

Absolute Mean Graceful Labeling of Corona Product of Various Families.

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Abstract

This paper investigates corona product of several graphs are absolute mean graceful graphs. We establish new results concerning absolute mean graceful graphs. The main objective of this work is to study the absolute mean graceful nature of graphs of the form $G \odot K_1$, where G belongs to specific graph families. We present explicit and systematic labeling constructions to show that the corona product of complete bipartite graphs $K_{m,n} \odot K_1$, alternate triangular graphs $AT_n \odot K_1$, alternate quadrilateral snake $AQS_n \odot K_1$, fan graphs $F_n \odot K_1$, jewel graphs with and without prime edge $J_n \odot K_1$ & $J_n^* \odot K_1$, H - graph of path graphs $H_n \odot K_1$, triangular book graphs $B(3, n) \odot K_1$ admits an absolute mean graceful labeling.

Keywords : *Graceful labeling, Absolute mean graceful labeling, Corona product.*

MSC : 05C78

1 Introduction

Graph labeling is a central topic in graph theory with applications in computer science, communication networks, and combinatorial optimization. Among its many variants, graceful labeling has been widely studied for its simplicity and theoretical significance. A recent extension, absolute mean graceful labeling, was introduced by Kaneria and Chudasama [8], who proved that several standard graphs are absolute mean graceful and also investigated such graphs in the context of duplication of graph elements. Kaneria et al. [9] further investigated absolute mean graceful graphs in the context of path unions of graphs. Akbari et al. [5] investigated that jewel and jellyfish-related graphs are absolute mean graceful, while the same authors in [4] examined absolute mean graceful graphs in the context of barycentric subdivision. In this paper, we discuss the absolute mean graceful labeling of corona products of various graph structures. For all undefined terminologies, readers may refer to [2] and [3].

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2 Preliminaries

We consider only finite, simple graphs. Let $G = (V, E)$ be a graph with vertex set $V(G)$ and edge set $E(G)$.

Definition 2.1. A function f is said to be absolute mean graceful labeling of a graph G , if $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$ is injective and edge labeling function $f^*: E(G) \rightarrow \{1, 2, \dots, q\}$ defined as $f^*(e) = \left\lfloor \frac{|f(u)-f(v)|}{2} \right\rfloor$ is bijective, $\forall e = uv \in E(G)$. A graph which admits an absolute mean graceful labeling is called absolute mean graceful graph.

Definition 2.2. In graph theory, the corona product of graphs G and H , denoted $G \odot H$, can be obtained by taking one copy of G , called the center graph, and a number of copies of H equal to the order of G . Then, each copy of H is assigned a vertex in G , and that one vertex is attached to each vertex in its corresponding H copy by an edge.

3 Main Results

Theorem 3.1. The graph $K_{m,n} \odot K_1$, corona product of complete bipartite $K_{m,n}$ and complete graph K_1 , is absolute mean graceful graph.

Proof. Let $G = K_{m,n} \odot K_1$ is a corona product of any complete bipartite graph $K_{m,n}$ with complete graph K_1 .

Let $V(K_{m,n}) = \{u_i, v_j / 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(K_{m,n}) = \{u_i v_j / 1 \leq i \leq m, 1 \leq j \leq n\}$.

Therefore,

$V(G) = \{u_i, u'_i, v_j, v'_j / 1 \leq i \leq m, 1 \leq j \leq n\}$ and

$E(G) = \{u_i v_j / 1 \leq i \leq m, 1 \leq j \leq n\}$
 $\cup \{u_i u'_i / 1 \leq i \leq m\} \cup \{v_j v'_j / 1 \leq j \leq n\}$.
 Clearly, $|V(G)| = 2(m+n)$ and $|E(G)| = q = mn + m + n$.

The vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$ defined as follows.

$$\begin{aligned}
 f(u_i) &= q - 1 - 2ni + 2n, 1 \leq i \leq m \\
 f(v_j) &= 2j - 2 - q, 1 \leq j \leq n \\
 f(u'_i) &= q - 2m - 2nj + 2j - 2, 1 \leq i \leq m \\
 f(v'_j) &= 4i - q - 3, 1 \leq i \leq n
 \end{aligned}$$

The labeling function f defined as above is one- one, as there is no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective.

Thus, f is an absolute mean graceful labeling for given graph. Therefore, the graph $K_{m,n} \odot K_1$ is an absolute mean graceful graph.

Example 3.2. Absolute mean graceful labeling for $K_{3,4} \odot K_1$ is shown in following Figure 1.

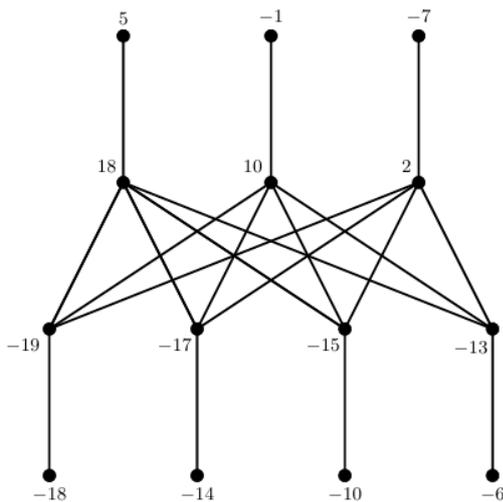


Figure 1 Absolute mean graceful labeling of $K_{3,4} \odot K_1$.

Theorem 3.3. The graph $AT_n \odot K_1$ corona product of alternate triangular graph and complete graph K_1 is absolute mean graceful graph.

Proof. Let $G = AT_n \odot K_1$ is a corona product of any alternate triangular graph AT_n with complete graph K_1 .

Let $V(AT_n) = \{u_i, v_i, x_i / 1 \leq i \leq n\}$ and $E(AT_n) = \{u_i v_i, v_i x_i, x_i u_i / 1 \leq i \leq n\} \cup \{x_i u_{i+1} / 1 \leq i \leq n-1\}$.

Therefore, $V(G) = \{u_i, v_i, x_i, u'_i, v'_i, x'_i / 1 \leq i \leq n\}$ and $E(G) = \{u_i x_i, v_i u_i, v_i x_i, u_i u'_i, v_i v'_i, x_i x'_i / 1 \leq i \leq n\} \cup \{x_i u_{i+1} / 1 \leq i \leq n-1\}$.

The vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$ defined as follows.

For $1 \leq i \leq n$,

$$\begin{aligned}
 f(u_i) &= q - 2i + 2, f(v_i) = n + 4i, \\
 f(x_i) &= 1 - q + 2i - 2, f(u'_i) = n + 4i - 5, \\
 f(v'_i) &= 2i - 5n, f(x'_i) = 3 - n - 4i
 \end{aligned}$$

The labeling function f defined as above is one- one, as there is no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective.

Thus, f is an absolute mean graceful labeling for given graph. Therefore, the graph $AT_n \odot K_1$ is an absolute mean graceful graph.

Example 3.4. Absolute mean graceful labeling for $AT_4 \odot K_1$ is shown in following Figure 2.

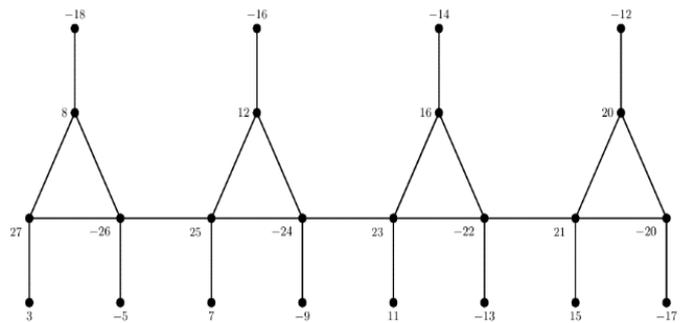


Figure 2 Absolute mean graceful labeling of $AT_4 \odot K_1$.

Theorem 3.5. The graph corona product of alternate quadrilateral snake graph $AQS_n \odot K_1$ and complete graph K_1 is absolute mean graceful graph.

Proof. Let $G = AQSn \odot K_1$ is a corona product of any alternate quadrilateral snake graph AQS_n with complete graph K_1 .

Let $V(AQS_n) = \{u_i, v_i, x_i, w_i / 1 \leq i \leq n\}$ and $E(AQS_n) = \{u_i v_i, v_i w_i, w_i x_i, x_i u_i / 1 \leq i \leq n\} \cup \{x_i u_{i+1} / 1 \leq i \leq n-1\}$.

Therefore, $V(G) = \{u_i, v_i, w_i, x_i, u'_i, v'_i, w'_i, x'_i / 1 \leq i \leq n\}$ and

$$\begin{aligned}
 E(G) = & \{u_i x_i, v_i w_i, v_i u_i, w_i x_i, u_i u'_i, v_i v'_i, w_i w'_i, x_i x'_i / 1 \leq i \leq n\} \\
 & \cup \{x_i u_{i+1} / 1 \leq i \leq n-1\}.
 \end{aligned}$$

To obtain vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$, we take following cases.

Case-1 : $n \equiv 0 \pmod{2}$

$$\begin{aligned}
 f(u_i) &= q - 2i + 2, 1 \leq i \leq n \\
 f(x_i) &= 1 - q + 2i - 2, 1 \leq i \leq n \\
 f(v_i) &= 4n - q + 2i - 4, 1 \leq i \leq n \\
 f(w_i) &= q - 4n - 2i + 2, 1 \leq i \leq n \\
 f(u'_i) &= q + 2i - 1 - 4n, 1 \leq i \leq n
 \end{aligned}$$

$$\begin{aligned}
f(x'_i) &= 1 - q - 2i + 4n, 1 \leq i \leq n \\
f(v'_i) &= q - 4n - 6i + 3, 1 \leq i \leq \frac{n}{2} \\
f(v'_i) &= q - 6n - 2i + 1, (\frac{n}{2} + 1) \leq i \leq n \\
f(w'_i) &= 4n - q + 6i - 1, 1 \leq i \leq \frac{n}{2} \\
f(w'_{\frac{n}{2}+1}) &= 7n - q + 1, \\
f(w'_i) &= 6n - q + 2i - 1, (\frac{n}{2} + 2) \leq i \leq n
\end{aligned}$$

Case-2 : $n \equiv 1 \pmod{2}$

$$\begin{aligned}
f(u_i) &= q - 2i + 2, 1 \leq i \leq n \\
f(x_i) &= 1 - q + 2i - 2, 1 \leq i \leq n \\
f(v_i) &= 4n - q + 2i - 4, 1 \leq i \leq n \\
f(w_i) &= q - 4n - 2i + 2, 1 \leq i \leq n \\
f(u'_i) &= q + 2i - 1 - 4n, 1 \leq i \leq n \\
f(x'_i) &= 1 - q - 2i + 4n, 1 \leq i \leq n \\
f(v'_i) &= q - 4n - 6i + 3, 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \\
f(v'_{\lfloor \frac{n}{2} \rfloor + 1}) &= q - 4n - 6\lfloor \frac{n}{2} \rfloor + 1, \\
f(v'_i) &= q - 4n - 4\lfloor \frac{n}{2} \rfloor + 3 - 2i, \lfloor \frac{n}{2} \rfloor + 2 \leq i \leq n \\
f(w'_i) &= 4n - q + 6i - 1, 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \\
f(w'_i) &= 4n - q + 4\lfloor \frac{n}{2} \rfloor + 2i + 1, \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n
\end{aligned}$$

The labeling function f defined as above is one- one, as there is no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective.

Thus, f is an absolute mean graceful labeling for given graph. Therefore, the graph $AQS_n \odot K_1$ is an absolute mean graceful graph.

Example 3.6. Absolute mean graceful labeling for $AQS_3 \odot K_1$ is shown in following Figure 3.

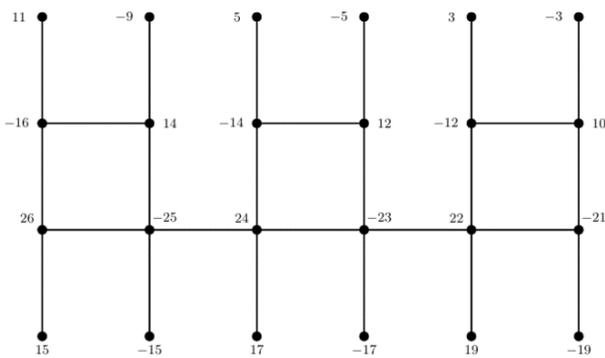


Figure 3 Absolute mean graceful labeling of $AQS_3 \odot K_1$.

Theorem 3.7. The graph $F_n \odot K_1$ corona product of fan graph F_n and complete graph K_1 is absolute mean graceful graph.

Proof. Let $G = F_n \odot K_1$ is a corona product of any fan graph F_n with complete graph K_1 .

Let $V(F_n) = \{u_1, u_2, \dots, u_n, x\}$ and $E(F_n) = \{u_i u_{i+1} / 1 \leq i \leq n - 1\} \cup \{x u_i / 1 \leq i \leq n\}$.

Therefore, $V(G) = \{u_i, u'_i / 1 \leq i \leq n\} \cup \{x, x'\}$ and $E(G) = \{u_{i-1} u_i, u_i u'_i / 1 \leq i \leq n\} \cup \{x u_i / 1 \leq i \leq n\} \cup \{x x'\}$.

To obtain vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$, we take following cases.

Case-1 : $n \equiv 0 \pmod{2}$

$$\begin{aligned}
f(u_i) &= (-1)^{i+1}(q - i + 1), 1 \leq i \leq n \\
f(u'_1) &= -1, \\
f(u'_i) &= (-1)^{i+1}(i - 2), 2 \leq i \leq \frac{n+2}{2} \\
f(u'_i) &= (-1)^{i+1}(n + i - 1), \frac{n+2}{2} \leq i \leq n \\
f(x) &= -(n + 2), \\
f(x') &= n - 2.
\end{aligned}$$

Case-2 : $n \equiv 1 \pmod{2}$

$$\begin{aligned}
f(u_i) &= (-1)^{i+1}(q - i + 1), 1 \leq i \leq n \\
f(u'_1) &= -1, \\
f(u'_i) &= (-1)^{i+1}(i - 2), 2 \leq i \leq \frac{n+1}{2} \\
f(u'_i) &= (-1)^{i+1}(n + i - 1), \frac{n+3}{2} \leq i \leq n \\
f(x) &= -(n + 2), \\
f(x') &= n - 2.
\end{aligned}$$

The labeling function f defined as above is one- one, as there is no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective.

Thus, f is an absolute mean graceful labeling for given graph. Therefore, the graph $F_n \odot K_1$ is an absolute mean graceful graph.

Example 3.8. Absolute mean graceful labeling for $F_7 \odot K_1$ is shown in following Figure 4.

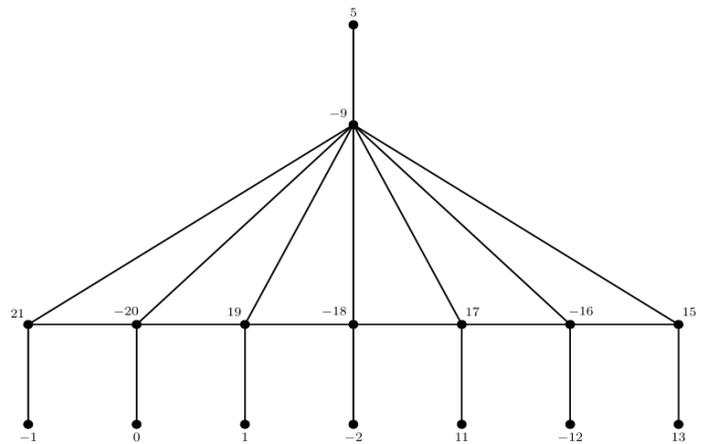


Figure 4 Absolute mean graceful labeling of $F_7 \odot K_1$.

Theorem 3.9. The graph $J_n \odot K_1$ corona product of jewel graph J_n and complete graph K_1 is absolute mean graceful graph.

Proof. Let $G = J_n \odot K_1$ is a corona product of any jewel graph J_n with complete graph K_1 .

Let $V(J_n) = \{u, v, x, y\} \cup \{v_i / 1 \leq i \leq n\}$ and $E(J_n) = \{ux, xv, uy, yv, xy\} \cup \{uv_i / 1 \leq i \leq n\} \cup \{vv_i / 1 \leq i \leq n\}$.

Therefore, $V(G) = \{x, x', y, y', u, u', v, v'\} \cup \{v_i, v'_i / \forall 1 \leq i \leq n\}$ and $E(G) = \{xx', uu', vv', yy', xy\} \cup \{v_i v'_i / 1 \leq i \leq n\} \cup \{uv_i / 1 \leq i \leq n\} \cup \{vv_i / 1 \leq i \leq n\}$.

The vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$ defined as follows.

$$\begin{aligned} f(x) &= -q, & f(x') &= q, \\ f(u) &= q - 2, & f(u') &= 10 - q, \\ f(v) &= q - 4, & f(v') &= 6 - q, \\ f(y) &= 4 - q, & f(y') &= 2 - q, \\ f(v_i) &= 8 - q + 4i, & 1 \leq i \leq n, \\ f(v'_i) &= 4 - n + 2i, & 1 \leq i \leq n. \end{aligned}$$

The labeling function f defined as above is one-one, as there is no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective.

Thus, f is an absolute mean graceful labeling for given graph. Therefore, the graph $J_n \odot K_1$ is an absolute mean graceful graph.

Example 3.10. Absolute mean graceful labeling for $J_3 \odot K_1$ is shown in following Figure 5.

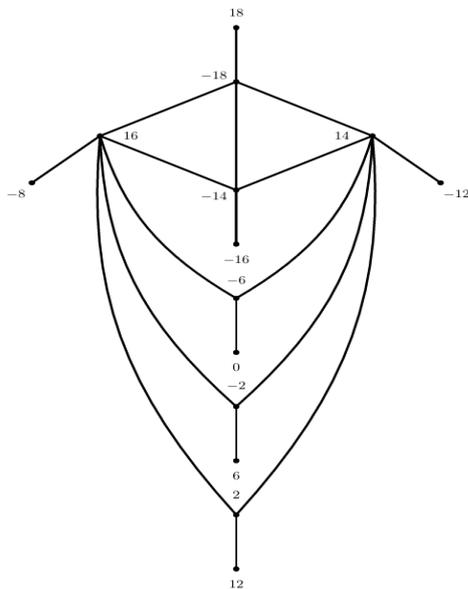


Figure 5 Absolute mean graceful labeling of $J_3 \odot K_1$.

Corollary 3.11. The graph $J_n^* \odot K_1$ corona product of jewel graph without prime edge J_n^* and complete graph K_1 is absolute mean graceful graph.

Proof. Let $G = J_n^* \odot K_1$ is a corona product of any jewel graph without prime edge with complete graph K_1 .

Let $V(J_n^*) = \{u, v, x, y\} \cup \{v_i / 1 \leq i \leq n\}$ and $E(J_n^*) = \{ux, xv, uy, yv\} \cup \{uv_i / 1 \leq i \leq n\} \cup \{vv_i / 1 \leq i \leq n\}$.

Therefore, $V(G) = \{x, x', y, y', u, u', v, v'\} \cup \{v_i, v'_i / \forall 1 \leq i \leq n\}$ and $E(G) = \{xx', uu', vv', yy'\} \cup \{v_i v'_i / 1 \leq i \leq n\} \cup \{uv_i / 1 \leq i \leq n\} \cup \{vv_i / 1 \leq i \leq n\}$.

The vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$ defined as follows.

$$\begin{aligned} f(x) &= -q, & f(x') &= q, \\ f(u) &= q - 2, & f(u') &= 10 - q, \\ f(v) &= q - 4, & f(v') &= 6 - q, \\ f(y) &= 4 - q, & f(y') &= 2 - q, \\ f(v_i) &= 8 - q + 4i, & 1 \leq i \leq n, \\ f(v'_i) &= 3 - n + 2i, & 1 \leq i \leq n. \end{aligned}$$

The labeling function f defined as above is one-one, as there is no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective.

Thus, f is an absolute mean graceful labeling for given graph. Therefore, the graph $J_n^* \odot K_1$ is an absolute mean graceful graph.

Example 3.12. Absolute mean graceful labeling for $J_3^* \odot K_1$ is shown in following Figure 6.

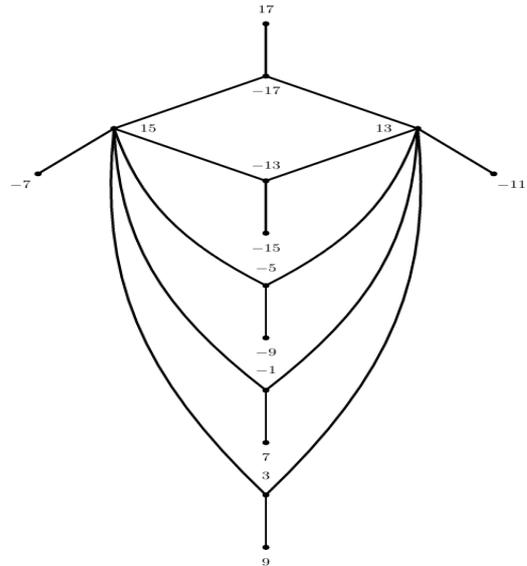


Figure 6 Absolute mean graceful labeling of $J_3^* \odot K_1$.

Theorem 3.13. The graph $H_n \odot K_1$ corona product of H -graph of path graph P_n and complete graph K_1 is absolute mean graceful graph.

Proof. Let $G = H_n \odot K_1$ is a corona product of any H -graph of path P_n with complete graph K_1 .

Let $V(H_n) = \{u_i, v_i / 1 \leq i \leq n\}$ and

Now if $n \equiv 0(\text{mod}2)$ then

$$E(H_n) = \{u_{i-1} u_i, v_{i-1} v_i / 1 \leq i \leq n\} \cup \{u_{\frac{n+1}{2}} v_{\frac{n}{2}}\}$$

and if $n \equiv 1(\text{mod}2)$ then

$$E(H_n) = \{u_{i-1} u_i, v_{i-1} v_i / 1 \leq i \leq n\} \cup \{u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\}.$$

Therefore, $V(G) = \{u_i, v_i, u'_i, v'_i / 1 \leq i \leq n\}$ and

if $n \equiv 0(\text{mod}2)$ then

$$E(G) = \{u_i u'_i, v_i v'_i, u_{i-1} u_i, v_{i-1} v_i, / 1 \leq i \leq n\} \cup \{u_{\frac{n+1}{2}} v_{\frac{n}{2}}\}$$

and if $n \equiv 1(\text{mod}2)$ then

$$E(G) = \{u_i u'_i, v_i v'_i, u_{i-1} u_i, v_{i-1} v_i, / 1 \leq i \leq n\} \cup \{u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\}.$$

The vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$ defined as follows.

$$\begin{aligned} f(u_i) &= (-1)^{i+1}(q-i+1), \\ f(v_i) &= (-1)^{n+i-1}(q-n-i+1), \\ f(u'_i) &= (-1)^{i+1}(i-2), \\ f(v'_i) &= (-1)^{n+i-1}(n+i-2) \end{aligned}$$

The labeling function f defined as above is one-one, as there is no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective.

Thus, f is an absolute mean graceful labeling for given graph. Therefore, the graph $H_n \odot K_1$ is an absolute mean graceful graph.

Example 3.14. Absolute mean graceful labeling for $H_5 \odot K_1$ is shown in following Figure 7.

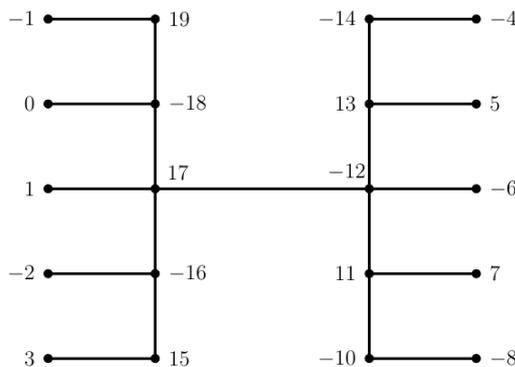


Figure 7 Absolute mean graceful labeling of $H_5 \odot K_1$.

Theorem 3.15. The graph $B(3, n) \odot K_1$ corona product of triangular book graph $B(3, n)$ and complete graph K_1 is absolute mean graceful graph.

Proof. Let $G = B(3, n) \odot K_1$ is a corona product of any triangular book graph with n pages $B(3, n)$ and complete graph K_1 .

Let $V(B(3, n)) = \{x, y\} \cup \{v_i / 1 \leq i \leq n\}$ and

$$E(B(3, n)) = \{xv_i, yv_i / 1 \leq i \leq n\} \cup \{xy\}$$

then $V(G) = \{x, y, x', y'\} \cup \{v_i, v'_i / 1 \leq i \leq n\}$

and

$$E(G) = \{xv_i, yv_i, v_i v'_i / 1 \leq i \leq n\} \cup \{xx', yy', xy\}$$

The vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$ defined as follows.

$$\begin{aligned} f(x) &= q, f(y) = q - 2, \\ f(x') &= q - 5, f(y') = q - 6, \\ f(v_i) &= 4i - q - 3, 1 \leq i \leq n \\ f(v'_i) &= 2 - q + 6i, 1 \leq i \leq n \end{aligned}$$

The labeling function f defined as above is one-one, as there is no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective.

Thus, f is an absolute mean graceful labeling for given graph. Therefore, the graph $B(3, n) \odot K_1$ is an absolute mean graceful graph.

Example 3.16. Absolute mean graceful labeling for $B(3, 5) \odot K_1$ is shown in following Figure 8.

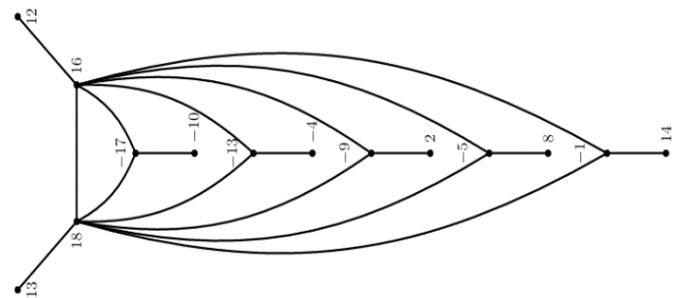


Figure 8 Absolute mean graceful labeling of $B(3, 5) \odot K_1$.

Corollary 3.17. The triangular book graph with bookmark $TB_n(y, x)(x, x')$ is absolute mean graceful graph.

Proof. Let $G = TB_n(y, x)(x, x')$ be any triangular book graph with bookmark.

Let $V(G) = \{x, y, x'\} \cup \{v_i / 1 \leq i \leq n\}$ and

$$E(G) = \{xv_i, yv_i / 1 \leq i \leq n\} \cup \{xx', xy\}.$$

The vertex labeling function $f: V(G) \rightarrow \{0, \pm 1, \pm 2, \dots, \pm q\}$ defined as follows.

$$f(x) = q, f(y) = q - 2, f(x') = q - 5, \\ f(v_i) = 4i - q - 3, 1 \leq i \leq n$$

The labeling function f defined as above is one-one, as there is no repeated vertex labels. It is easy to check that the edge labeling function f^* is bijective.

Thus, f is an absolute mean graceful labeling for given graph. Therefore, the graph $TB_n(y, x)(x, x')$ is an absolute mean graceful graph.

Example 3.18. Absolute mean graceful labeling for $TB_5(y, x)(x, x')$ is shown in following Figure 9.

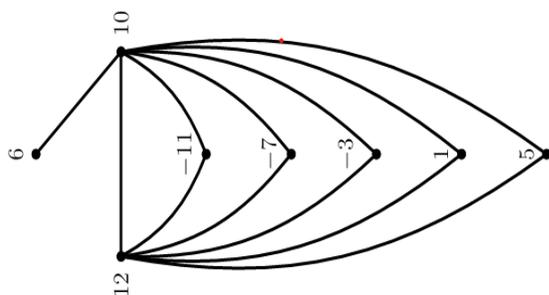


Figure 9 Absolute mean graceful labeling of $TB_5(y, x)(x, x')$.

4 Conclusion

In this work, we show that the corona product of different graph structures can be given an absolute mean graceful labeling. In particular, we look at the corona product of some graphs with the complete graph K_1 and find that these are absolute mean graphs. This gives a new challenge for researchers to check if the same result holds for all K_n and for other types of graph structures.

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