(s(t)) - sign(s(t_k))\right)$$

whereas γ_1, γ_2 and β are positive design constants. This mechanism will provide adjustable decay rate for ϑ. During the sliding phase higher decay rate can be achieved in comparison to that during reaching phase.

Thus, under the action of control law (8). System dynamics demonstrates the desired performance with sliding variable converging to a residual set contained in sliding manifold. Next sections illustrate the performance analysis in analytical aspects and via numerical simulation.

4. Stability Analysis

This section presents the analytical validation of the system performance using the concept of Lyapunov analysis.

Analysing the system behaviour at an instant $t \in [t_k, t_{k+1})$. Under the action of control term (8), derivative of the sliding variable (4) can be expressed as

$$\dot{s}(t) = \Big\{\left(\lambda_1 e_2(t) + \lambda_2 e_3(t) + \cdots + \lambda_{n-1}e_n(t)\right) - \left(\lambda_1 e_2(t_k) + \lambda_2 e_3(t_k) + \cdots + \lambda_{n-1}e_n(t_k)\right) + \overset{n}{y}_d(t_k) - \overset{n}{y}_d(t) - f(x(t_k)) + f(x(t)) + d(t) - k_1 s(t) - k_1|s(t))|^{\frac{1}{2}} sign(s(t)) + k_1 s(t) - k_1 s(t_k) + k_1|s(t))|^{\frac{1}{2}} sign(s(t)) - k_1|s(t_k)|^{\frac{1}{2}} sign(s(t_k)) + v(t_k)\Big\} \quad (12)$$

$$\dot{s}(t) = \Big\{\textstyle\sum_{i=1}^{n-1}\lambda_i\left(e_{i+1}(t) - e_{i+1}(t_k)\right) + \overset{n}{y}_d(t_k) - \overset{n}{y}_d(t) - f(x(t_k)) + f(x(t)) - k_1 s(t) + d(t) - k_1|s(t))|^{\frac{1}{2}} sign(s(t)) + k_1 s(t) - k_1 s(t_k) + k_1|s(t))|^{\frac{1}{2}} sign(s(t)) - k_1|s(t_k)|^{\frac{1}{2}} sign(s(t_k)) + v(t_k)\Big\} \quad (13)$$

To streamline the process of stability analysis, variable $\zeta(t)$ is introduced

$$\zeta(t) = \begin{bmatrix}\zeta_1(t)\\ \zeta_2(t)\end{bmatrix} = \begin{bmatrix}s(t) + |s(t))|^{\frac{1}{2}} sign(s(t))\\ v(t)\end{bmatrix} \quad (14)$$

Its differentiation results in

$$\dot{\zeta}(t) = \begin{bmatrix}\dot{\zeta}_1(t)\\ \dot{\zeta}_2(t)\end{bmatrix} = \begin{bmatrix}\left(1 + |s(t))|^{\frac{-1}{2}}\right)\dot{s}(t)\\ \dot{v}(t)\end{bmatrix} \quad (16)$$

Substitution of $\dot{v}(t_k)$(8) in (16) results in

$$\dot{\zeta}(t) = \left(1 + |s(t))|^{\frac{-1}{2}}\right)\begin{bmatrix}\dot{s}(t)\\ \dfrac{\dot{\left(1+|s(t_k)|^{\frac{-1}{2}}\right)}}{\left(1+|s(t))|^{\frac{-1}{2}}\right)}\zeta_1(t_k)\end{bmatrix} \quad (17)$$

In terms of $\zeta_1(t)$ and $\zeta_2(t)$, the dynamics in (13) can be expressed as

$$\dot{s}(t) = \Big\{\textstyle\sum_{i=1}^{n-1}\lambda_i\left(e_i(t) - e_i(t_k)\right) + \overset{n}{y}_d(t_k) - \overset{n}{y}_d(t) - f(x(t_k)) + f(x(t)) + d(t) - k_1\zeta_1(t) + k_1\left(\zeta_1(t) - \zeta_1(t_k)\right) + \zeta_2(t_k)\Big\} = \{\gamma - k_1\zeta_1(t) + k_1\left(\zeta_1(t) - \zeta_1(t_k)\right) + \zeta_2(t_k) - \zeta_2(t) + \zeta_2(t)\} \quad (18)$$

where $\gamma = \sum_{i=1}^{n-1}\lambda_i\left(e_{i+1}(t) - e_{i+1}(t_k)\right) + \overset{n}{y}_d(t_k) - \overset{n}{y}_d(t) + d(t) - f(x(t_k)) + f(x(t))$

Substitution of (18) in (17) results in

$$\dot{\zeta}(t) = \left(1+|s(t))|^{\frac{-1}{2}}\right)\begin{bmatrix}-k_1 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}\zeta_1(t)\\ \zeta_2(t)\end{bmatrix} + \left(1 + |s(t))|^{\frac{-1}{2}}\right)\begin{bmatrix}\gamma + k_1(\zeta_1(t)-\zeta_1(t_k)) + \zeta_2(t_k) - \zeta_2(t)\\ \zeta_1(t) - \frac{\left(1+|s(t_k)|^{\frac{\dot{-1}}{2}}\right)}{\left(1+|s(t))|^{\frac{-1}{2}}\right)}\zeta_1(t_k)\end{bmatrix}$$

(19)

To examine the stability of the system under consideration, Lyapunov function of the form is considered

$V(t) = \zeta^T(t)P\zeta(t)$(20)

where P is a positive definite, symmetric, matrix

$P = \begin{bmatrix}P_{11} & 0\\ 0 & P_{22}\end{bmatrix}$(21)

Differentiating the Lyapunov function (20) along the system trajectories and subsequently substituting $\dot{\zeta}(t)$ (19)

$$\dot{V}(t) = \dot{\zeta}^T(t)P\zeta(t) + \zeta^T(t)P\dot{\zeta}(t)$$

$$\dot{V}(t) = \left(1+|s(t))|^{\frac{-1}{2}}\right)[\zeta_1(t) \quad \zeta_2(t)]\left\{\begin{bmatrix}-k_1 & 1\\ -1 & 0\end{bmatrix}^T\begin{bmatrix}P_{11} & 0\\ 0 & P_{22}\end{bmatrix} + \begin{bmatrix}-k_1 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}P_{11} & 0\\ 0 & P_{22}\end{bmatrix}\right\}\begin{bmatrix}\zeta_1(t)\\ \zeta_2(t)\end{bmatrix} + 2\left(1 + |s(t))|^{\frac{-1}{2}}\right)\left\{\left(\gamma + k_1\left(\zeta_1(t)-\zeta_1(t_k) + (\zeta_2(t_k)-\zeta_2(t))\right)\right)P_{11}\zeta_1(t) + \left(\zeta_1(t) - \frac{\left(1+|s(t_k)|^{\frac{\dot{-1}}{2}}\right)}{\left(1+|s(t))|^{\frac{-1}{2}}\right)}\zeta_1(t_k)\right)P_{22}\zeta_2(t)\right\}$$

$$\dot{V}(t) \le -\left(1+|s(t))|^{\frac{-1}{2}}\right)\zeta(t)^T Q\zeta(t) + 2\left(1+|s(t))|^{\frac{-1}{2}}\right)\left\{\left(\sum_{i=1}^{n-1}\lambda_i|(e_{i+1}(t)-e_{i+1}(t_k))| + \left|\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\right| + |f(x(t)) - f(x(t_k))| + \delta_d + k_1|(\zeta_1(t)-\zeta_1(t_k))| + |(\zeta_2(t_k)-\zeta_2(t))|\right)|\zeta_1(t)|P_{11} + |\zeta_1(t)||\zeta_2(t)|P_{22} + \frac{\left(1+|s(t_k)|^{\frac{\dot{-1}}{2}}\right)}{\left(1+|s(t))|^{\frac{-1}{2}}\right)}|\zeta_1(t_k)||\zeta_2(t)|P_{22}\right\}(22)$$

Under the condition $\|\zeta(t)\| \ge |\zeta_1(t)|$ and $\|\zeta(t)\| \ge |\zeta_2(t|$, dynamics in (22) can be expressed as

$$\dot{V}(t) \le \left(1+|s(t))|^{\frac{-1}{2}}\right)(-\zeta(t)^T Q\zeta(t) + 2\|\zeta(t)\|^2 P_{22}) + 2\left(1 + |s(t))|^{\frac{-1}{2}}\right)\|\zeta(t)\|\left\{P_{11}\left(\sum_{i=1}^{n-1}\lambda_i|(e_{i+1}(t)-e_{i+1}(t_k))| + \left|\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\right| + \delta_d + |f(x(t)) - f(x(t_k))| + k_1|(\zeta_1(t)-\zeta_1(t_k))| + |(\zeta_2(t_k)-\zeta_2(t))|\right) + \frac{\left(1+|s(t_k)|^{\frac{\dot{-1}}{2}}\right)}{\left(1+|s(t))|^{\frac{-1}{2}}\right)}|\zeta_1(t_k)|P_{22}\right\}(23)$$

Following inequalities are now considered to facilitate the further analysis

$k_1|(\zeta_1(t)-\zeta_1(t_k))| + |(\zeta_2(t_k)-\zeta_2(t))| \le k_1|\zeta_1(t)| + k_1|\zeta_1(t_k)| + |\zeta_2(t)| + |\zeta_2(t_k)| \le (k_1+1)\|\zeta(t)\| + k_1|\zeta_1(t_k)| + |\zeta_2(t_k)|$(25)

$|f(x(t)) - f(x(t_k))| \le L_1\sum_{i=1}^{n}|(e_i(t) - e_i(t_k))| + L_1\sum_{i=1}^{n}\left|\left(\overset{i-1}{y}_d(t) - \overset{i-1}{y}_d(t_k)\right)\right|$(26)

$\sum_{i=1}^{n-1}\lambda_i|(e_{i+1}(t) - e_{i+1}(t_k))| \le \mu\sum_{i=1}^{n}|(e_i(t) - e_i(t_k))|$(27)

where $\mu = sup(\lambda_1 \quad \cdots \quad \lambda_n)$

and the terms used in triggering rule (10) are now designed as

$\sigma = \frac{-1}{(L_1+\mu)}$ and

$\theta = \frac{-P_{22}}{(P_{11}L_1+P_{11}\mu)}\frac{\left(1+|s(t_k)|^{\frac{-1}{2}}\right)}{\left(1+|s(t))|^{\frac{-1}{2}}\right)}$(30)

Substitution of inequalities (25-27) and triggering rule (10) in (23) results in

$$\dot{V}(t) \le \left(1+|s(t))|^{\frac{-1}{2}}\right)\|\zeta(t)\|^2(-\lambda_{min}(Q) + 2P_{22} + 2P_{11}(k_1+1)) + 2\left(1 + |s(t))|^{\frac{-1}{2}}\right)\|\zeta(t)\|\left\{P_{11}\left|\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\right| + L_1P_{11}\sum_{i=1}^{n}\left|\left(\overset{i-1}{y}_d(t) - \overset{i-1}{y}_d(t_k)\right)\right| + P_{11}\delta_d + (P_{11}L_1 + P_{11}\mu)\left(\frac{\rho}{2} + \frac{\rho}{2}e^{-\beta t} + \vartheta e^{-\alpha t}\right)\right\}(31)$$

$$\dot{V}(t) \le \left(1+|s(t))|^{\frac{-1}{2}}\right)\|\zeta(t)\|\left(\left(-\lambda_{min}(Q)+2\,P_{22}+2\,P_{11}(k_1+1)\right)\|\zeta(t)\| \;+2\emptyset\right) \quad (32)$$

Thus, $\dot{V}$ is negative outside a compact set defined as

$$\Omega = \left\{\zeta(t)|\|\zeta(t)\| \le \frac{\emptyset}{(\lambda_{min}(Q)-2\,P_{22}-2\,P_{11}(k_1+1))}\right\}(33)$$

where

$$\emptyset = 2\left\{P_{11}\left|\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\right| + L_1P_{11}\sum_{i=1}^{n}\left|\left(\overset{i-1}{y}_d(t) - \overset{i-1}{y}_d(t_k)\right)\right| + P_{11}\delta_d + (P_{11}L_1+P_{11}\mu)\left(\frac{\rho}{2}+\frac{\rho}{2}+\vartheta\right)\right\}$$

With appropriately adjusted parameters, the set can be reduced to an arbitrarily small value.

Now, on the basis of above analysis it can be stated that, for a networked closed loop system containing the nonlinear dynamics (1) under the action of event triggered super twisting sliding mode control strategy (8) with event triggering rule (10), all the closed loop signals are ultimate upper bounded. Boundedness of the closed loop signals is ensured over the entire span of operation including the update instants.

Next section presents the analytical proof for nonexistence Zeno behaviour.

5. Inter-execution time

Zeno execution which means infinite number of triggering events in finite time is an undesired phenomenon in networked control system. Zeno execution often results in loss of synchronization in control signal which therefore may not get updated at the triggering instants. This condition results in detuning of system states and sometimes even leads to instability. This context therefore requires the avoidance of Zeno existence which can be assured by ensuring the existence of a positive lower bound on the duration between any two consecutive triggering instants [10,12].

For the closed loop system (1) with control term (8) and triggering rule (10), there exist a positive lower bound Δton inter-execution time

$$(t_{k+1}-t_k) \ge \Delta t \quad (34)$$

Considering the event triggering mechanism (10)

$$E(t) = \sum_{i=1}^{n}|e_i(t)-e_i(t_k)|\,; E(t_k)=0\,, \forall t \in [t_k, t_{k+1})(35)$$

Its differentiation and subsequent substitution of system dynamics (5) and control term (8) results in

$$\frac{dE(t)}{dt} = \frac{d\sum_{i=1}^{n}\lambda_i|e_i(t)-e_i(t_k)|}{dt} \quad (36)$$

$$\frac{dE(t)}{dt} \le \sum_{i=1}^{n}\left|\frac{d}{dt}e_i(t) - \frac{d}{dt}e_i(t_k)\right| \le \sum_{i=1}^{n}\left|\frac{d}{dt}e_i(t)\right| \le \sum_{i=1}^{n-1}\left|\frac{d}{dt}e_i(t)\right| + \left|\frac{d}{dt}e_n(t)\right|$$

$$\frac{dE(t)}{dt} \le \sum_{i=1}^{n-1}|e_{i+1}(t)| + \left|\overset{n}{y}_d(t) + f(x(t)) + u\right| \le (1+\mu)\sum_{i=1}^{n-1}|e_{i+1}(t)| + \left|\sum_{i=1}^{n-1}\lambda_i\left(e_{i+1}(t)-e_{i+1}(t_k)\right) + \overset{n}{y}_d(t_k) - \overset{n}{y}_d(t) - f(x(t_k)) + f(x(t)) - k_1s(t_k) - k_1|s(t_k)|^{\frac{1}{2}}sign(s(t_k)) + v(t_k)\right|$$

$$\frac{dE(t)}{dt} \le (1+\mu)\sum_{i=1}^{n-1}|e_{i+1}(t)| + \mu\sum_{i=1}^{n}\left|\left(e_i(t)-e_i(t_k)\right)\right| + \left|\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\right| + \left|f(x(t)) - f(x(t_k))\right| + k_1|s(t_k)| + |v(t_k)| + k_1|s(t_k)|^{\frac{1}{2}}$$

$$\frac{dE(t)}{dt} \le (1+\mu)\sum_{i=1}^{n-1}|e_{i+1}(t)| + \mu\sum_{i=1}^{n}\left|\left(e_i(t)-e_i(t_k)\right)\right| + \left|\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\right| + L_1\sum_{i=1}^{n}\left|\left(e_i(t)-e_i(t_k)\right)\right| + L_1\sum_{i=1}^{n}\left|\left(\overset{i-1}{y}_d(t) - \overset{i-1}{y}_d(t_k)\right)\right| + k_1|s(t_k)| + |v(t_k)| + k_1|s(t_k)|^{\frac{1}{2}} \quad (37)$$

As all the closed loop signals and system nonlinearities are bounded (33), it implies the existence of a positive constant ρ_{max} such that

$$(1+\mu)\sum_{i=1}^{n-1}|e_{i+1}(t)| + \left|\overset{n}{y}_d(t_k) - \overset{n}{y}_d(t)\right| + L_1\sum_{i=1}^{n}\left|\left(\overset{i-1}{y}_d(t) - \overset{i-1}{y}_d(t_k)\right)\right| + k_1|s(t_k)| + |v(t_k)| + k_1|s(t_k)|^{\frac{1}{2}} \le \rho_{max}$$

Substituting in (37)

$$\frac{d\sum_{i=1}^{n}|(e_i(t)-e_i(t_k))|}{dt} \le (\mu + L_1)\sum_{i=1}^{n}\left|\left(e_i(t) - e_i(t_k)\right)\right| + \rho_{max} \quad (38)$$

Integrating both sides of (38)

$$\sum_{i=1}^{n}\left|\left(e_i(t) - e_i(t_k)\right)\right| \le \frac{\rho_{max}}{(\mu+L_1)}e^{(\mu+L_1)(t-t_k)} - \frac{\rho_{max}}{(\mu+L_1)} \quad (39)$$

As all the signals used in construction of triggering rule (10) are bounded, it allows the following consideration of the upper bound (10)

$$\epsilon = max\left(\frac{\rho}{2} + \frac{\rho}{2}e^{-\beta t} + \vartheta e^{-\alpha t} + \sigma(k_1|\zeta_1(t_k)| + |\zeta_2(t_k)|) + \theta|\zeta_1(t_k)|\right)$$

Substituting in (39)

$$\epsilon \le \frac{\rho_{max}}{(\mu+L_1)}\left(e^{(\mu+L_1)(t_{k+1}-t_k)} - 1\right) \quad (40)$$

This equation results in following inequality

$$(t_{k+1} - t_k) \ge \frac{1}{(\mu+L_1)}\ln\left(\frac{\epsilon(\mu+L_1)}{\rho_{max}} + 1\right) \quad (41)$$

Thus, inter execution time is lower bounded and absence of Zeno existence is validated [10,12].

Next section will illustrate the effectiveness of the control design with the help of simulation study.

6. Simulation

This section illustrates the simulation study carried out for validation of the control scheme (8). Considering a strict feedback nonlinear system with following dynamics

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + u + d \end{cases} \quad (42)$$

where x_1 and x_2 are the system states and nonlinear dynamics is $f(x) = 1.25\cos(x_1x_2) + 1.1x_1^2(1-x_2^2)$.

Event triggered control scheme and event triggering rule developed in this work is applied to (8,10) enforce the system state $x_1(t)$ to track the desired trajectory. Desired trajectory is taken as $y_d = 1.8\sin(t)$. System response and control term are depicted in Fig. 1 ,2 and 3 respectively. To demonstrate the robustness of the proposed controller, white gaussian nose with power 1dbWis inserted in the system and the system response is illustrated in Fig. 4,5,6 and 7respectively.Simulation in both the cases is carried out with same system settings.

Simulation is carried out following initial condition of the system and controller gain and other parameter settings Initial system setting: $x(0) = [0.3 \quad 0]^T$,controller gain parameters: $k_1 = 3.42$; $\lambda_1 = 2.42$, triggering rule parameters: $\alpha = .72, \gamma_1 = .65, \gamma_2 = .42, \rho = 1.3$ As illustrated in Fig. 1, state variables are bounded and state variable $x_1(t)$ tracks the desired trajectorywith an acceptable level of tracking performance. Tracking error is bounded within the finite limits and does not exhibit any abrupt variation. As shown in Fig.2 and 3, an admissible event triggered control term evolved over the entire duration of simulation. Proposed control scheme successfully avoids the Zeno behaviour with a permissible triggering sequence. Minimum and maximum inter-execution interval are about 0.09 sec and 0.442 sec respectively and average triggering instants around 9 over a duration of 2 sec.

To assess the robustness of the control scheme (8), simulation is carried out by injecting white gaussian noise in same channel with the input. System response is shown in Fig. 4 ,5, 6 and 7. As revealed from the figures, system response is almost identical to the response in Fig. 1 which reflects the robustness for matched disturbances.

7. Conclusion

An event triggered super twisting sliding mode control strategy is presented in this paper. Control scheme presents an a meliorated chattering scenario along with robustness for disturbances. Event triggering mechanism efficiently avoids the Zeno behaviour and successfully optimizes the utilization of computation and communication resources by allowing optimal triggering instants. Control scheme, under the condition of event triggering, not only ensures the system stability but also the desired system performance by confining the tracking error within the acceptable bounds. Triggering scheme presented is designed with an objective to confine the sliding variable within the sliding manifold during the sliding phase. Simulation results illustrate the desired tracking performance along with reduced chattering and enhanced robustness with respect to matched external disturbances

Conflicts of Interest

"There is no conflict of interest "

Author Contributions

Concept and formulation: Ajay Kulkarni and Nitesh Kumar Soni. Simulation and interpretation: Ajay Kulkarni, Nitesh Kumar Soni and Sachin Puntambekar. Manuscript drafting: Nitesh Kumar Soni and Sachin Puntambekar. Approval of the final version of the manuscript: Ajay Kulkarni. Accuracy and integrity: Ajay Kulkarni, Nitesh Kumar Soni and Sachin Puntambekar.

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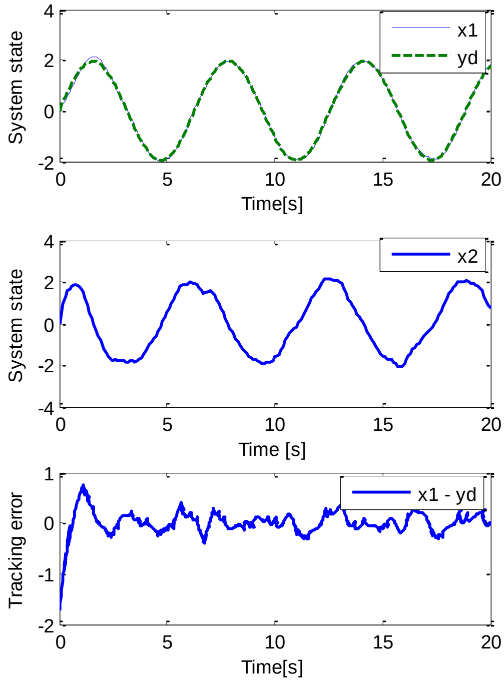

Figure 1. Trajectory tracking performance (a)State and desired trajectories, (b) State Variable (c) Tracking error

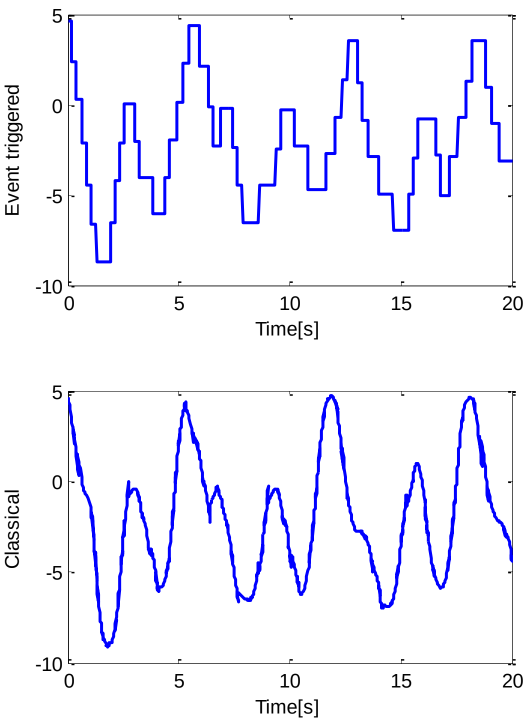

Figure 2. Control effort a) Event Triggered b) Classical view

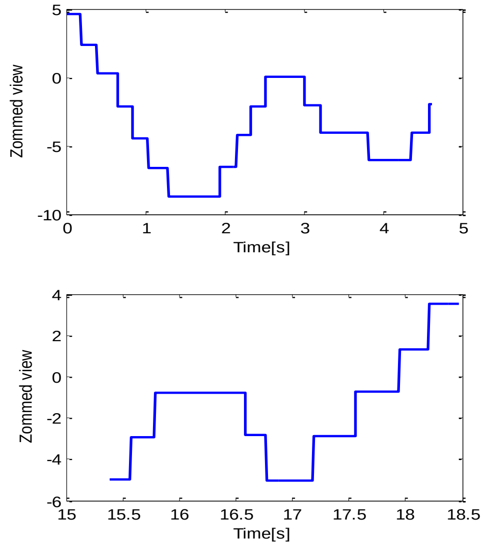

Figure 3. Zoomed view of control term in two time frames (a) 0-4.5 sec (b)15.4 – 18.4 sec

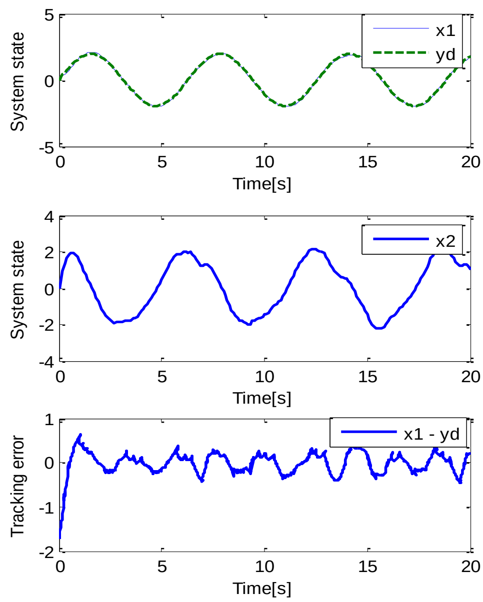

Figure 4. Trajectory tracking performance (a)State and desired trajectories, (b) State Variable (c) Tracking error

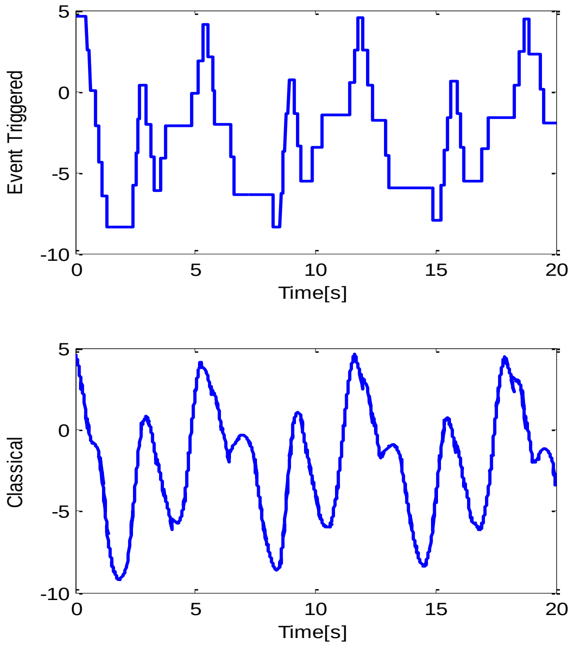

Figure 5. Control effort a) Event Triggered b) Classical view

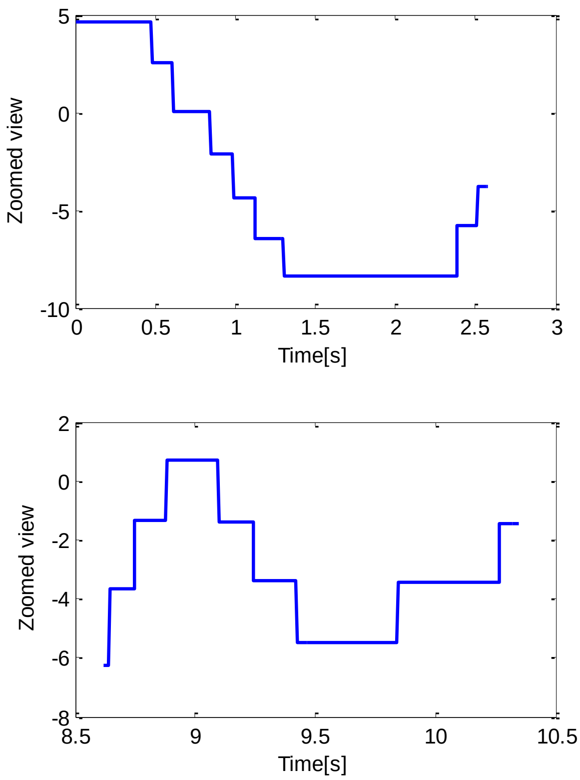

Figure 6. Zoomed view of control term in two time frames (a) 0-2.6 sec (b)8.7 – 10.3 sec

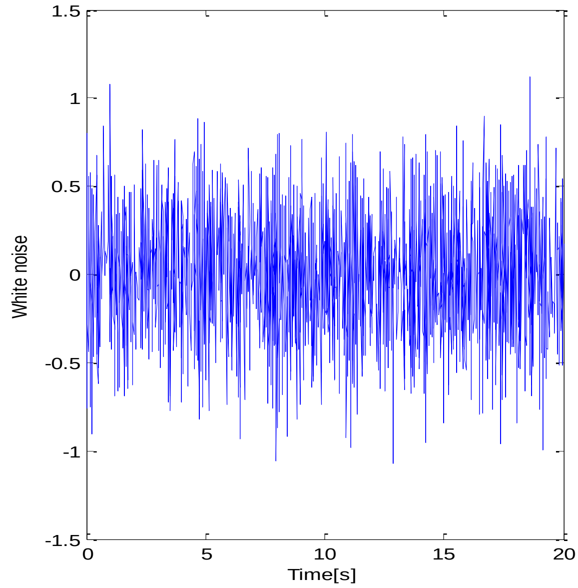

Figure 7. White Gaussian noise injected into the system