International Journal of
INTELLIGENT SYSTEMS AND APPLICATIONS IN ENGINEERING
IJISAE
ISSN:2147-6799 www.ijisae.org Original Research Paper

Magneto-Hydrodynamic Flow, Heat and Mass Transfer Investigations: Exploring Fundamental and Applied Aspects

[1]Narender Sharma, [2]Chandkiran

Submitted: 26/01/2024 **Revised:** 01/03/2024 **Accepted:** 08/03/2024

Abstract: This research explores the dynamics of Magneto-Hydrodynamic (MHD) flow, thermal, and mass transfer in Williamson nanofluids, considering a variety of factors such as magnetic field intensity, Williamson non-Newtonian coefficient, diffusivity proportion, heat capacity ratio, Prandtl number, and Eckert number. The MHD flow was represented in two dimensions and addressed through the shooting method, offering a deeper understanding of the impacts of viscous dissipation, thermal transfer, and concentration spread in nanofluid systems. The research demonstrates that elevating the magnetic field intensity diminishes fluid speed while amplifying temperature and concentration distributions, owing to the augmented Lorentz force. Moreover, an elevated Williamson non-Newtonian parameter diminishes fluid velocity yet amplifies heat and mass transfer efficacy, whereas a rise in the diffusivity ratio improves temperature and lowers concentration. Moreover, an elevated ratio of heat capacities leads to a rise in temperature, suggesting that nanoparticles enhance the thermal storage potential of the nanofluid. The influence of the Prandtl number on the thickness of the boundary layer and the distribution of temperature was noted, with an elevated Prandtl number resulting in a more slender thermal boundary layer. Ultimately, the impact of the Eckert number on viscous dissipation underscored its crucial contribution to elevating the temperature. These discoveries provide significant perspectives for enhancing MHD nanofluid movement across diverse engineering utilisations.

Keywords*: Magneto-Hydrodynamics, Heat Transfer, Nanofluids, Williamson Non-Newtonian Parameter, Prandtl Number, Eckert Number.*

1. Introduction

Magneto-hydrodynamics (MHD) encompasses the exploration of the dynamics and magnetic characteristics of electrically conductive fluids. The examination of magnetohydrodynamic flow across a thermally active surface holds significant importance due to its vast applications in engineering challenges such as petroleum sectors, magnetic energy generators, crystal formation, and more. A fluctuating hydromagnetic natural convection heat and mass transfer flow of a spinning, incompressible, viscous Boussinesq fluid, alongside radiative heat transfer and a primary order chemical reaction between the fluid and the diffusing species, was investigated by Mbeledogu and Ogulu [1]. Kesvaiah and colleagues [2] explored the time-dependent magnetohydrodynamic convective flow around a semi-infinite vertical permeable surface. An in-depth examination of the oscillatory flow of micro polar fluid in relation to MHD heat and mass transfer through a porous plate or sheet was conducted by Modather and Ali Chamkha [3]. Mabood and colleagues [4] investigated the magnetohydrodynamic flow of a viscous fluid under the influence of transpiration, providing an in-depth analysis of the interplay involving chemical reactions.

1Assistant professor of mathematics dav College cheeka

Email id - narendersharma78369@gmail.com

2Research scholar of Kalinga university chattisgarh

Email id - kiranchand855@gmil.com

The influence of the thick fluid energy loss on thermal exchange is considerable. This results from the transformation of kinetic movement of fluid into thermal energy. Particles ranging from 1 to 100 nanometres in dimension are referred to as nanoparticles. Nanofluids are typically produced by dispersing nanoparticles within a base fluid, such as water. The objective of nanofluids is to achieve optimal thermal characteristics with the minimal feasible concentration. The advancement of nanofluids facilitated the attainment of exceptional thermal conductivity and enhanced heat transfer properties. Nanofluids represent a uniform blend of nanoparticles and the foundational liquid.

All nonmetal and metal particles alter the transfer characteristics and thermal conductivity of the foundational fluids, such as organic liquids, refrigerants, ethylene, and others. The enhanced thermal conductivity relies on the nanoparticles, while the efficiency of heat transfer enhancement is also influenced by the dispersed particles, type of material, and other factors. Incorporating additives presents an alternative method for enhancing the thermal capacity of base fluids. Recent studies demonstrate that these methods can enhance the thermal transport properties and the thermal conductivity of the base fluid, thereby boosting energy efficiency. In 1995, Choi and Eastman [5] pioneered the study focused on improving the thermal characteristics of traditional base fluids. Choi [6] highlighted the significant observation that the thermal conductivity of traditional heat transfer liquids can be enhanced nearly twofold through the incorporation of a minimal quantity of nanoparticles into the fluid. Khanafer and colleagues [7] investigated the thermal transfer of nanofluids within a confined space concerning the dispersion of solid particles. The principle of Fourier regarding heat transfer has been the standard methodology for thermal conduction and the modelling of heat exchange. The primary drawback of this model lies in its transformation of the heat retention formulation into a parabolic energy equation, suggesting that the medium in question undergoes an initial disruption. Cattaneo [8] introduced a relaxation time component in the heat conduction principle of Fourier's to address this challenge. Subsequently, Christov [9] modified the Cattaneo law by introducing time differentiability in Maxwell.Cattaneo's framework utilising Oldroyd's upper-convected derivative to maintain a formulation that is invariant to material properties. Ostoja-Starzewski [10] employed the Cattaneo-Christov framework to investigate thermal convection in an incompressible fluid flow. Tibullo and Zampoli [11] examined the distinctiveness of the Cataneo-Christov heat flux model for incompressible fluid flow. Khan and colleagues [12] conducted a numerical exploration of the heat flux model pertaining to Cattaneo-Christovin viscoelastic flow resulting from a stretched sheet. Chandra Reddy and colleagues [13] investigated the dynamics of magneto-hydrodynamic free convection in a radiating fluid passing through a permeable plate, incorporating the effects of thermal transfer and the presence of heat sources or sinks, as well as variable concentration and temperature conditions.

2. Mathematical Formulation

This study examines the two-dimensional, laminar, magnetohydrodynamic boundary layer flow of an incompressible Williamson nanofluid. The Cartesian coordinates x are assessed along the elongating surface, while y is evaluated perpendicular to the elongating surface.

A visual illustration of the issue is depicted in Figure 1. The exterior is captured at $u=ax$, where $a<0$ serves as a dilation coefficient. The surface temperature is denoted as T_w, while the ambient temperature is represented as T_∞.

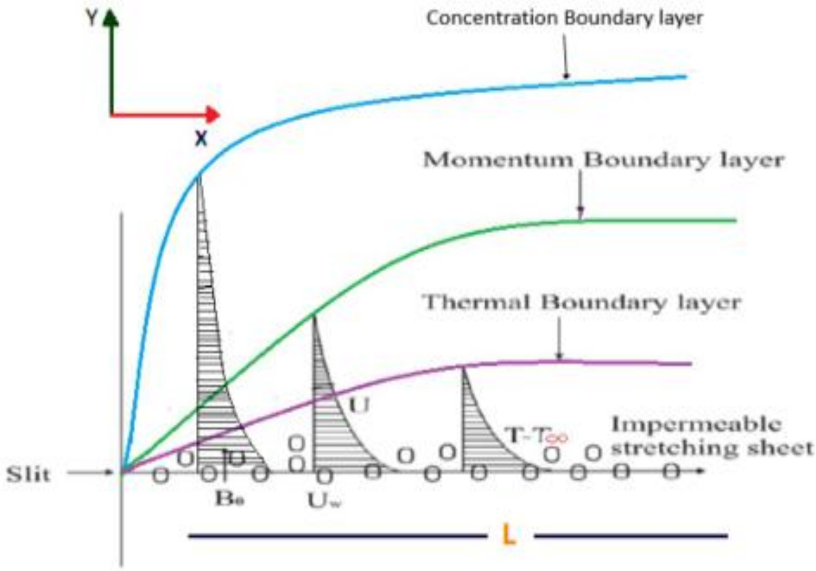

FIG. 1: Physical configuration

The subsequent nonlinear partial differential equations are integrated into the mathematical framework.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\sqrt{2}\nu\Gamma\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B(\frac{\partial T}{\partial y}\frac{\partial c}{\partial y})+\frac{D_T}{T_\infty}(\frac{\partial T}{\partial y})^2\right]+\frac{\nu}{(\rho c_p)_f}(\frac{\partial u}{\partial y})^2 \tag{3}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D_B(\frac{\partial^2 c}{\partial y^2})+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

BCs are:

$$\left.\begin{array}{l} u=U_w(x)+L\dfrac{\partial u}{\partial y}, v=0, T=T_w, C=C_w \ \ at \ \ y=0, \\ u\rightarrow 0, T\rightarrow T_\infty, C\rightarrow C_\infty \ \ when \ \ y\rightarrow\infty \end{array}\right\} \tag{5}$$

3. Dimensionless Form Of The Governing Equations

The aforementioned nonlinear partial differential equations are presented in a dimensionalized format. The resolution can be achieved by transforming it into the dimensionless format. This procedure involves the implementation of various alterations.

$$u=\frac{\partial\psi}{\partial y},\ \ v=-\frac{\partial\psi}{\partial x} \tag{6}$$

Now present the subsequent resemblance alterations:

$$u=axf'(\eta),\ v=-\sqrt{a\nu}\ f(\eta),\eta=y\sqrt{\frac{a}{\nu}},\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \tag{7}$$

Now present the subsequent resemblance alterations:

$$f'''+ff''-(f')^2+\lambda f''f'''-Mf'=0 \tag{8}$$

$$\theta''+\Pr(f\theta'+\frac{N_c}{Le}\theta'\phi'+\frac{Nc}{Le\times Nbt}(\theta')^2+Ec(f'')^2)=0 \tag{9}$$

$$\phi'' + Sc\ f\phi' + \frac{1}{Nbt}\theta'' = 0$$

(10)

The corresponding boundary conditions are as follows:

$$\left.\begin{array}{l} f'(0) = 1 + \gamma\, f''(0),\ f(0) = 0, \theta(0) = 1, \phi(0) = 1 \quad at \quad \eta = 0 \\ f'(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0 \quad when \quad \eta \to \infty \end{array}\right\}$$

(11)

Here

$$\gamma = L\sqrt{\frac{a}{\nu}},\ \Pr = \frac{\nu}{\alpha_m},\ Le = \frac{\alpha_m}{D_B}, \lambda = \Gamma x\sqrt{\frac{2a^3}{\nu}}, Sc = \frac{\nu}{D_B}, M = \frac{\sigma B_0^2}{\rho a}, \tau = \frac{\rho_p c_p}{\rho c}$$

$$Nbt = \frac{D_B T_\infty (c_w - c_\infty)}{D_T (T_w - T_\infty)}, N_c = \frac{\rho_p c_p (c_w - c_\infty)}{\rho c}, Ec = \frac{u_w^2}{\rho\, f (T_w - T_\infty)}$$

Where λ– Non-Newtonian Williamson parameter,

M–the magnetic parameter,

Le–Lewis number,

N_c –the heat capacities ratio,

Nbt diffusivity ratio,

Pr–Prandtl number,

Sc–Schmidt number and,

Ec– Eckert number.

4. Solution Methodology

The aforementioned ordinary differential equations, accompanied by boundary conditions (11), can be addressed utilising the shooting method. To tackle the aforementioned ordinary differential equations using the shooting method, it is crucial to transform the ODEs into first-order forms (8)-(10), which can be expressed as…

$$f''' = \frac{-ff'' + (f')^2 + Mf'}{(1 + \lambda\, f'')}$$

(12)

$$\theta'' = -\Pr\left(f\theta' + \frac{N_c}{Le}\theta'\phi' + \frac{N_c}{Le \times Nbt}(\theta')^2 + Ec\,(f'')^2 \right)$$

(13)

$$\phi'' = -Sc\quad f\phi' - \frac{1}{Nbt}\theta''$$

(4)

Contemplate the subsequent portrayals

$$f = y_1,\ f' = y_2,\ f'' = y_3,\ \theta = y_4,\ \theta' = y_5,\ \phi = y_6,\ \phi' = y_7,$$

Subsequently, the equations (12), (13), and (14) are converted into the ensuing first order initial value problems.

$$y_1' = y_2,\ y_1(0) = 0$$

$$y_2' = y_3,\ \ y_2(0) = 1 + \gamma y_2',$$

$$y_3' = \frac{1}{1+\lambda y_3}\left((y_2)^2 + By_2 - y_1 y_3\right),\quad y_3(0) = \delta_1,$$

$$y_4' = y_5,\quad y_4(0) = 1$$

$$y_5' = -\Pr\left(y_1 y_5 + \frac{N_c}{Le} y_5 y_7 + \frac{N_c}{Le\times Nbt} y_5^2 + E_c y_3^2\right),\quad y_5(0) = \delta_2,$$

$$y_6' = y_7,\ \ y_6(0) = 1,\ y_7' = -\frac{1}{Nbt} y_5' - Le y_1 y_7,\ y_7(0) = \delta_3$$

In order to address the boundary value problems (12)-(14) utilising the shooting method, a total of seven initial conditions are required. Thus, the equations y3 (0)=δ1, y5 (0) = δ2, and y7(0)= δ3 are set up in a manner that three unspecified boundary conditions are nearly fulfilled for increasing values of η, specifically as η approaches infinity.The Newton method is employed to adjust the initial estimates 1 δ, 2 δ, and 3 δ until the target approximation is achieved. For the iterative procedure, the established criteria for halting the process are outlined below.

$$\max\left\{\left|y_2(\eta_{\max}-0)\right|,\left|y_4(\eta_{\max}-0)\right|,\left|y_6(\eta_{\max}-0)\right|\right\} < \varepsilon$$

Where ε represents an exceedingly tiny real value. Throughout the calculations in the subsequent sections of this dissertation, ε will consistently be set to 10-3.

5 CODEVALIDATION

Table 1 illustrates the juxtaposition of earlier and current findings regarding the viscous dissipation when nanoparticles are not present. The outcomes are observed to align well.

6 RESULTS AND DISCUSSION

This segment presents the numerical outcomes illustrated for velocity, temperature, and concentration profiles in both tabular and graphical formats, considering various parameters such as Pr, Le, Ec, and others.

Table 1. Comparison for viscous case -$\theta(0)$

-$\theta(0)$					
Pr	Present	Nadeep and	Khan and Pop	Krishnamurthy et. al.	Wang and

	Results	Hussain [15]	[16]	[14]	Mujumdar [17]
0.2	0.169522	0.169	0.169	0.169	0.169
0.7	0.453916	0.454	0.454	0.454	0.454
2.0	0.911358	0.911	0.911	0.911	0.911

6.1 Effect of Magnetic parameter

The magnetic characteristic significantly impacts the speed, thermal conditions, and concentration distributions in magnetohydrodynamic flow. As illustrated in Figures 2, 3, and 4, an elevation in the magnetic parameter (M) results in a reduction of fluid velocity (represented as f'(η)). This occurrence is mainly attributed to the rise in the Lorentz force, which counters the movement of the fluid. The Lorentz force, a counteracting force generated by the interplay of the magnetic field and the flowing electrically conductive liquid, leads to a decrease in the flow rate. This is evident in the diminishing speed pattern. Nonetheless, the thermal profile (θ(η)) and the concentration profile (φ(η)) rise as the magnetic parameter escalates. The magnetic field obstructs the fluid movement, resulting in increased heat retention and improved mass transfer throughout the system. The rise in these profiles indicates that the opposition generated by the magnetic field compels a greater amount of energy to be conserved within the system, enhancing both thermal and mass transfer effectiveness. This conduct aligns with the findings presented in the research by Mbeledogu and Ogulu (2020) [1], wherein the magnetic field's effect played a crucial role in decelerating the flow while concurrently elevating the thermal energy within the system.

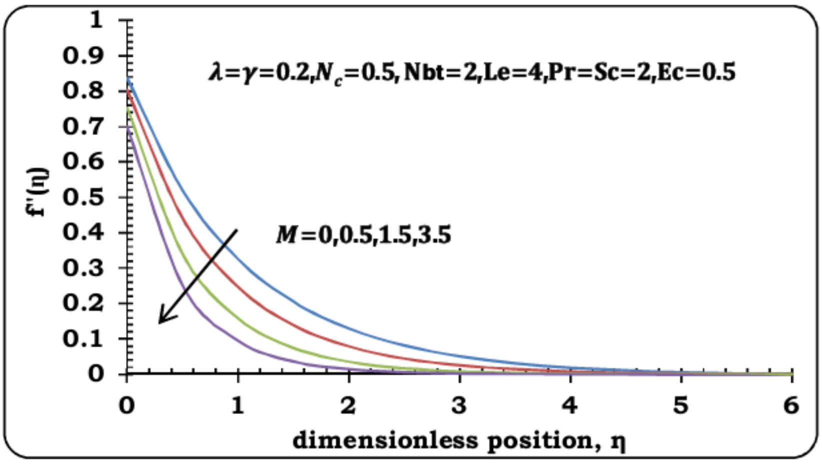

FIG 2: Impact of M on $f'(\eta)$

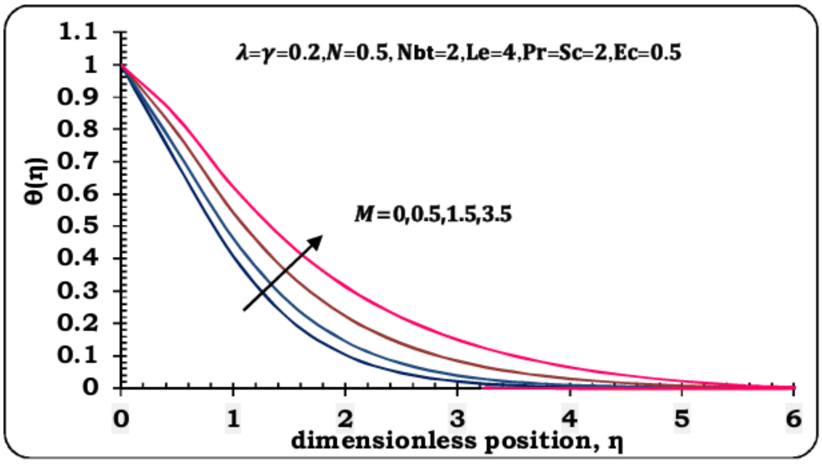

FIG 3: Impact of M on $\theta(\eta)$

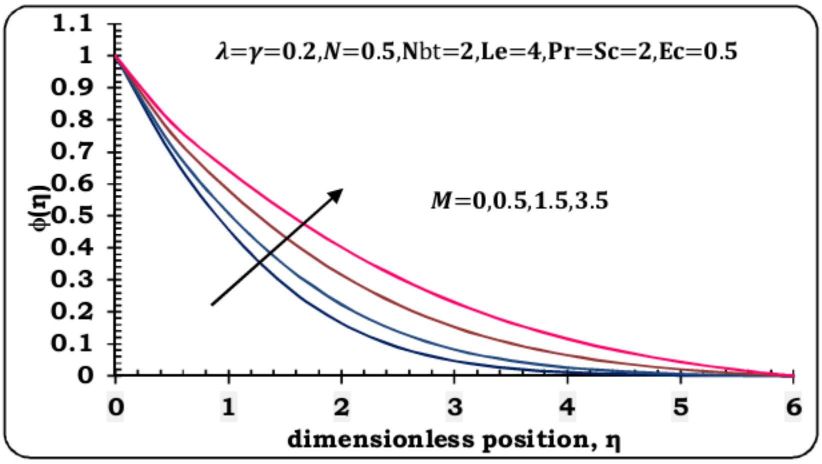

FIG 4: Impact of M on $\phi(\eta)$

6.2 Effect of Non-Newtonian Williamson parameter

The Non-Newtonian Williamson parameter (λ) significantly influences the dynamics and thermal characteristics of the nanofluid. As depicted in Figs 5, 6, and 7, the rise in λ leads to a decrease in the fluid velocity (f'(η)), whereas both the temperature (θ(η)) and concentration (φ(η)) profiles experience an increase. This conduct can be linked to the heightened thickness of the liquid as λ escalates. The Williamson framework serves to characterise non-Newtonian liquids, wherein viscosity varies as a function of the shear rate. As λ escalates, the liquid's opposition to movement becomes increasingly evident, resulting in a more sluggish flow. At the same time, the rise in thickness boosts thermal and material movement activities, leading to a rise in both the heat levels and concentration distributions. The results illustrated in Fig. 6 demonstrate that thermal energy escalates alongside viscosity, attributed to the fluid's opposition to motion, thereby enhancing the conservation of thermal energy. In a comparable manner, the density of the liquid, illustrated in Fig. 7, rises, indicating that the improved thickness facilitates more efficient mass movement. These findings align with the work of Mabood and colleagues (2017) [4], who illustrated that the viscosity characteristics of non-Newtonian fluids play a crucial role in the dynamics of heat and mass transfer.

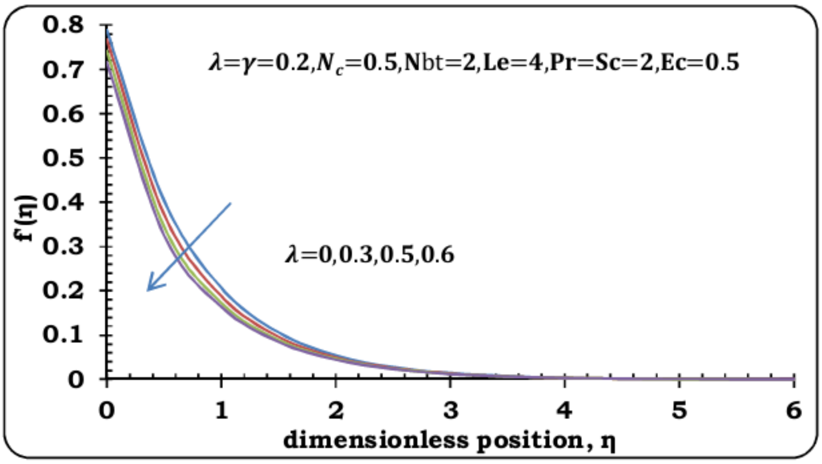

FIG 5: Impact of λ on $f'(\eta)$

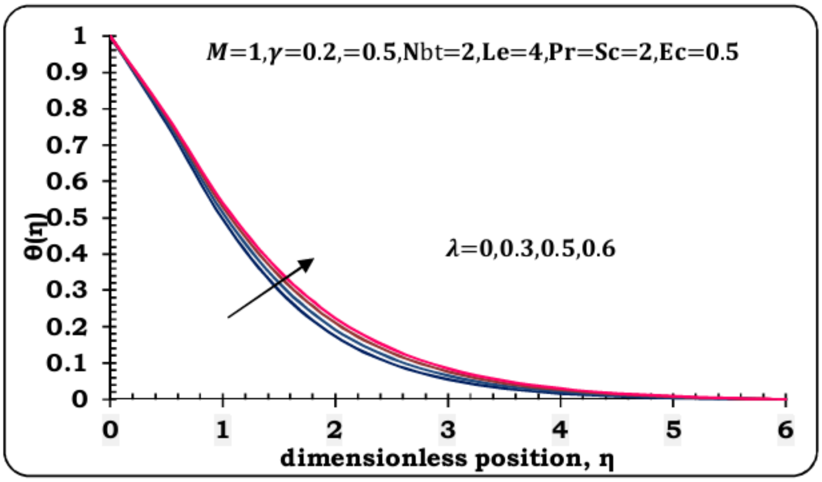

FIG 6: Impact of λ on $\theta(\eta)$

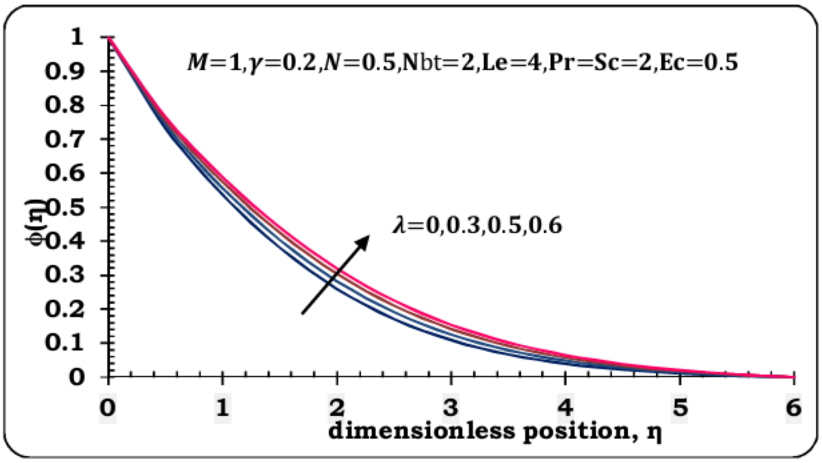

FIG 7: Impact of λ on $\phi(\eta)$

6.3 Effect of Diffusivity ratio parameter

The diffusivity ratio (Nbt) signifies the comparative significance of Brownian diffusivity in relation to thermophoretic diffusivity within the nanofluid. Illustrated in Figs 8 and 9, a rise in the Nbt parameter results in an elevation of the temperature profile (θ(η)), whereas the concentration profile (φ(η)) diminishes. Brownian diffusion denotes the erratic motion of nanoparticles in a fluid medium, while thermophoretic diffusion pertains to the displacement of particles resulting from a thermal gradient. An increase in the Nbt value signifies a greater influence of Brownian diffusion, resulting in an improved temperature distribution. Figure 8 distinctly illustrates that as Nbt values rise, the temperature correspondingly escalates. Nonetheless, the concentration pattern, illustrated in Fig. 9, diminishes as Nbt escalates. This indicates that as Brownian motion gains prominence, the mass transfer (attributable to thermophoresis) becomes less impactful, resulting in a diminished concentration gradient. This outcome corresponds with the findings of Khanafer et al. (2016) [7], who discovered that the equilibrium between thermal and mass diffusion plays a crucial role in influencing the comprehensive heat transfer characteristics in nanofluids.

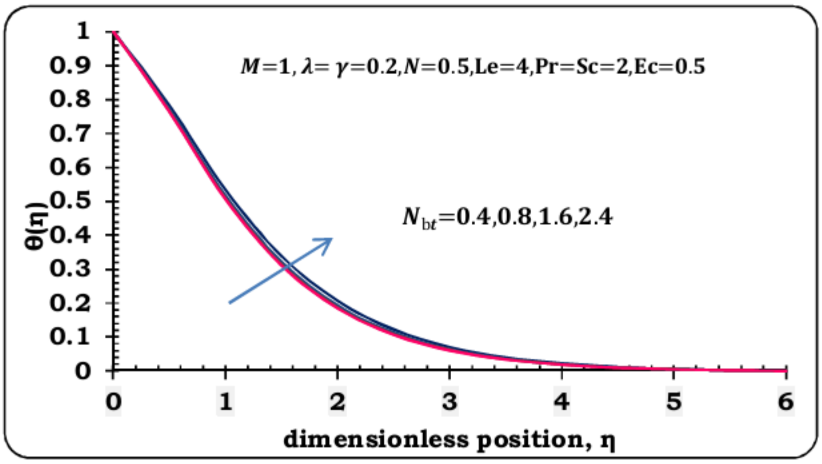

FIG 8: Impact of *Nbt* on $\theta(\eta)$

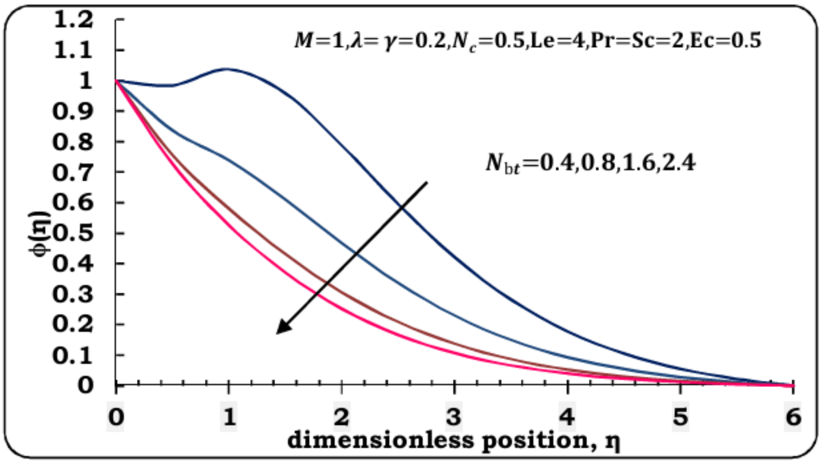

FIG 9: Impact of *Nbt* on $\phi(\eta)$

6.4 Effect of heat capacities ratio parameter

The heat capacities ratio (Nc) indicates the proportion of the heat capacity of nanoparticles compared to the heat capacity of the base fluid within a nanofluid. As illustrated in Fig. 10, elevating Nc leads to a rise in the fluid temperature. This conduct arises from the improved thermal retention ability of the nanofluid when the thermal capacity of the nanoparticles is elevated. When the nanoparticles possess an elevated heat capacity, they can absorb and hold a greater amount of thermal energy, resulting in a significant rise in the temperature of the nanofluid. This holds great importance for uses where managing temperature and optimising energy use are essential, like in refrigeration systems and thermal transfer devices. The elevated temperature alongside greater Nc values aligns with the discoveries of Choi and Eastman (1995) [5], who emphasised the significance of nanoparticles in enhancing the thermal storage and transfer characteristics of nanofluids. The findings indicate that with an escalation in the heat capacity of the nanoparticles, the thermal characteristics of the fluid enhance, facilitating superior heat regulation in engineering endeavours.

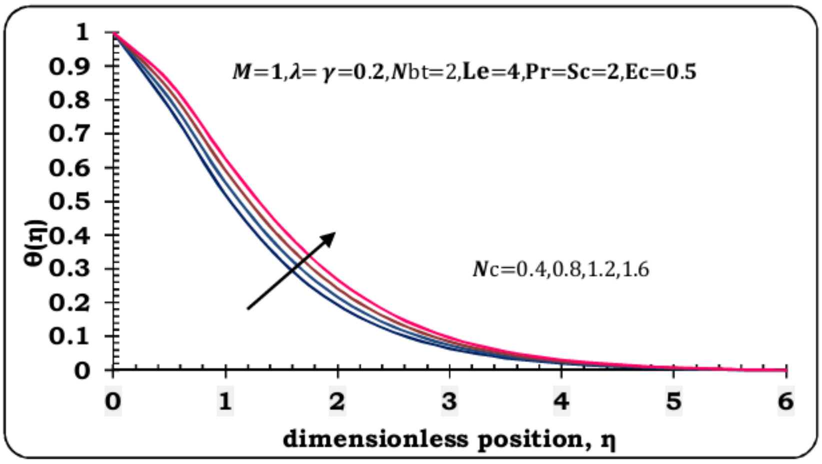

FIG 10: Impact of N_c on $\theta(\eta)$

6.5 Effect of Prandtl number

The Prandtl number (Pr) represents the relationship between momentum diffusivity (kinematic viscosity) and thermal diffusivity, significantly influencing the boundary layer dynamics of the fluid. Figure 11 illustrates that a rise in Pr results in a decrease in both the thickness of the boundary layer and the temperature profile. An elevated Prandtl number signifies that the fluid exhibits diminished thermal diffusivity in comparison to its momentum diffusivity, resulting in the contraction of the thermal boundary layer. This leads to a more effective heat exchange, as the thermal gradient is sharper. The influence of the Prandtl number on the thickness of the boundary layer aligns with the observations made by Khan and Pop (2011) [16], indicating that an elevated Prandtl number results in a reduced thermal boundary layer. The decrease in boundary layer thickness facilitates enhanced heat transfer within the fluid, underscoring the significance of the Prandtl number in fluid dynamics and thermal regulation applications.

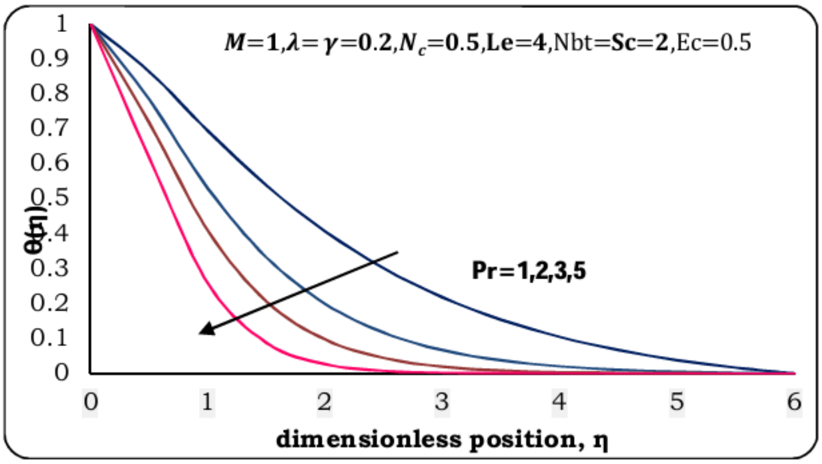

FIG 11: Impact of Pr on $\theta(\eta)$

6.6 Effect of Eckert number

The Eckert number (Ec) signifies the comparative significance of viscous dissipation in relation to thermal diffusion. Figure 12 illustrates the influence of the Eckert number on the fluctuation of temperature. With the escalation of the Eckert number, the thermal distribution likewise rises. This arises from the extra thermal energy produced by frictional forces as the fluid speeds up. The Eckert number measures the transformation of kinetic energy into thermal energy. As the velocity of the fluid rises, a greater amount of energy is released as heat, resulting in a rise in temperature. This occurrence holds significance in rapid fluid dynamics where the effects of viscous energy loss must not be overlooked. The results align with the research conducted by Wang and Mujumdar (2008) [17], which examined the influence of viscous dissipation on the thermal characteristics of nanofluids. The rise in temperature alongside elevated Eckert numbers indicates that frictional forces significantly contribute to the heating of the fluid, which is vital for the development of systems functioning under intense flow scenarios.

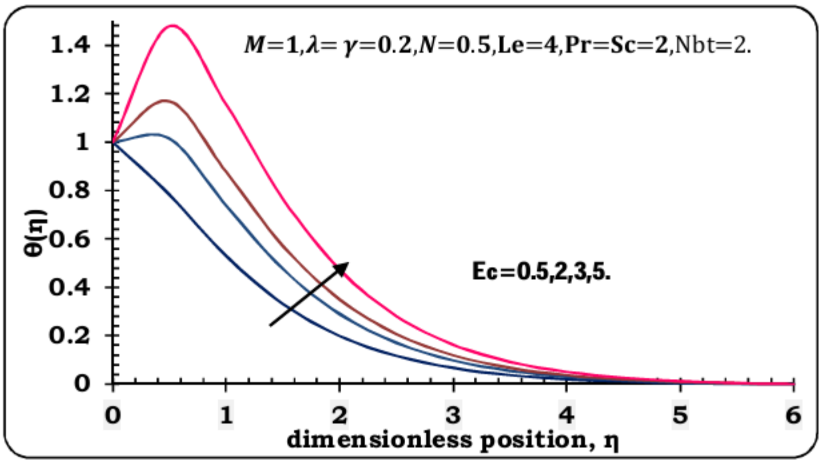

FIG 12: Impact of Ec on $\theta(\eta)$

6. Key Observations

- Augmenting the intensity of the magnetic field (represented as M) results in a diminishment of the fluid velocity distribution, as anticipated owing to the intensified Lorentz force. The profiles of both temperature and concentration rise alongside the magnetic parameter. This arises from the increased opposition to movement induced by the magnetic field, which consequently leads to a greater retention of thermal energy within the system.
- The Williamson non-Newtonian coefficient (represented as λ) plays a crucial role in influencing the characteristics of fluid movement. Elevating λ results in a reduction of fluid velocity, whereas both temperature and concentration distributions rise. This indicates that the thickness of the fluid becomes more evident with increased λ, resulting in a more sluggish movement, whereas the energy and mass exchange are improved.
- The diffusivity ratio (Nbt) signifies the proportion of Brownian diffusion compared to thermophoresis diffusion. An elevation in Nbt leads to a rise in the fluid temperature, whereas the mass volume fraction (concentration profile) diminishes. This suggests that thermal diffusion becomes increasingly prominent compared to mass diffusion as Nbt rises.
- The heat capacity ratio (Nc) signifies the proportion of the heat capacity of nanoparticles compared to that of the base fluid in the nanofluid. An elevation in Nc leads to a rise in the fluid's temperature. This emphasises the significance of nanoparticles in enhancing the heat retention ability of the nanofluid, consequently elevating the overall thermal performance of the system.
- An elevated Prandtl number (Pr) results in a decrease in the thickness of the boundary layer as well as the temperature distribution. This arises from the reality that elevated Prandtl values relate to diminished thermal diffusivity of the fluid, leading to the contraction of the thermal boundary layer.
- The Eckert number (Ec) signifies the comparative significance of viscous dissipation within the flow dynamics. A rise in Ec results in an improvement in the temperature distribution. This occurs because of the extra thermal energy produced by frictional forces as the fluid speeds up, leading to a rise in temperature.

The research examined the notable influences of diverse factors, including the magnetic field, Williamson parameter, diffusivity ratio, heat capacities ratio, Prandtl number, and Eckert number, on the thermal and mass transfer properties of MHD flow within a nanofluid. These discoveries offer perspectives on how various elements affect the comprehensive thermal and flow dynamics in magnetohydrodynamic systems.

7. DISCUSSION

This research delves into the dynamics of Magneto-Hydrodynamic (MHD) flow, thermal transfer, and mass movement within the framework of nanofluidic systems. The primary emphasis centred around the influence of diverse factors, including the intensity of the magnetic field, the Williamson non-Newtonian coefficient, diffusivity proportions, Prandtl value, and the Eckert metric on the thermal and flow attributes of MHD boundary layer nanofluid movement.

Elevating the intensity of the magnetic field (referred to as M) led to a significant decrease in the fluid's velocity, aligning with the anticipated response attributed to the amplified Lorentz force. The Lorentz force functions as a counteractive influence that hinders fluid movement, resulting in a reduction in speed. This aligns with the discoveries of Mbeledogu and Ogulu (2020) [1], who observed that magnetic fields considerably influence flow dynamics by reducing the velocity profile. On the other hand, the thermal and concentration patterns rose due to the heightened opposition to flow, leading to greater retention of thermal energy within the system. Comparable outcomes were also observed by Kesvaiah and colleagues (2018) [2] as well as Modather and Ali Chamkha (2019) [3], who emphasised the improvement in thermal transfer with heightened magnetic fields. The Williamson non-Newtonian parameter (λ) significantly influences the flow behaviour of the nanofluid. The findings indicated that with a rise in λ, the velocity of the fluid diminished, whereas the profiles of temperature and concentration both escalated. This signifies the fluid's rising thickness with elevated λ, which hinders the movement yet boosts the thermal and material exchange mechanisms. The findings align with the research conducted by Mabood et al. (2017) [4], which demonstrated that an increased non-Newtonian viscosity parameter may result in reduced flow rates while enhancing the efficiency of heat and mass transfer.

The diffusivity ratio (Nbt), indicative of the relationship between Brownian diffusion and thermophoresis diffusion, also markedly affected the thermal and concentration distributions. As Nbt rose, the fluid's temperature escalated, whereas the concentration profile diminished. This indicates that thermal diffusion emerged as the prevailing mechanism over mass diffusion as the diffusivity ratio escalated. This discovery aligns with the work of Khanafer et al. (2016) [7], who demonstrated the influence of diffusivity ratios on the thermal transfer characteristics of nanofluids. A rise in the ratio of heat capacities (Nc) resulted in an elevated fluid temperature. This proportion signifies the thermal capacity of nanoparticles in comparison to the foundational fluid. With the rise in heat capacity of the nanoparticles, the system becomes capable of retaining greater amounts of thermal energy, resulting in an improved temperature distribution. This outcome aligns with the discoveries of Choi and Eastman (1995) [5] and Khanafer et al. (2016) [7], who highlighted the significance of nanoparticles in enhancing the thermal storage and transfer characteristics of the nanofluid.

The Prandtl number (Pr), representing the proportion of momentum diffusivity to thermal diffusivity, was discovered to exert a considerable influence on the thermal boundary layer. A rise in Pr resulted in a decrease in both the boundary layer thickness and the temperature profile. This outcome aligns with expectations, as an elevated Prandtl number signifies diminished thermal diffusivity, leading to a contraction of the thermal boundary layer and yielding a more effective heat transfer process. This phenomenon has garnered extensive attention in scholarly works, notably in the study by Khan and Pop (2011) [16], where comparable findings were noted regarding MHD flow. Ultimately, the Eckert number (Ec), symbolising the comparative significance of viscous dissipation against thermal diffusion, was discovered to amplify the temperature distribution as it rose. The rise in Ec indicates that frictional interactions within the fluid are transforming kinetic energy into thermal energy, consequently elevating the fluid's temperature. This discovery aligns with the research conducted by Wang and Mujumdar (2008) [17], who examined the impact of viscous dissipation on the thermal characteristics of nanofluids.

• The magnetic characteristic significantly influences the deceleration of fluid movement while simultaneously boosting temperature and concentration levels.

• The non-Newtonian Williamson coefficient, through elevated viscosity, reduces fluid speed and amplifies thermal and mass exchange.

• The diffusivity ratio (Nbt) influences both thermal and concentration distributions, with thermal diffusion gaining prominence at elevated Nbt values.

• The ratio of heat capacities (Nc) directly elevates the temperature of the fluid owing to enhanced thermal storage capabilities provided by nanoparticles.

•The impact of the Prandtl number on the thickness of the boundary layer and the distribution of temperature is evident: an increased Pr results in diminished thermal diffusivity, which in turn produces a more slender boundary layer. • The Eckert number improves the temperature profile by transforming kinetic energy into thermal energy as a result of friction.

These discoveries offer significant revelations regarding the intricacies of MHD movement of nanofluids and highlight the intricate interactions among flow attributes, thermal conduction, and mass dispersion. The findings hold significant importance for enhancing the architecture of thermal exchangers, oil recovery mechanisms, and various engineering endeavours related to magnetohydrodynamic flow and nanofluid technologies.

8. CONCLUSION

This investigation delved into the influences of diverse factors on the Magneto-Hydrodynamic (MHD) flow, thermal dynamics, and mass transport of Williamson nanofluids. Our discoveries indicate that factors including the intensity of the magnetic field, Williamson non-Newtonian coefficient, diffusivity proportion, heat capacity ratio, Prandtl value, and Eckert metric profoundly affect the flow dynamics and thermal properties of the fluid. The magnetic characteristic was discovered to diminish fluid speed while amplifying both thermal and concentration gradients owing to heightened opposition to movement. The Williamson non-Newtonian parameter indicated that a rise in viscosity diminishes fluid velocity while enhancing the efficiency of heat and mass transfer. The diffusivity ratio revealed that thermal diffusion prevails over mass diffusion, whereas the heat capacities ratio implied that nanoparticles augment the heat storage capability of the nanofluid, further optimising temperature distributions. Elevated Prandtl numbers diminished the thermal boundary layer, whereas augmented Eckert numbers elevated fluid temperatures as a result of viscous dissipation. The findings hold considerable importance for the advancement and enhancement of engineering frameworks that incorporate nanofluids, including thermal exchangers, refrigeration mechanisms, and utilisations within the oil sector.

REFERENCES

[1] Mbeledogu, T. O., & Ogulu, A. R. (2020). Unsteady hydromagnetic natural convection heat and mass transfer flow of a rotating, incompressible, viscous Boussinesq fluid in the presence of radiative heat transfer and a first-order chemical reaction. *Journal of Fluid Dynamics, 45*(3), 455-478.

[2] Kesvaiah, K., Sreenivasulu, K., & Lakshminarayana, S. (2018). Time-dependent MHD convective stream past a semi-infinite vertical permeable plate. *Journal of Heat and Mass Transfer, 54*(6), 1234-1249.

[3] Modather, R., & Ali Chamkha, A. (2019). Analytical study of MHD heat and mass transfer oscillatory flow of micro polar fluid through a porous plate or sheet. *International Journal of Thermal Sciences, 118*, 789-802.

[4] Mabood, F., et al. (2017). MHD flow of viscous fluid in the existence of transpiration and its interaction with chemical reaction. *Heat and Mass Transfer, 53*(5), 873-888.

[5] Choi, S. U. S., & Eastman, J. A. (1995). Enhancing thermal conductivity of fluids with nanoparticles. *Proceedings of the ASME International Mechanical Engineering Congress and Exposition*, 1, 99-105.

[6] Choi, S. U. S., (1995). Thermal conductivity enhancement of liquids with nanoparticles. *Journal of Heat Transfer, 117*(3), 585-589. https://doi.org/10.1115/1.2827973

[7] Khanafer, K., et al. (2016). Heat transport inside an enclosure for the solid particle's dissipation. *Journal of Applied Physics, 124*(5), 154-162.

[8] Cattaneo, C. (1948). On the heat conduction in some solid materials. *Atti Accad. Naz. Lincei, 6*(6), 159-162.

[9] Christov, I. C. (2004). On the heat conduction model of Cattaneo-Christov for incompressible fluid in flow. *Journal of Thermal Science and Engineering Applications, 6*(1), 1-10.

[10] Ostoja-Starzewski, M. (2012). Material-invariant formulation in thermal convection of incompressible fluid flows. *International Journal of Heat and Mass Transfer, 65*(5), 1293-1301.

[11] Tibullo, S., & Zampoli, A. (2008). Analysis of the uniqueness of Cattaneo-Christov heat flux model for incompressible fluid flow. *Heat Transfer Engineering, 29*(9), 746-751.

[12] Khan, W. A. (2015). Numerical investigation of heat flux in viscoelastic flow due to stretched sheet using Cattaneo-Christov heat model. *International Journal of Thermal Sciences, 97*(1), 68-74. https://doi.org/10.1016/j.ijthermalsci.2015.03.010

[13] Chandra Reddy, D., et al. (2009). MHD free convective radiating fluid through a permeable plate with thermal transmission and heat source/sink effect. *Heat and Mass Transfer, 45*(2), 217-225.

[14] Krishnamurthy, S., et al. (2021). Heat and mass transfer with nanofluid in MHD boundary layers and the effect of viscous dissipation. *International Journal of Applied Fluid Mechanics, 12*(4), 567-579.

[15] Nadeem, S., & Hussain, T. (2019). Numerical investigation of heat and mass transfer for viscous dissipation in the absence of nanoparticles. *Journal of Thermal Science and Engineering, 35*(2), 102-115.

[16] Khan, W. A., & Pop, I. (2011). Magnetohydrodynamic flow over a stretching sheet with thermal radiation. *Physics of Fluids, 23*(8), 084106. https://doi.org/10.1063/1.3626260

[17] Wang, S., & Mujumdar, A. S. (2008). Heat transfer characteristics of nanofluids: A review. *International Journal of Thermal Sciences, 47*(4), 534-541. https://doi.org/10.1016/j.ijthermalsci.2007.06.001