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Solving General and Special Cases of Heat-like Equations with Non Local Conditions Using the Homotopy Perturbation Method

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Abstract: This paper presents novel approaches to solving general and special cases of heat-like equations across one and two dimensions, incorporating both initial and non-local boundary conditions. Utilizing the Homotopy Perturbation Method (HPM), we demonstrate the efficacy of this technique in tackling these complex problem sets. Our results show high accuracy, with HPM offering a continuous solution-unlike the discrete approximations provided by finite difference methods. Our findings underscore HPM's potency as a versatile mathematical tool applicable to a wide array of linear and nonlinear problems spanning various scientific and technological domains.

Keywords: *Homotopy perturbation method (HPM), Heat-like equations, Non-local boundary conditions, Partial differential equations.*

1. Introduction

In recent years, several promising analytic methods have emerged for addressing initial boundary value problems. Notable among these are series solution techniques, including the Adomain decomposition method [8], Homotopy Analysis Method [9], Variation Iteration Method [7], and Homotopy Perturbation Method [6]. These methods have garnered significant attention for their ability to provide approximate and analytical solutions to partial differential equations, driven by demands across various industrial and scientific applications. Extensive research has been devoted to both the theoretical underpinnings and numerical methodologies for solving initial boundary value problems, as evidenced by numerous studies (see, for instance, [1-4] and references therein). Perturbation methods represent one widely applied technique in this domain.He [6] introduced a new perturbation technique, the Homotopy Perturbation Method (HPM), which combines traditional perturbation methods with the homotopy technique. Unlike conventional perturbation methods, HPM constructs a homotopy with an embedding parameter $p \in [0,1]$, treated as a small parameter.

This method has attracted considerable attention in recent years, with many researchers incorporating it into their investigations involving differential equations. He [5], for instance, successfully applied HPM to solve initial boundary value problems governed by nonlinear differential equations, demonstrating its efficiency and simplicity.

The primary objective of this study is to leverage the Homotopy Perturbation Method (HPM) to solve heat-like equations in both general and special forms, featuring variable coefficients and subject to non-local boundary conditions across one-dimensionaland two-dimensional scenarios.

2. Analysis of the method

To illustrate the basic ideas, let X and be Y two topological spaces.

If f and g are continuous maps of X into Y, it is said that f is homotopic to g if there is continuous map $F : X \times [0,1] \to Y$ such that $F(x,0) = f(x)$ and $F(x,1) = g(x)$ for each $x \in X$, then the map is called homotopy between f and g.

We consider the following nonlinear partial differential equation

$$A(u) - f(r) = 0, in\ \Omega \quad (1)$$

Subject to the boundary conditions

$$B\left(u, \frac{\partial u}{\partial \eta}\right) = 0, \text{on} \Gamma \quad (2)$$

Where $A$is a general differential operator,f is a known analytic function, $\Gamma$is the boundary of the domain $\Omega$and $\frac{\partial}{\partial \eta}$ denotes directional derivative in outward normal direction to Ω. The operator A, is generally divided into two parts,$L$and N, where $L$is linear, while $N$is nonlinear. Using $A = L + N$; (1) canbe rewritten as follows:

$$L(v) + N(v) - f(r) = 0 \quad (3)$$

By the homotopy technique, we construct a homotopy defined as:

$$H(v,p): \Omega \times [0,1] \longrightarrow \mathbb{R} \quad (4)$$

This satisfies

$H(v,p) = (1-p)\big(L(v) - L(u_0)\big) +$

$p\big(A(v) - f(r)\big) = 0, p \in [0,1], r \in \Omega$ (5)

Or

$H(v,p) = L(v) - L(u_0) + pL(u_0) +$

$p\big(N(v) - f(r)\big),\ p \in [0,1], r \in \Omega$ (6)

Where $p \in [0,1]$ is an embedding parameter, u_0 is an initial approximation of equation (1), which satisfies the boundary conditions. It follows from equation(6) that:

$H(v,0) = L(v) - L(u_0) = 0$ (7)

$H(v,1) = A(v) - f(r) = 0$
(8)

The changing process of p from 0 to 1 monotonically is a trivial problem.

$\mathrm{H(v,0) = L(v) - L(u_0) = 0}$ is continuously transformed to the original problem

$H(v,1) = A(v) - f(r) = 0$ (9)

In topology, this process is known as continuous deformation. $\mathrm{L(v) - L(u_0) = 0}$and $\mathrm{A(v) - f(r)}$are called homotopic. We use the embedding parameter $p$as a small parameter, and assume that the solution of equation (6)can be written as power series of p :

$v = p^0 v_0 + p^1 v_1 + p^2 v_2 + p^3 v_3 + \cdots + p^n v_n + \cdots$(10)

Setting $p = 1$, we obtained the approximate solution of equation (11) as:

$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + \cdots + v_n + \cdots$ (11)

The series of equation (1) is convergent for most of the cases, but the rate of theconvergence depends on the nonlinear operator $N(v)$. He (1999) has suggested that the second derivative of $N(v)$with respect to $v$should be small because the parameter may be relatively large i.e$p \to 1$and the norm of $\mathrm{L}^{-1}\left(\frac{\partial \mathrm{N}}{\partial \eta}\right)$ must be smaller than one in order the series to converge.

3. Convergence analysis

Lemma

Suppose that L^{-1} exist then the exact solution satisfy

$u = L^{-1}\big(f(r)\big) - \sum_{i=0}^{+\infty} L^{-1}(N(v_i))$ (12)

Proof

Rewriting equation (6), in the following from

$L(v) = L(u_0) + p[f(r) - N(v) - L(u_0)]$ (13)

Applying the inverse operator$\mathrm{L}^{-1}$to both sides of(13), we had:

$v = u_0 + p\big[\mathrm{L}^{-1}\big(f(r)\big) - \mathrm{L}^{-1}(N(v)) - u_0\big]$ (14)

We write (10) in compact form as, follows

$v = \sum_{i=0}^{+\infty} p^i v_i$ (15)

Substituting into the right hand side of (14), we got:

$$v = u_0 + p\left[\mathrm{L}^{-1}\big(f(r)\big) - \mathrm{L}^{-1}(N\left(\sum_{i=0}^{+\infty} p^i v_i\right)) - u_0\right]$$

From (11), we obtained the exact solution

$$u = \lim_{p \to 1} v = \mathrm{L}^{-1}\big(f(r)\big) - \mathrm{L}^{-1}(N\left(\sum_{i=0}^{+\infty} v_i\right)$$
$$= \mathrm{L}^{-1}\big(f(r)\big) - \sum_{i=0}^{+\infty} \mathrm{L}^{-1}(N(v_i))$$

In order to study the convergence of the method, we present the sufficient condition of the convergence in the following.

Theorem

Supposing that $X$and$Y$are Banach spaces and $N: X \to Y$is a contraction non linear mapping, that is

$\forall u, v \in X: \|\mathrm{N}(u) - N(v)\| \le L\|u - v\|, 0 < L < 1$ (16)

Then, according to Banach's theorem$N$ has a unique fixed point$w$, that is $N(w) = w$. Supposing that the sequence generated by homotopy perturbation method can be written as:

$V_\mathrm{n} = N(V_{n-1}),\ \mathrm{V}_{n-1} = \sum_{\mathrm{i}=0}^{\mathrm{n}-1} \mathrm{v_i},\ \mathrm{n} = 1, 2, 3$ (17)

And$V_0 = \mathrm{v}_0 \in B_r(\mathrm{v})$

Where $B_r(\mathrm{v}) = \{u \in X, \|\mathrm{v} - u\| < r\}$

Then we have (i)$V_n = B_r(\mathrm{v})$

(ii) $\lim_{n \to +\infty} \mathrm{v}_n = \mathrm{v}$

Proof

(i) By induction, for n=1, we had

$\|V_1 - \mathrm{v}\| = \|N(V_0) - N(\mathrm{v})\| \le \mathrm{L}\|\mathrm{v}_0 - \mathrm{v}\|$

Assume that

$\|V_{n-1} - \mathrm{v}\| \le \mathrm{L}^{\mathrm{n}-1}\|\mathrm{v}_0 - \mathrm{v}\|$

Then

$\|V_n - \mathrm{v}\| \le \|N(V_{n-1}) - N(\mathrm{v})\|$

$\le \mathrm{L}\|\mathrm{v}_{\mathrm{n}-1} - \mathrm{v}\|\mathrm{L}^{\mathrm{n}}\|\mathrm{v}_0 - \mathrm{v}\|$

Using (i), we obtained:

$\|V_n - \mathrm{v}\| \le \mathrm{L}^\mathrm{n}\|\mathrm{v}_0 - \mathrm{v}\| \le \mathrm{L}^\mathrm{n} \mathrm{r} \Rightarrow \mathrm{V_n} \in \mathrm{B_n}(\mathrm{v})$

(ii) Because $\|V_{n-1} - \mathrm{v}\| \le \mathrm{L}^{\mathrm{n}-1}\|\mathrm{v}_0 - \mathrm{v}\|$

And

$\lim_{n\to+\infty} L^n = 0, \lim_{n\to+\infty} \|V_n - \text{v}\| = 0$

That is $\lim_{n\to+\infty} V_n = \text{v}$

4. General and special examples

4.1. Example 01

We consider the problem

$$\frac{\partial \text{u}}{\partial \text{t}} = x^\alpha + \frac{1}{\alpha(\alpha-1)} x^2 \frac{\partial^2 u}{\partial x^2}, 0 \le x \le 1;\ t > 0;\ \alpha \in \mathbb{R}_+^* \quad (18)$$

With the initial condition

$$u(x,0) = 0$$

And the boundary conditions

$$u(0,t) = \int_0^1 \text{u}(x,t)dx + g_1 = \frac{1}{\alpha+1}(e^t - 1), g_1 = 0$$

$$u(1,t) = \int_0^1 \text{u}(x,t)dx + g_2 = \frac{1}{\alpha+1} e^t, g_2 = \frac{1}{\alpha+1} \quad (19)$$

For solving this problem, we constructed the HPM as follows:

$$H(v,p) = (1-p)\left(\frac{\partial v}{\partial \text{t}} - \frac{\partial \text{u}_0}{\partial \text{t}}\right) + p\left(\frac{\partial v}{\partial t} - \frac{1}{\alpha(\alpha-1)} x^2 \frac{\partial^2 v}{\partial x^2} - x^\alpha\right) = 0 \quad (20)$$

The components v_i of (11) are obtained as follows:

$$\frac{\partial v_0}{\partial t} - \frac{\partial u_0}{\partial t} = 0, v_0 = u_0 = u(x,0) = 0 \quad (21)$$

$$\frac{\partial v_1}{\partial t} - \frac{1}{\alpha(\alpha-1)} \text{x}^2 \frac{\partial^2 v_0}{\partial x^2} - x^\alpha = 0, v_1(x,0) = 0 \quad (22)$$

$$\frac{\partial^2 v_0}{\partial x^2} = 0 \Rightarrow \frac{\partial v_1}{\partial t} = x^\alpha$$

Hence $v_1 = x^\alpha t$

Then, we obtained:

$$\frac{\partial v_2}{\partial t} - \frac{1}{\alpha(\alpha-1)} \text{x}^2 \frac{\partial^2 v_1}{\partial x^2} = 0, v_2(x,0) = 0 \quad (23)$$

$$v_2 = x^\alpha \frac{t^2}{2!}$$

For the next component:

$$\frac{\partial v_3}{\partial t} - \frac{1}{\alpha(\alpha-1)} \text{x}^2 \frac{\partial^2 v_2}{\partial x^2} = 0, v_3(x,0) = 0 \Rightarrow v_3 = x^\alpha \frac{t^3}{3!} \quad (24)$$

And so on, we obtained the approximate solution as follows:

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + v_3 + \cdots + v_n + \cdots$$

And this leads to the following solution

$$u(x,t) = x^\alpha(e^t - 1) \quad (25)$$

We can, immediately observe that this solution is exact.

4.2. Example 02

We consider this special problem

$$\frac{\partial \text{u}}{\partial \text{t}} = x^{12} + \frac{1}{132} x^2 \frac{\partial^2 u}{\partial x^2}, 0 \le x \le 1;\ t > 0$$

Subject to the initial condition

$$u(x,0) = 0$$

And the boundary conditions

$$u(0,t) = \int_0^1 \text{u}(x,t)dx + g_1 = \frac{1}{13}(e^t - 1), g_1 = 0$$

$$u(1,t) = \int_0^1 \text{u}(x,t)dx + g_2 = \frac{1}{13} e^t, g_2 = \frac{1}{13}$$

We constructed the HPM, so the components v_i of (11) are obtained as follows:

$$\frac{\partial v_0}{\partial t} - \frac{\partial u_0}{\partial t} = 0, v_0 = u_0 = u(x,0)$$

$$\frac{\partial v_1}{\partial t} - x^{12} - \frac{1}{132}(\text{x}^2 \frac{\partial^2 v_0}{\partial x^2}) -= 0, v_1(x,0) = 0$$

$$v_1 = x^{12} t$$

$$\frac{\partial v_2}{\partial t} - \frac{1}{132}(\text{x}^2 \frac{\partial^2 v_1}{\partial x^2}) = 0, v_2(x,0) = 0$$

$$v_2 = x^{12} \frac{t^2}{2!}$$

$$\frac{\partial v_3}{\partial t} - \frac{1}{132}(\text{x}^2 \frac{\partial^2 v_2}{\partial x^2}) = 0, v_3(x,0) = 0$$

$$v_3 = x^{12} \frac{t^3}{3!}$$

Repeating the above process gives the remainder components as:

$$v_n = x^{12} \frac{t^n}{n!}$$

Using equations in the above, we got:

$$u(x,t) = x^{12}(e^t - 1)$$

By induction, for n=1-3 we had

$$\|\text{v}_1 - \text{v}\| = \|N(v_0) - N(v)\|$$

$$\le \|\text{N}\| \|\text{v}_0 - \text{v}\|$$

$$\leq \left(\int_0^1 x^{12} dx\right) \|v_0 - v\|$$

$$\leq \left[\frac{x^{13}}{13}\right]_0^1 \|v_0 - v\|$$

$$\leq \frac{1}{13}\|v_0 - v\|$$

$$\|v_2 - v\| = \|N(v_0 + v_1) - N(v)\|$$

$$\leq \|N\|\|v_1 - v\|$$

$$\leq \left(\int_0^1 x^{12} dx\right)\left(\frac{1}{13}\|v_0 - v\|\right)$$

$$\leq \left(\frac{1}{13}\right)^2 \|v_0 - v\|$$

$$\|v_3 - v\| = \|N(v_0 + v_1 + v_2) - N(v)\|$$

$$\leq \|N\|\|v_2 - v\|$$

$$\leq \left(\int_0^1 x^{12} dx\right)\left(\left(\frac{1}{13}\right)^2 \|v_0 - v\|\right)$$

$$\leq \left(\frac{1}{13}\right)^3 \|v_0 - v\|$$

Than

$$\|v_n - v\| = \|N(v_0 + v_1 + v_2 + \cdots + v_{n-1}) - N(v)\|$$

$$\leq \|N\|\|v_{n-1} - v\|$$

$$\leq \left(\int_0^1 x^{12} dx\right)\left(\left(\frac{1}{13}\right)^{n-1} \|v_1 - v\|\right)$$

$$\leq \left(\frac{1}{13}\right)^n \|v_0 - v\|$$

So

$$\|v_n - v\| \leq \left(\frac{1}{13}\right)^n \|v_0 - v\|$$

And

$$\lim_{n\to+\infty}\left(\frac{1}{13}\right)^n = 0, \lim_{n\to+\infty}\|v_n - v\| = 0$$

That is $\lim_{n\to+\infty} v_n = v$

Table1. Example 2

$h_x = 0.1, h_t = 0.004, 3 - Iterates$

x_i	u_{ex}	u_{hpm}	$\left\|u_{ex} - u_{hpm}\right\|$
0.0	0.0000	0.0000	0.0000
0.1	4.0080e-015	4.0080e-015	0.0000
0.2	1.6417e-011	1.6417e-011	0.0000
0.3	2.1300e-009	2.1300e-009	0.0000
0.4	6.7243e-008	6.7243e-008	0.0000
0.5	9.7852e-007	9.7852e-007	0.0000
0.6	8.7246e-006	8.7246e-006	0.0000
0.7	5.5476e-005	5.5476e-005	0.0000
0.8	2.7543e-004	2.7543e-004	0.0000
0.9	0.0011	0.0011	0.0000
1.0	0.0040	0.0040	0.0000

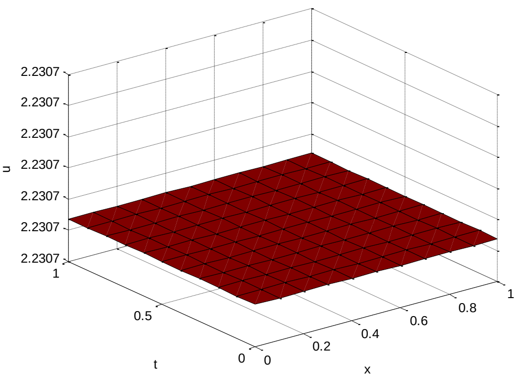

Fig.1. Example 2.Variation of $u = x^{12}(e^t - 1)$ for different values of$x$and t

4.3. Example 3

Consider the following two dimensional heat-like equations:

$$\frac{\partial u}{\partial t} = x^{\alpha}y^{\beta} + \frac{1}{(\alpha^2+\beta^2)-(\alpha+\beta)}\left(x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2}\right), 0 \leq x, y \leq 1 \quad t > 0;\ \alpha, \beta \in \mathbb{R}_+^* \qquad (26)$$

Subject to the initial condition:

$$u(x, y, t) = 0 \qquad (27)$$

And the boundary conditions:

$$u(0, y, t) = \int_0^1\int_0^1 u(x, y, t)dxdy + g_1$$

$$= \frac{1}{(\alpha+1)(\beta+1)}(e^t - 1), g_1 = 0$$

$$u(1,y,t)=\int_0^1\int_0^1 u(x,y,t)dxdy+g_2$$

$$=\frac{1}{(\alpha+1)(\beta+1)}(e^t-1)+\frac{1}{2}t\,,g_2=\frac{1}{2}t$$

$$u(x,0,t)=\int_0^1\int_0^1 u(x,y,t)dxdy+g_3$$

$$=\frac{1}{(\alpha+1)(\beta+1)}e^t\,,g_3=\frac{1}{(\alpha+1)(\beta+1)}$$

$$u(x,1,t)=\int_0^1\int_0^1 u(x,y,t)dxdy+g_4$$

$$=\frac{1}{(\alpha+1)(\beta+1)}(e^t+3)\,,g_4=\frac{4}{(\alpha+1)(\beta+1)}$$

According to the HPM, we had

$$H(v,p)=(1-p)\left(\frac{\partial v}{\partial t}-\frac{\partial u_0}{\partial t}\right)+$$
$$p\left(\frac{\partial v}{\partial t}-\frac{1}{(\alpha^2+\beta^2)-(\alpha+\beta)}\left(x^2\frac{\partial^2 v}{\partial x^2}+y^2\frac{\partial^2 v}{\partial y^2}\right)-x^\alpha y^\beta\right)=0$$

Solving the equation (26) with the initial condition (27), yields:

$$\frac{\partial v_0}{\partial t}-\frac{\partial u_0}{\partial t}=0, v_0=u_0=u(x,y,0)=0$$

$$\frac{\partial v_1}{\partial t}-x^\alpha y^\beta-\frac{1}{(\alpha^2+\beta^2)-(\alpha+\beta)}\left(x^2\frac{\partial^2 v_0}{\partial x^2}+y^2\frac{\partial^2 v_0}{\partial y^2}\right)=0,$$

$$v_1(x,y,0)=0$$

$$v_1=x^\alpha y^\beta t \qquad (28)$$

$$\frac{\partial v_2}{\partial t}-x^\alpha y^\beta-\frac{1}{(\alpha^2+\beta^2)-(\alpha+\beta)}\left(x^2\frac{\partial^2 v_1}{\partial x^2}+y^2\frac{\partial^2 v_1}{\partial y^2}\right)=0,$$

$$v_2(x,y,0)=0$$

$$v_2=x^\alpha y^\beta\frac{t^2}{2!} \qquad (29)$$

$$\frac{\partial v_3}{\partial t}-x^\alpha y^\beta-\frac{1}{(\alpha^2+\beta^2)-(\alpha+\beta)}\left(x^2\frac{\partial^2 v_2}{\partial x^2}+y^2\frac{\partial^2 v_2}{\partial y^2}\right)=0,$$

$$v_3(x,y,0)=0$$

$$v_3=x^\alpha y^\beta\frac{t^3}{3!} \qquad (30)$$

Repeating the above process gives the remainder components as:

$$v_n=x^\alpha y^\beta\frac{t^n}{n!} \qquad (31)$$

And so on, we obtained the approximate solution

$$au_{nhpm}=x^\alpha y^\beta\left[\left(1+\frac{t}{1!}+\frac{t^2}{2!}+\frac{t^3}{3!}+\cdots+\frac{t^n}{n!}+\cdots\right)-1\right]$$

Form this result we deduce that the series solution converge to the exact one:

$$u(x,y,t)=x^\alpha y^\beta(e^t-1) \qquad (32)$$

4.4. Example 4

Consider the following two dimensional heat-like equations

$$\frac{\partial u}{\partial t}=x^{12}y^{12}+\frac{1}{264}\left(x^2\frac{\partial^2 u}{\partial x^2}+y^2\frac{\partial^2 u}{\partial y^2}\right),0\le x,y\le 1,t>0$$

With the initial condition

$$u(x,y,t)=0$$

And the boundary conditions

$$u(0,y,t)=\int_0^1\int_0^1 u(x,y,t)dxdy+g_1$$

$$=\frac{1}{169}(e^t-1)\,,g_1=0$$

$$u(1,y,t)=\int_0^1\int_0^1 u(x,y,t)dxdy+g_2$$

$$=\frac{1}{169}(e^t-1)+\frac{1}{2}t\,,g_2=\frac{1}{2}t$$

$$u(x,0,t)=\int_0^1\int_0^1 u(x,y,t)dxdy+g_3$$

$$=\frac{1}{169}e^t\,,g_3=\frac{1}{169}$$

$$u(x,1,t)=\int_0^1\int_0^1 u(x,y,t)dxdy+g_4$$

$$=\frac{1}{169}(e^t+3)\,,g_4=\frac{4}{169}$$

According to the HPM, so the components$v_i$of (11) are obtained as follows:

$$\frac{\partial v_0}{\partial t}-\frac{\partial u_0}{\partial t}=0, v_0=u_0=u(x,y,0)=0$$

$$\frac{\partial v_1}{\partial t}-x^{12}y^{12}-\frac{1}{264}\left(x^2\frac{\partial^2 v_0}{\partial x^2}+y^2\frac{\partial^2 v_0}{\partial y^2}\right)=0,$$

$$v_1(x,y,0)=0$$

$$v_1=x^{12}y^{12}t$$

$$\frac{\partial v_2}{\partial t} - x^{12}y^{12} - \frac{1}{264}\left(x^2\frac{\partial^2 v_1}{\partial x^2} + y^2\frac{\partial^2 v_1}{\partial y^2}\right) = 0,$$

$$v_2(x,y,0) = 0$$

$$v_2 = x^{12}\mathrm{y}^{12}\frac{t^2}{2!}$$

$$\frac{\partial v_3}{\partial t} - x^{12}y^{12} - \frac{1}{264}\left(x^2\frac{\partial^2 v_2}{\partial x^2} + y^2\frac{\partial^2 v_2}{\partial y^2}\right) = 0,$$

$$v_3(x,y,0) = 0$$

$$v_3 = x^{12}\mathrm{y}^{12}\frac{t^3}{3!}$$

Repeating the above process gives the remainder components as:

$$v_n = x^{12}\mathrm{y}^{12}\frac{t^n}{n!}$$

Using equations in the above, we got:

$$u(x,y,t) = x^{12}\mathrm{y}^{12}(e^t - 1)$$

Table 2. Example 4

$h_x = 0.1, h_t = 0.004, 3 - Iterates$

x_i	u_{ex}	u_{hpm}	$\left\|u_{ex} - u_{hpm}\right\|$
0.0	0.0000	0.0000	0.0000
0.1	4.0080e-027	4.0080e-027	0.0000
0.2	6.7243e-020	6.7243e-020	0.0000
0.3	1.1320e-015	1.1320e-0	0.0000
0.4	1.1282e-012	1.1282e-012	0.0000
0.5	2.3890e-010	2.3890e-010	0.0000
0.6	1.8991e-008	1.8991e-008	0.0000
0.7	7.6786e-007	7.6786e-007	0.0000
0.8	1.8927e-005	1.8927e-005	0.0000
0.9	3.1970e-004	3. 1970e-004	0.0000
1.0	0.0040	0.0040	0.0000

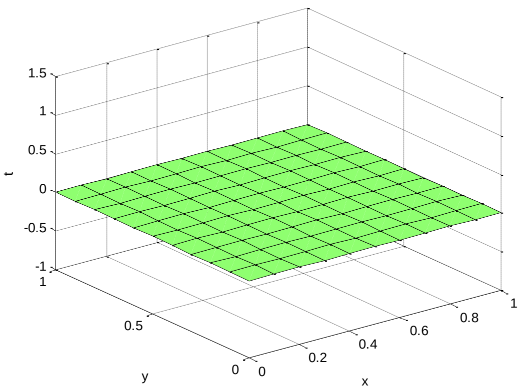

Fig.2. Example 4.Variation of $u = x^{12}y^{12}(e^t - 1)$ for different values of x, y and t

5. Conclusion

This study introduces novel types of heat-like equations featuring non-local conditions, with solutions tackled using the Homotopy Perturbation Method (HPM). Our method yields a rapidly converging series solution, requiring only a few terms for accurate results. Comparative analyses with recent findings employing finite difference schemes demonstrate the effectiveness of HPM, with our case studies showing strong agreements with exact solutions.

Notably, our iterative approach eliminates the need for linearization, discretization, transformation, or restrictive assumptions, highlighting the method's versatility and ease of implementation. Unlike traditional techniques, HPM does not rely on Adomian's polynomials for solving nonlinear problems, presenting a distinct advantage. The stability and convergence of the method are evident from our results, reaffirming its efficacy in addressing a broad spectrum of linear and nonlinear problems across various domains.

In summary, the Homotopy Perturbation Method emerges as a powerful tool for tackling complex mathematical problems, offering efficient solutions with broad applicability and simplified implementation.

Conflicts of interest

The authors declare no conflicts of interest.

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