International Journal of
INTELLIGENT SYSTEMS AND APPLICATIONS IN ENGINEERING
IJISAE
ISSN:2147-6799 www.ijisae.org Original Research Paper

# Topology-Based Analysis of Edge Coloring in Complement Fuzzy Graphs Using α-Cuts

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**Submitted:** 01/11/2024 **Revised:** 10/12/2024 **Accepted:** 20/12/2024

**Abstract:** A topological graph is a representation of a graph in the plane, where the vertices of the graphs are represented by distinct points and the edges are represented by Jordan arcs joining the corresponding pair of points representing the points and arcs are called the vertices and the edges of the topological graph. Graphcolouring is an assignment of colors to each vertex such that no two adjacent vertices have same colour. Colourings of Fuzzy graphs are used in real life applications to solve combinatorial optimization like traffic light system, examination programming etc.This paper investigates the edge coloring of complement fuzzy graphs using the α-cut technique, with a focus on topological aspects. The α-cut method is applied to fuzzy graph values, enabling an analysis of edge coloring in the complement fuzzy graph. The study explores key topological properties such as chromatic number and edge independence, providing insights into graph coloring within fuzzy environments. This approach has potential applications in network design, image processing, and decision systems.

***Keywords:*** *Chromatic number, Complement fuzzy graph, Edge Colouring, Fuzzy graph, Graph Colouring, Topology, $\alpha$ - cut*

## A) Introduction:

Graph theory has long been a powerful tool for modeling relationships in various domains, from computer networks to social systems. The traditional approach, based on crisp edges and vertices, is extended in fuzzy graph theory to handle uncertainty and partial truth by introducing fuzzy membership functions that represent the degree of connection between vertices. This extension allows more nuanced modeling of real-world problems where relationships are not binary but have varying degrees of strength.

One important concept in fuzzy graph theory is the complement fuzzy graph, which inverts the fuzzy membership values of the original graph. This offers a new perspective on graph structures and allows for the analysis of alternative relationships in a fuzzy context. In the context of edge coloring, the goal is to assign colors to edges in such a way that no two edges sharing a common vertex are assigned the same color, and this must be done while considering the fuzzy nature of the graph.

The alpha-cut technique plays a crucial role in this process. It is a method used to discretize fuzzy values, enabling thes transformation of fuzzy graphs into a series of crisp graphs at different levels of granularity. By adjusting the alpha-cut value, one can explore how the structure and properties of the graph change, making it a valuable tool for graph coloring tasks in fuzzy environments.

This paper focuses on the topological analysis of edge coloring in complement fuzzy graphs using the alpha-cut method. The research explores how the chromatic number, edge independence, and other topological properties behave under different alpha-

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cut values. By incorporating these concepts, we aim to offer a deeper understanding of the interplay between fuzzy graphs, edge coloring, and topology, ultimately contributing to fields such as network design, image processing, and decision-making systems, where uncertainty and fuzzy relationships are prevalent.

This study presents a framework for understanding and analyzing the edge coloring problem in complement fuzzy graphs, providing insights into the role of topology and alpha-cuts in shaping the graph's structure and coloring properties.

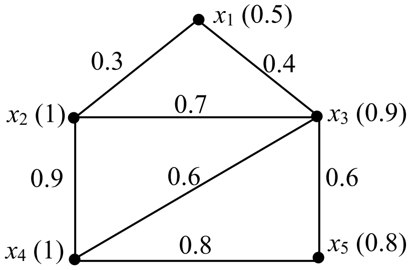
**B) Preliminaries**

**Fuzzy graph**

A Fuzzy graph is an ordered triplet $G = (v, \sigma, \mu)$ is a non empty set v together with a pair of functions $\sigma : v \to [0,1]$ and μ is a fuzzy relation on σ.

ie) $\mu(u, v) \le \sigma(u) \wedge \sigma(v)$.

We call μ is the fuzzy vertex set of G and σ is the fuzzy edge set of G respectively.




**α - cutset of fuzzy set**

α - cutset of fuzzy set A is defined as $A_\alpha$ is made up of members whose membership is not less than α

$A_\alpha = \{x \in X / \mu_A(x) \ge \alpha\}$. αcutset of fuzzy set is crisp set.

**α - cut set of fuzzy graph**

The α - cut set of fuzzy graph defined as $G_\alpha = (V_\alpha, E_\alpha)$ where $V_\alpha = \{v \in V : \sigma \ge \alpha\}$ and $E_\alpha$ ={e∈E:μ≥$\alpha$}

**Adjacent vertices**

Two vertices u and v in G are called adjacent if $\frac{1}{2}[\sigma(u) \wedge \sigma(v)] \le \mu(u, v)$

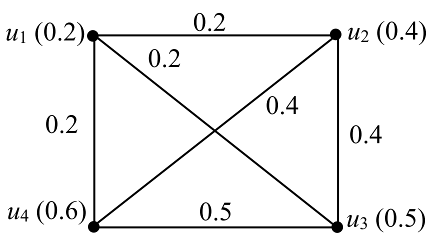
**Adjacent edges**

Two edges are adjacent, if they share a common vertex.
(If both edges have the same vertex, they are adjacent)

**Complete fuzzy graph**

A fuzzy graph G = (σ,μ) is said to be complete if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ ғ u,v∈V

Example of complete fuzzy graph



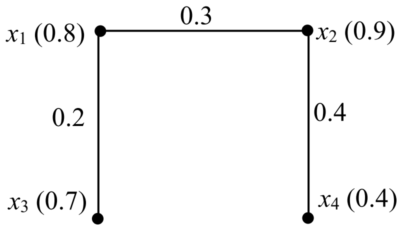

**Example of complete fuzzy graph**

**Complement Fuzzy graph:**

The complement of Fuzzy graph G = (σ,μ) is $G^c = (\sigma^c, \mu^c)$ where $\sigma^c = \sigma$ and

$\mu^c(x,y)_{=0}$ if $\mu(x,y) > 0$ and $\mu^c(x,y) = \sigma(x) \wedge \sigma(y)$ from the definition $G^c$ is a fuzzy graph even if G is not and $(G^c)^c = G$ iff G is a strong fuzzy graph also automorphism group of G and $G^c$ are not identical.

Complement of a fuzzy graph G:(σ,μ) is the fuzzy graph $G^c = (\sigma^c, \mu^c)$ where $\sigma^c \cong \sigma$ and $(x,y) = \sigma(x) \wedge \sigma(y) - \mu(x,y)$ ∀ $x,y \in$ X.



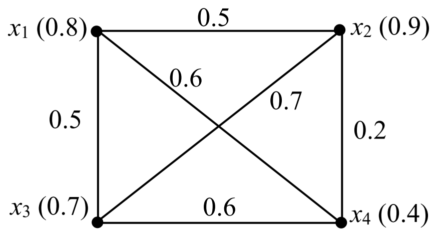




**Figure 1: Fuzzy graph (G)** **Figure 2: Complement of Fuzzy graph ($G^c$)**

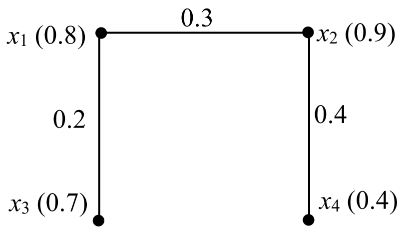



**Figure 3: $(G^c)^c$ Complement of complement fuzzy graph**

$$(x,y) = \sigma^c(x) \wedge \sigma^c(y) - \mu^c(x,y)$$

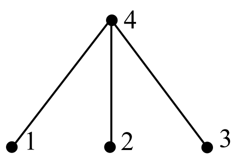
Hence $G^c = G$

**Colouring Function**

Graph colouring is an assignment of colors to each vertex of a graph G such that no two adjacent vertices have same colour.

**Adjacency matrix**

The adjacency matrix of a simple labeled graph is matrix with rows and columns labeled by graph vertices with 1 or 0 in position $(v_i, v_j)$ according to whether $v_i$ and $v_j$ are adjacent or not.




$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

**Example graph with its adjacency matrix**

**Topological space**

A topology on a nonempty set X is a collection of subsets of X, called open sets such that,

a) The empty set ∅ and the set X are open

b) The union of an orbitrary collection of open sets is open

c) The intersection of a finite number of open sets is open

A subset A of X is closed as iff its complement $A^c = X/A$ is open

More formally, a collection τ of subset of X is a topology on X if,

a) ϕ, X∈τ

b) If $G_\alpha \in \tau$ for α∈ A then $\bigcup_{\alpha\in A} G_\alpha \in \tau$

c) If $G_i \in \tau$ for i= 1,2 …….n then $\bigcap_{i=1}^{n} G_i \in \tau$

We call the pair (*x*, τ) as a topological space if ***τ*** is a clear from the context, then we often refer to X is topological space.

**C) Motivation of our approach**

In our approach, we have focused on the problem of Edge colouring using the concept of fuzzy chromatic numbers through the α - cuts of the complement fuzzy graphs for topology.

**D) Our proposed algorithm**

In our proposed algorithm, we have introduced a colouring function of complement fuzzy graph to color all the Edges of graph of G and then we find the chromatic number of graphs G. The function is based on α-cut of graph G. We have taken $G_\tau^c$ as a complement fuzzy graph from the topological space. A complement fuzzy graph $G_\tau^c = (V_F, E_F)$ its chromatic number is a fuzzy number $\chi(G_\tau^c) = \{x_{\alpha,\alpha}\}$ where $x_\alpha$ is the chromatic number of $G_\tau^c$ and α values are the different membership value of vertex and edge of graph $G_\tau^c$. Chromatic number is fuzzy number of topology $\tau(x(G_\tau^c)) = \{x_{\alpha,\alpha}\}$.

We used α values are the different membership value of vertex and edge of complement fuzzy graph for topology set we find the minimum number of colors needed to color the complement fuzzy graph $G_{\tau\alpha}^c$ for topology. Then we find the fuzzy chromatic number is a fuzzy number which is calculated by its α-cut for topology.

**E) Algorithm – color Assign**

Step 1 - Start

Step 2 - Let us consider a fuzzy graph for a $G_\tau$ topological graph.

Step 3 - Prepare the adjacency matrix from $G_\tau$

Step 4 - Find the complement fuzzy graph $G_\tau^c$

Step 5 - Find the α-cut values from $G_\tau^c$

Step 6 - Find the α-cut values in the complement fuzzy graph prepare the membership value where we considered α≤1

Step 7 - Remove the selected α-cut from $G_\tau^c$

Step 8 - Find the chromatic number

Step 9 - Repeat step 5,6 and 7 untill all the α-cut values are substituted in $G_\tau^c$

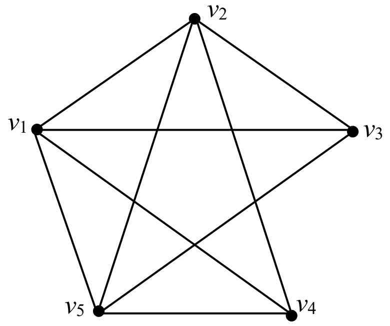
Step 10 - List out all the chromatic number of the complement fuzzy graph

Step 11 - Stop

**F) Working of our algorithm**

Let τ be the topological space on a finite set X. Elements of τ are vertices of graphs and any two distinct vertices are adjacent it one of the set is subset of other element.

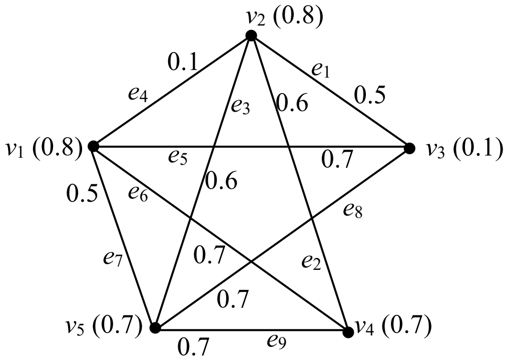
$x = \{1, 2, 3\}$ $\tau = \{\phi, x, \{1\}, \{2\}, \{1, 2\}\}$




**Topological Graph $G_\tau$**

Step 1: Start

Step 2: Let us consider a fuzzy graph $G_\tau$.




**Fuzzy Graph $G_\tau$**

Let us considered a fuzzy graph $G_\tau = \{v, \sigma, \mu\}$ where v has five vertices. The membership values of the vertices are σ = {0.7,0.8}.

The graph G has 9 edges and membership values of the edges are in μ.

Step 3: Prepare the adjacency matrix from $G_\tau$

| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ |
|---|---|---|---|---|---|
| $V_1$ | 0.0 | 0.1 | 0.7 | 0.7 | 0.5 |
| $V_2$ | 0.1 | 0.0 | 0.5 | 0.6 | 0.6 |
| $V_3$ | 0.7 | 0.5 | 0.0 | 0.0 | 0.7 |
| $V_4$ | 0.7 | 0.6 | 0.0 | 0.0 | 0.7 |
| $V_5$ | 0.5 | 0.6 | 0.7 | 0.7 | 0.0 |

Adjacency matrix I membership value of edges.

| | | | | | |
|---|---|---|---|---|---|
| $V_1$ | 0.0 | $e_4$ | $e_5$ | $e_6$ | $e_7$ |
| $V_2$ | $e_4$ | 0.0 | $e_1$ | $e_2$ | $e_3$ |
| $V_3$ | $e_5$ | $e_1$ | 0.0 | 0.0 | $e_8$ |

| | | | | | |
|---|---|---|---|---|---|
| $V_4$ | $e_6$ | $e_2$ | 0.0 | 0.0 | $e_9$ |
| $V_5$ | $e_7$ | $e_3$ | $e_8$ | $e_9$ | 0.0 |

Adjacency matrix II – Name of edge between the vertices.

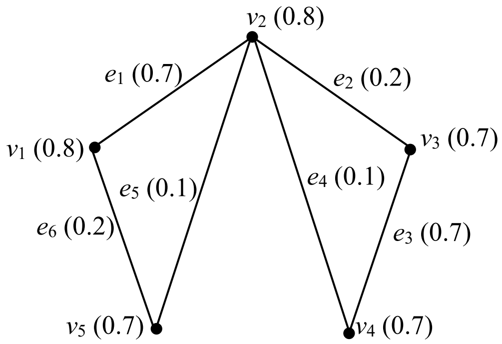
Adjacency matrix I – represent the membership value of edges.

Adjacency matrix II – represent the name of the edge between the vertices.

For example the edge between two vertices $V_1$and $V_2$ is $e_4$ and membership value of that edge is 0.1.

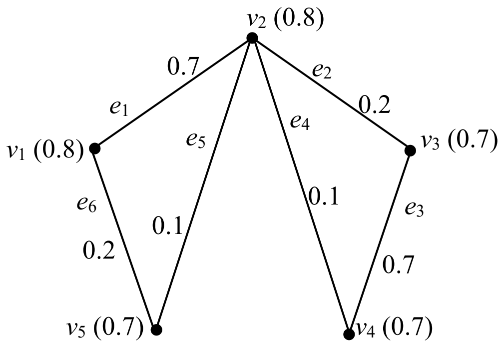
Let fuzzy graph $G_\tau = (\mathrm{V_F}, \mathrm{E_F})$ be a fuzzy graph where $V_F = \{(V_1,0.8),(V_2,0.8),(V_3,0.7),(V_4,0.7),(V_5,0.7)\}$ and $E_F = \{(e_1,0.5)(e_2,0.6)(e_3,0.6)(e_4,0.1)(e_5,0.7),(e_{6,}0.7)(e_7,0.5)(e_8,0.7)(e_9,0.7)\}$

Step 4: Find the complement fuzzy graph $\mathrm{G}_\tau^c$




**$\mathrm{G}_\tau^c$ (Complement of fuzzy graph)**

Step 5: Find the α-cut values from $\mathrm{G}_\tau^c$




In this fuzzy graph there are 4 - α cuts {0.1,0.2,0.7,0.8}

Step6: For each α-cut values in the complement fuzzy graph prepare the membership value where we considered α≤ 1.

Step7: Remove the selected α-cut from $\mathrm{G}_\tau^c$

Step 8: Find the chromatic number.

For each value of α,we find the graph $\mathrm{G}_{\tau\alpha}^c$ and its fuzzy chromatic number for topology.

i) For α = 0.1, complement fuzzy graph $\mathrm{G}_\tau^c = (\nu,\sigma,\mu)$ where σ = {0.1,0.2,0.7,0.7,0.7,0.8,0.8} and

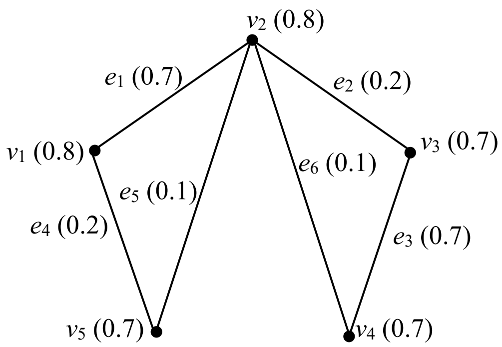
$\mu$ =

| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ |
|---|---|---|---|---|---|
| $V_1$ | 0.0 | 0.7 | 0.0 | 0.0 | 0.2 |
| $V_2$ | 0.7 | 0.0 | 0.2 | 0.1 | 0.1 |
| $V_3$ | 0.0 | 0.0 | 0.0 | 0.7 | 0.0 |
| $V_4$ | 0.0 | 0.1 | 0.7 | 0.0 | 0.0 |
| $V_5$ | 0.2 | 0.1 | 0.0 | 0.0 | 0.0 |

=

| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ |
|---|---|---|---|---|---|
| $V_1$ | 0 | $e_1$ | 0 | 0 | $e_6$ |
| $V_2$ | $e_1$ | 0 | $e_2$ | $e_4$ | $e_5$ |
| $V_3$ | 0 | 0 | 0 | $e_3$ | 0 |
| $V_4$ | 0 | $e_4$ | $e_3$ | 0 | 0 |
| $V_5$ | $e_6$ | $e_5$ | 0 | 0 | 0 |

For $\alpha = 0.1$, $G^c_{\tau(0.1)} = (V_{0.1}, E_{0.1})$ where $V_{0.1} = \{v_1, v_2, v_3, v_4, v_5\}$ and $E_{0.1} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$




Here we need minimum 4 colours to colour all the vertices of the graph $G^c_{\tau(0.1)}$ properly. So, the chromatic number of $G^c_{\tau(0.1)}$ is 4

$$\alpha \Rightarrow 0.1 \Rightarrow X_{0.1} = X\left(G^c_{\tau(0.1)}\right) = 4.$$

Step 9: Repeat step 5,6,7 and 8 until find the all $\alpha$-cut values are substituted in $G^c_\tau$

ii) For $\alpha$ =0.2 complement fuzzy graph $G^c_\tau = (\nu, \sigma, \mu)$ where $\sigma$ = {0.2,0.7,0.7,0.7,0.8,0.8} and

$\mu$ =

| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ |
|---|---|---|---|---|---|
| $V_1$ | 0.0 | 0.7 | 0.0 | 0.0 | 0.2 |
| $V_2$ | 0.7 | 0.0 | 0.2 | 0.0 | 0.0 |
| $V_3$ | 0.0 | 0.0 | 0.0 | 0.7 | 0.0 |
| $V_4$ | 0.0 | 0.0 | 0.7 | 0.0 | 0.0 |
| $V_5$ | 0.2 | 0.0 | 0.0 | 0.0 | 0.0 |

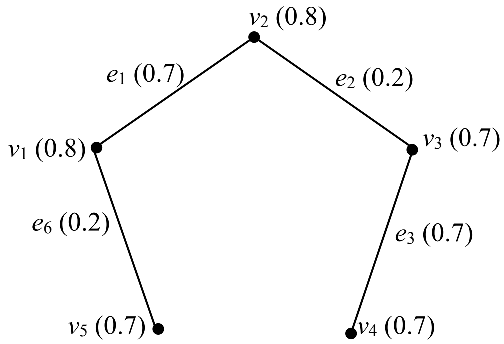
Adjacency matrix – I ; membership value of edge.

| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ |
|---|---|---|---|---|---|
| $V_1$ | 0 | $e_1$ | 0 | 0 | $e_6$ |
| $V_2$ | $e_1$ | 0 | $e_2$ | 0 | 0 |
| $V_3$ | 0 | 0 | 0 | $e_3$ | 0 |
| $V_4$ | 0 | 0 | $e_3$ | 0 | 0 |
| $V_5$ | $e_6$ | 0 | 0 | 0 | 0 |

Membership value of edges after remove $e_4$ and $e_5$

For each α = 0.2, $G^c_{\tau(0.2)} = (V_{0.2}, E_{0.2})$ where $V_{0.2} = \{v_1, v_2, v_3, v_4, v_5\}$ and $E_{0.2} = \{e_1, e_2, e_3, e_6\}$.

Were we need minimum 2 colours to colour all the edges of graph $G^c_{\tau(0.2)}$ properly so the chromatic number of $G^c_{\tau(0.2)}$ is 2. For α = 0.2 $\Rightarrow \chi_{0.2} = \chi\left(G^c_{\tau(0.2)}\right) = 2$.




**Complement fuzzy graph with α-cut = 0.2**

ii) α = 0.7, complement fuzzy graph $G^c_\tau = (\nu, \sigma, \mu)$ where σ = {0.7,0.7,0.7,0.8,0.8} and

μ =

| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ |
|---|---|---|---|---|---|
| $V_1$ | 0.0 | 0.7 | 0.0 | 0.0 | 0.0 |
| $V_2$ | 0.7 | 0.0 | 0.0 | 0.0 | 0.0 |
| $V_3$ | 0.0 | 0.0 | 0.0 | 0.7 | 0.0 |
| $V_4$ | 0.0 | 0.0 | 0.7 | 0.0 | 0.0 |
| $V_5$ | 0.2 | 0.0 | 0.0 | 0.0 | 0.0 |

Adjacency Matrix.

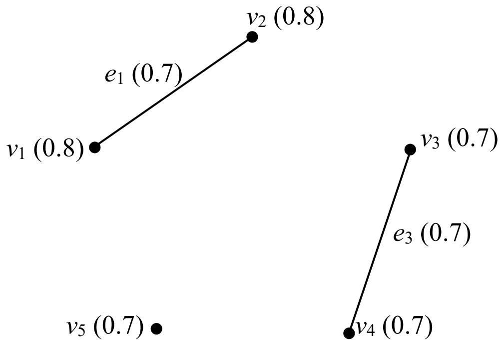
| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ |
|---|---|---|---|---|---|
| $V_1$ | 0 | $e_1$ | 0 | 0 | 0 |
| $V_2$ | 0 | 0 | 0 | 0 | 0 |
| $V_3$ | 0 | 0 | 0 | $e_3$ | 0 |
| $V_4$ | 0 | 0 | $e_3$ | 0 | 0 |
| $V_5$ | $e_6$ | 0 | 0 | 0 | 0 |

Adjacency matrix for membership value of edges (after remove $e_6$ and $e_2$)

For each α = 0.7, $G^c_{\tau(0.7)} = (V_{0.7}, E_{0.7})$ where $V_{0.7} = \{v_1, v_2, v_3, v_4, v_5\}$ and $E_{0.2} = \{e_1, e_2\}$

Here we need minimum 2 colours to colour all the edges of graph $G^{c}_{\tau(0.7)}$ properly. So the chromatic number $G^{c}_{\tau(0.7)}$ is 1.

For $\alpha = 0.7$, $\chi_{0.7} = \chi\left(G^{c}_{\tau(0.7)}\right) = 1$




iii) $\alpha$= 0.8, complement fuzzy graph $G^{c}_{\tau} = (\nu, \sigma, \mu)$ where $\sigma$ = {0.8} and

| | $V_1$ | $V_2$ |
|---|---|---|
| $V_1$ | 0.0 | 0.0 |
| $V_2$ | 0.0 | 0.0 |

Adjacency Matrix

| | $V_1$ | $V_2$ |
|---|---|---|
| $V_1$ | 0.0 | 0.0 |
| $V_2$ | 0.0 | 0.0 |

Adjacency matrix for membership value of edge (after remove $e_1$)

For each $\alpha = 0.8$, $G^{c}_{\tau(0.8)} = (V_{0.8}, E_{0.8})$ where $V_{0.8} = \{v_1, v_2\}$ and $E_{0.8} = \{e_1\}$. Here we need minimum zero colour to colour all the edges of graph $G^{c}_{\tau(0.8)}$ properly. So the chromatic number of $G^{c}_{\tau(0.8)}$ is zero. For $\alpha$ = $0.8 \Rightarrow \chi_{0.8} = \chi\left(G^{c}_{\tau(0.8)}\right) = 0$

Now, the chromatic number $\chi_{\alpha}$ for any $\alpha$ it can be shown that the chromatic number of complement fuzzy graph $G^{c}_{\tau}$ *is* $\chi\left(G^{c}_{\tau}\right) = \{(4,0.1), (2,0.2), (1,0.7), (0,0.8)\}$

**G) Conclusion and Future work**

In this paper, we find the fuzzy chromatic number based on $\alpha$ - cut of a complement fuzzy graph for a topology. Here the chromatic number of complement fuzzy graph will decrease when the value of $\alpha$ - cut of the complement fuzzy graph will increase, In future work, we would like to extends this work by applying this algorithm in vertex colouring of complement fuzzy graph for topology.

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