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Role of Z Fuzzy Relation in Approximate Reasoning

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Abstract: The development of Fuzzy sets were as a generalization of usual sets to take into account of uncertainty and vagueness which are in daily life. In 2011 Zadeh further extended the concept of z-numbers which model the 'fuzziness' of information and addition including reliability factor. An z-number is an ordered pair of fuzzy numbers where the first component gives the information and the second component deals with the reliability of information.Fuzzy relations have been well studied and they play a vital role in fuzzy logic. In this paper we extend the concept of fuzzy relation to Z fuzzy relation and show how z fuzzy if-then rules can be described. The generalized compositional rule of inference is also developed. It is applicable in theory of approximate reasoning, medical diagnosis etc.

Keywords: *Z fuzzy Relation, Z fuzzy if-then rules, Compositional rule of inference on z-numbers.*

Introduction

Fuzzy versions of various mathematical concepts have been developed over the last few decades. Fuzzy relations have been well studied and they play a vital role in fuzzy logic. In this chapter, we show how z fuzzy if-then rules can be described by z fuzzy relation. The generalized compositional rule of inference is also developed here.

Z-fuzzy if-then rule

First consider the 'if-then' rule in a crisp context.
Crisp if-then rule format;
If X∈A then Y∈B (A, B are crisp sets)

Example:

If X is real then Y is non negative. Here A is the set of real numbers and B is the set of non negative numbers.
Next look at fuzzy-case
Fuzzy if-then rule format;
If x ∈ $\tilde{A}$ then y ∈ $\tilde{B}$

Example:

If X is small then Y is negligible.
Here A is the fuzzy set of small numbers, B is the fuzzy set of negligible numbers.
Now let us look at z- fuzzy case;

Z-fuzzy if-then rule format;

If X isz (A,B) then Y isz (C,D)
(Note: X isz (A,B) means X is 'equal to the z-number' (A,B))

Example:

Consider the following two statements
If the monsoon is normal the yield of paddy will be good -- fuzzy proposition⟶ (1)
If it is very likely that this year monsoon is normal, then it is likely that the paddy yield is good $-z\ fuzzy\ proposition \longrightarrow$ (2)

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Let:
X denote the amount of rainfall received in this monsoon.
Fuzzy set A - represent 'normal' monsoon

Fuzzy set B - represent 'very likely'
Y denote the amount of yield of paddy in this season
Fuzzy set C - represent 'good' yield
Fuzzy set D - represent 'likely'
So fuzzy proposition (1) can be stated as

$$If\ X\ is\ A\ then\ Y\ is\ C$$

The z fuzzy proposition (2) can be stated as
If monsoon isz (normal, very likely) then paddy yield isz (good, likely).
That is (2) can be represented by z fuzzy if then rule.

$$If\ X\ isz\ (A,B)\ then\ Y\ isz\ (C,D)$$

MODELING IF-THEN RULES USING RELATIONS

Case i: FUZZY RELATION

Consider the fuzzy proposition:
$If\ X\ is\ A\ then\ Y\ is\ B.$
Then the relationship function R can be calculated as follows:-
$R(x,y) = \mathcal{I}(A(x), B(y))$ where $\mathcal{I}$ is a suitably chosen implication operator.
For e.g.:- we can choose the Lukasiewicz implication

$$\mathcal{I}(a,b) = min\ (1, 1\text{-}a\text{+}b)$$

Example:

X = $\{x_1, x_2, x_3\}$; Y = $\{y_1, y_2\}$;
A = $\frac{.5}{x_1} + \frac{1}{x_2} + \frac{.6}{x_3}$ and
B = $\frac{1}{y_1} + \frac{.4}{y_2}$

$R(x_1, y_1) = \mathcal{I}[(A(x_1), B(y_1)] = \mathcal{I}[.5,1]$
$= min\ (1,1\text{-}.5\text{+}1) = min(1,1.5) = 1$
$R(x_1, y_2) = \mathcal{I}[(A(x_1), B(y_2)] = \mathcal{I}[.5,.4]$
$= min\ (1,1\text{-}.5\text{+}.4) = min(1,1.5) = .9$

Then R is represented by matrix

$$\begin{pmatrix} 1 & .9 \\ 1 & .4 \\ 1 & .8 \end{pmatrix}$$

Case ii: Z-FUZZY RELATION

Consider the case of Z-fuzzy proposition:
$If\ X\ isz\ (A,B) then\ Y\ isz\ (C,D).$
where B and D are real numbers belonging to [0,1]. We can calculate the z-fuzzy relationship function $R(x,y)$ = $(R_1(x,y), R_2(x,y))$ as follows
$R_1(x,y) = \mathcal{I}(A(x), C(y))$
$R_2(x,y) = \min(B, D)$

Example:

X = $\{x_1, x_2, x_3\}$; Y = $\{y_1, y_2\}$;
A: $\frac{1}{x_1} + \frac{.9}{x_2} + \frac{0}{x_3}$;
B = 0.9;
C: $\frac{1}{y_1} + \frac{.1}{y_2}$;
D = 0.8

Example:

Consider the following $X = \{x_1, x_2, x_3, x_4, x_5\}$, $Y = \{y_1, y_2, y_3, y_4, y_5\}$
Rule base is:

If X isz (A_1, B_1) then Y isz $(C_1, D_1) \longrightarrow$ (1)
If X isz (A_2, B_2) then Y isz $(C_2, D_2) \longrightarrow$ (2)
If X isz (A_3, B_3) then Y isz $(C_3, D_3) \longrightarrow$ (3)

$A_1: \frac{1}{x_1} + \frac{.9}{x_2}$; $B_1 = .9$; $C_1: \frac{.8}{y_4} + \frac{1}{y_5}$; $D_1 = .8$

$A_2: \frac{1}{x_3} + \frac{.8}{x_4}$; $B_2 = .7$; $C_2: \frac{.7}{y_2} + \frac{.7}{y_3}$; $D_2 = .8$

$A_3 : \frac{.9}{x_5}$; $B_3 = .8$; $C_3: \frac{1}{y_1}$; $D_3 = 1$

Therefore,

The z fuzzy relationship function of equations **(**1), (2), (3) is given by the following table

x_i/y_i	y_1	y_2	y_3	y_4	y_5
x_1	(0,.8)	(0,.8)	(0,.8)	(.8,.8)	(1,.8)
x_2	(0,.8)	(0,.8)	(0,.8)	(.9,.8)	(1,.8)
x_3	(0,.7)	(.7,.7)	(.7,.7)	(0,.7)	(0,.7)
x_4	(0,.7)	(.9,.7)	(.9,.7)	(0,.7)	(0,.7)
x_5	(1,.8)	(0,.8)	(0,.8)	(0,.8)	(0,.8)

COMPOSITIONAL RULE OF INFERENCE

Suppose we know R for the fuzzy proposition 'If X is A then Y is B'. Then if we know that X is A'. What can we conclude about Y?

We need a answer of the form

$If\ X\ is\ A'\ then\ Y\ is\ B'$

So how to compute B'given R and A'?

B$'$ (y) = $Sup_{x \in X}$ min [A' (x), $R(x, y)$]

The above is known as 'Compositional Rule of Inference'.

In Z-fuzzy case:

Suppose $R(x, y)$ = $(R_1(x, y), R_2(x, y))$ the z-fuzzy relationship function corresponding to the z-fuzzy if-then rule

'If X isz (A,B) then Y isz (C,D)' is known.

Further we know that X isz (A', B'). Then to find C', D' such that

If X isz (A', B') then Y isz (C', D')

We proceed as follows;

C'(y) $=$ $Sup_{x \in X}\ min$ [A'(x), $R(x, y)$]

D'= $min\ (B',\ Inf_x(R_2(x, y)))$

The above is the 'COMPOSITIONAL RULE FOR Z-FUZZY RELATIONS'.

In finite case we have

If X is a finite set $\{x_1, x_2, \ldots, x_n\}$ and Y is the finite set $\{y_1, \ldots y_m\}$ then

C'(y) $=$ max {$min[A'(x_1)$, R(x_1,y)], $min[A'(x_2)$, R(x_2,y)],…, $min[A'(x_n)$, R(x_n,y)]}

D' $=$ $min(B', R_2(x_1, y), R_2(x_2, y), \ldots, R_2(x_n, y))$

Note: Here the Sup-min, Overall min composition has been used. But other ZFR compositions also can be used.

Example:

Let

$$R = \begin{bmatrix} (.8,.9) & (.2,1) \\ (.6,.8) & (.3,.9) \\ (.9,1) & (.1,.8) \end{bmatrix}$$

$A' = \frac{.9}{x_1} + \frac{1}{x_2} + \frac{0}{x_3}$; $B' = .8$

Then

$C'(y_1) = Sup_{x_i} \min[A'(x_i), R_1(x_i, y_i)]$

$= max[min[A'(x_1), R_1(x_1, y)],\ min[A'(x_2), R_2(x_2, y)],\ min[A'(x_3), R_3(x_3, y)]]$

$= \ max\ \ [min(.9, .8), min(1, .6), min(0, .9)]$

$= \ max\ \ [.8, .6, 0]$

$= .8$

$D' = \ min\ (B', Inf_{\{(x,y)|A'(x) > 0\}} R_2(x, y))$

$\therefore D' = \min\ (.8, Inf(.9, 1, .8, .9)\ )$

$= \ min\ (.8, .8\ ) = .8$

$\therefore$ We get the required fuzzy set with their reliability, that is

C' = $\frac{.8}{y1} + \frac{.3}{y2}$; D' = .8

Conclusion

This method have been successfully advanced via fuzzy relation dominated with z-number by our proposing second component, the reliability of the rule base systems which is imprecise in our routine vague linguistic dialogues in life. This would have been given a huge potential, strong and strength for the relations of objects, persons, etc. For getting the accuracy diagnosis, this compositional rule of inference is involved in many fields especially in medical diagnosis and so on.

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