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The Forcing Steiner Hop Domination Number of a Graph

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Abstract: Let W is a γ_{hs}-set of G. A subset T of W is called a forcing subset of W if W is the unique γ_{hs}-set containing T. The minimum cardinality of T is the forcing Steiner hop domination number of W and is denoted by $f_{\gamma_{hs}}(G)$. The forcing Steiner hop domination number of G is $f_{\gamma_{hs}}(G) = min\{f_{\gamma_{hs}}(G)\}$, where the minimum is taken over all γ_{hs}-sets of G. It is shown that for every pair of integers a and b with $0 \le a \le b$ and $b \ge a+2$, there exists a connected graph G such that $f_{\gamma_{hs}}(G) = a$ and $\gamma_{hs}(G) = b$.

Keywords and Phrases: *Steiner number, hop domination number, Steiner hop domination number, forcing Steiner hop domination number.*

AMS Subject Classification: 05C69, 05C12.

1. Introduction

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by n and m respectively. For basic definitions and terminologies, we refer to [3]. Two vertices u and v are said to be *adjacent* if uv is an edge of G. For any vertex v in a graph G, the number of vertices adjacent to v is called the *degree* of v in G, denoted by $deg_G(v)$. A vertex of degree 1 is called a *pendent vertex* or an *end vertex* of G. A vertex v is called a *universal vertex* if $deg_G(v) = n-1$. For any set S of vertices of G, the *induced subgraph* $\langle S\rangle$ is the maximal subgraph of G with vertex set S and $E(\langle S\rangle) = \{uv \in E(G): u, v \in S\}$. A vertex $v \in G$ is said to be *extreme* if the subgraph induced by its neighbourhood is complete. The distance $d(u, v)$ between u and v is the length of a shortest $u-v$ paths in G and a $u-v$ path of length $d(u, v)$ is called a *u-v geodesic*.

Many authors have studied this concept with respect to several parameters like domination, Steiner, etc. In this paper, we study the concept of the forcing Steiner hop domination number of a graph for some certain classes of graphs and their general properties. This concept helps in designing communication networks, social networking and problems with facility location.

For a nonempty set W of vertices in a connected graph G, the *Steiner distance* $d(W)$ of W is the minimum size of a connected subgraph of G containing W. Necessarily, each such subgraph is a tree and is called a *Steiner tree* with respect to W or a *Steiner W-tree*. For a given set $W \subseteq V(G)$, there may be more than one Steiner W-tree in G. Let $S(W)$ denote the set of vertices that lies in Steiner W- trees. Let G be a connected graph with at least 2 vertices. A set $W \subseteq V(G)$ is called a Steiner set of G if $S(W) =$

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$V(G)$. The Steiner number $s(G)$ is the minimum cardinality of its *Steiner sets* and any Steiner set of cardinality $s(G)$ is a minimum Steiner set of G. The Steiner number of a graph was studied in [4-8, 11-13].

A set $S \subseteq W$ of a graph G is a *hop dominating set* of G if for every $v \in V - S$, there exists $u \in S$ such that $d(u, v) = 2$. The minimum cardinality of the hop dominating set is called *the hop domination number* and is denoted by$\gamma_h(G)$, this concept was studied in [1,2,10].

The forcing Steiner number of a graph was introduced and studied in [4] and further studied by many authors in several parameters arising in graphs [9,11,12].

The following Theorem is used in the sequel.

Theorem 1.1[4] Each extreme (end) vertex of a connected graph G belongs to every Steiner set of G.

2. The forcing Steiner hop domination number of a graph

Definition 2.1. A subset W of vertices in a graph G is called the *Steiner hop dominating set* of G if W is both a Steiner set and a hop dominating set of G. The minimum cardinality of a Steiner hop dominating set of G is its *Steiner hop domination number* $\gamma_{hs}(G)$. A Steiner hop domination set of size $\gamma_{hs}(G)$ is said to be a γ_{hs}-set of G.

Definition 2.2. Let W is a γ_{hs}-set of G. A subset T of W is called a forcing subset of W if W is the unique γ_{hs}-set containing T. The minimum cardinality of T is the forcing Steiner hop domination number of W and is denoted by $f_{\gamma_{hs}}(G)$. The forcing Steiner hop domination number of G is $f_{\gamma_{hs}}(G) = min\{f_{\gamma_{hs}}(G)\}$, where the minimum is taken over all γ_{hs}-sets of G.

Example 2.3. For the graph G given in Figure 1, $W_1 = \{v_1, v_5, v_8, v_{12}\}$ and $W_2 = \{v_3, v_7, v_{10}, v_{12}\}$ are the only two minimum γ_{hs}-sets of G such that $f_{\gamma_{hs}}(W_1) = f_{\gamma_{hs}}(W_2) = 1$ so that $f_{\gamma_{hs}}(G) = 1$.

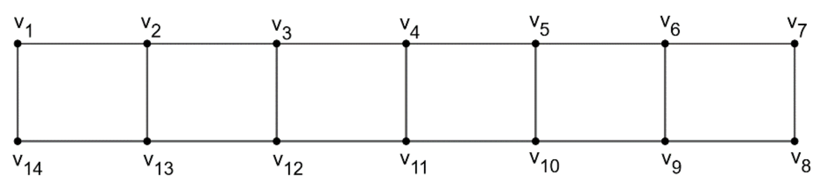


Figure 1. The graph G with $f_{\gamma_{hs}}(G) = 1$.

Theorem 2.4. For any connected graph G, $0 \leq f_{\gamma_{hs}}(G) \leq \gamma_{hs}(G)$.

The following Theorem characterize the graph G for which the bounds in Theorem 2.4 is obtained and also graphs for which $f_{\gamma_{hs}}(G) = 1$. The proofs of the following theorems are straightforward. Thus, we omit the proofs.

Theorem 2.5. Let G be a connected graph. Then

(a) $f_{\gamma_{hs}}(G) = 0$ if and only if G has a unique γ_{hs}-set.

(b) $f_{\gamma_{hs}}(G) = 1$ if and only if G has at least two γ_{hs}-sets one of which is a unique γ_{hs}-set containing one of its elements.

(c) $f_{\gamma_{hs}}(G) = \gamma_{hs}(G)$ if and only if γ_{hs}-set of G is the unique γ_{hs}-set containing any of its proper subsets.

Definition 2.6. A vertex v of a graph G is said to be a Steiner hop dominating vertex of G if V belongs to every γ_{hs}-set of G.

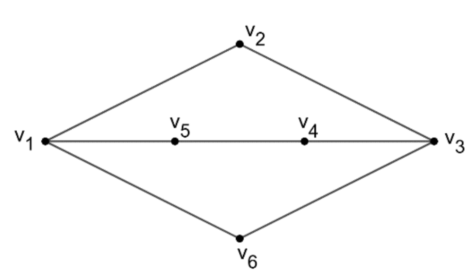


Figure 2

Remark 2.7. Each extreme vertex of G is a Steiner hop dominating vertex of G. In fact, there are Steiner hop dominating vertices that are not extreme vertices of G.

Example 2.8. For the graph G given in Figure 2, $S = \{v_1, v_3\}$ are the Steiner hop dominating vertices of G which are not extreme vertices of G.

Theorem 2.9. Let G be a connected graph and S be the set of all γ_{hs}-set of G. Then $f_{\gamma_{hs}}(G) \leq \gamma_{hs}(G) - |S|$.

Remark 2.10. The bound in Theorem 2.9 is sharp. For the graph G given in Figure 2, $W_1 = \{v_1, v_3, v_4\}$ and $W_2 = \{v_1, v_3, v_5\}$are the only two γ_{hs}-sets of G such that $f_{\gamma_{hs}}(G) = 1$. In addition, $S\{v_1, v_3\}$ is the set of all Steiner hop dominating vertices of G. Now, $\gamma_{hs}(G) - |S|.= 3 - 2 = 1$. Thus, $f_{\gamma_{hs}}(G) = \gamma_{hs}(G) - |S|$. Also, the bound in Theorem 2.9 is strict. For the graph $G = P_9$ with vertex set $V(G) = \{v_1, v_2, v_3, \ldots, v_8, v_9\}$, $W_1 = \{v_1, v_4, v_7, v_8, v_9\}$ and $W_2 = \{v_1, v_2, v_3, v_4, v_9\}$ are the only two γ_{hs}-sets of G such that $\gamma_{hs}(G) = 5$ and $f_{\gamma_{hs}}(G) = 2$. In addition, $S = \{v_1, v_9\}$ is the set of all Steiner hop dominating vertices of G. Now $\gamma_{hs}(G) - |S| = 5 - 2 = 3$. Thus, $f_{\gamma_{hs}}(G) \leq \gamma_{hs}(G) - |S|$.

Corollary 2.11. Let G be a connected graph with k extreme vertices. Then $f_{\gamma_{hs}}(G) \leq \gamma_{hs}(G) - k$.

Proof: This follows from Theorems 1.1 and 2.9.

Theorem 2.12. Let G be either the complete graph K_n $(n \geq 2)$ or the star graph $K_{1,n-1}$ $(n \geq 2)$ or the complete bipartite graph $K_{r,s}$ $(2 \leq r \leq s)$ or the fan graph $K_1 + P_{n-1}$ $(n \geq 3)$ or the wheel graph $K_1 + C_{n-1}$ $(n \geq 4)$. Then $f_{\gamma_{hs}}(G) = 0$.

Proof: Since $W = V(G)$ is the unique γ_{hs}-set of G, the result follows from Theorem 2.5(a).

Theorem 2.13. For the path P_n $(n \geq 2)$,

$$f_{\gamma_{hs}}(G) = \begin{cases} 0 \; if \; n = 2 \; or \; 3 \; or \; 4 \; or \; 6s+1 \; or \; 6s+4 \\ 1 \; if \; n = 5 \\ 2 \; if \; n = 6s \; or \; 6s+2 \; or \; n = 6s+3 \; or \; 6s+5, where \; s \geq 1 \end{cases}$$

Proof: Let $G = P_n$ be $v_1, v_2, v_3, \ldots, v_n$ be the set of all vertices.

Case 1(a): Let $n = 2$. The result follows from Theorem 2.12.

Case 1(b): Let $n = 3$. Then $W = \{v_1, v_2, v_3\}$ is the unique γ_{hs}-set of G so that $f_{\gamma_{hs}}(G) = 0$.

Case 1(c): Let $n = 4$. Then $W_1 = \{v_1, v_4\}$ is the unique γ_{hs}-set of G so that $f_{\gamma_{hs}}(G) = 0$.

Case 1(d): Let $n = 6s + 1$ $(s \geq 1)$. Then $W_2 = \{v_1, v_4, v_7, \ldots, v_{6s+1}\}$ is the unique γ_{hs}-set of G so that $f_{\gamma_{hs}}(G) = 0$.

Case 1(e): Let $n = 6s + 4$ $(s \geq 1)$. Then $W_3 = \{v_1, v_4, v_7, \ldots, v_{6s+4}\}$ is the unique γ_{hs}-set of G so that $f_{\gamma_{hs}}(G) = 0$.

Case 2: Let $n = 5$. Then $W_4 = \{v_1, v_4, v_5\}$ and $W_5 = \{v_1, v_2, v_5\}$ are the only two γ_{hs}-sets of G so that $f_{\gamma_{hs}}(W_4) = f_{\gamma_{hs}}(W_5) = 1$. Hence, it follows that $f_{\gamma_{hs}}(G) = 1$.

Case 3: $n = 6s$ or $6s + 2$ or $6s + 3$ or $6s + 5$ where $(s \geq 1)$.

Case 3(a): Let $n = 6s$ $(s \geq 1)$. Then $\gamma_{hs}(G) = 2s + 2$. In this case G has more than one γ_{hs}-sets. Let W_6 be a γ_{hs}-set of G. It is easily verified that there is no singleton subset of W_6 and so $f_{\gamma_{hs}}(G) \geq 2$. Let $W_6 = \{v_1, v_4, v_7, \ldots, v_{6s-2}\} \cup \{v_{6s-1}, v_{6s}\}$. Then W_6 is the unique γ_{hs}-set containing $\{v_{6s-2}, v_{6s-1}\}$ so that $f_{\gamma_{hs}}(G) = 2$. This is true for all γ_{hs} -sets of G. Therefore, $f_{\gamma_{hs}}(G) = 2$.

Case 3(b): Let $n = 6s + 2$ $(s \geq 1)$. Then $\gamma_{hs}(G) = 2s + 2$. In this case G has more than one γ_{hs}-sets. Let W_7 be a γ_{hs}-set of G. It is easily verified that there is no singleton subset of W_7 and so $f_{\gamma_{hs}}(G) \geq 2$. Let $W_7 = \{v_1\} \cup \{v_2, v_5, v_8, \ldots, v_{6s+2}\}$ Then W_7 is the unique γ_{hs}-set containing $\{v_2, v_5\}$ so that $f_{\gamma_{hs}}(G) = 2$. This is true for all γ_{hs}-sets of G. Therefore, $f_{\gamma_{hs}}(G) = 2$.

Case 3(c): Let $n = 6s + 3$ $(s \geq 1)$. Then $\gamma_{hs}(G) = 2s + 3$. In this case G has more than one γ_{hs}-sets. Let W_8 be a γ_{hs}-set of G. It is easily verified that there is no singleton subset of W_8 and so $f_{\gamma_{hs}}(G) \geq 2$. Let $W_8 = \{v_1, v_2\} \cup \{v_3, v_6, v_9, \ldots, v_{6s+3}\}$. Then W_8 is the unique γ_{hs}-set containing $\{v_3, v_6\}$ so that $f_{\gamma_{hs}}(G) = 2$. This is true for all γ_{hs} -sets of G. Therefore, $f_{\gamma_{hs}}(G) = 2$.

Case 3(d): Let $n = 6s + 5(s \geq 1)$. Then $\gamma_{hs}(G) = 2s + 3$. In this case G has more than one γ_{hs}-sets. Let W_9 be a γ_{hs}-set of G. It is easily verified that there is no singleton subset of W_9 and so $f_{\gamma_{hs}}(G) \geq 2$. Let $W_9 = \{v_1\} \cup \{v_2, v_5, v_8, \ldots, v_{6s+5}\}$. Then W_9 is the unique γ_{hs}-set containing $\{v_2, v_5\}$ so that $f_{\gamma_{hs}}(G) = 2$.

This is true for all γ_{hs}-sets of G. Therefore, $f_{\gamma_{hs}}(G) = 2$.

Theorem 2.14. For the cycle C_n $(n \geq 3)$,

$$f_{\gamma_{hs}}(G) = \begin{cases} 0 \; if \; n = 3 \; or \; n = 4 \\ 1 \; if \; n = 6s \; or \; 6s+3 \\ 2 \; if \; n = 6s-1 \; or \; 6s+1 \\ 3 \; if \; n = 6s+2 \; or \; n = 6s+4, where \; s \geq 1 \end{cases}$$

Proof: Let $G = C_n$ be $v_1, v_2, v_3, \dots, v_n, v_1$.

Case 1: $n = 3$ *or* $n = 4$. The result follows from Theorem 2.12.

Case 2(a): Let $n = 6s$ $(s \geq 1)$. Then $\gamma_{hs}(G) = 2s$. In this case G has more than one γ_{hs}-sets. The γ_{hs}-sets of G are $W_1 = \{v_1, v_4, \dots, v_{6s-2}\}$, $W_2 = \{v_2, v_5, \dots, v_{6s-1}\}$ and $W_3 = \{v_3, v_6, \dots, v_{6s}\}$. Now $f_{\gamma_{hs}}(W_i) = 1$ for $i = 1$ to 3 so that $f_{\gamma_{hs}}(G) = 1$.

Case 2(b): Let $n = 6s + 3$ $(s \geq 1)$. Then $\gamma_{hs}(G) = 2s + 1$. In this case G has more than one γ_{hs}-sets. The γ_{hs}-sets of G are $W_4 = \{v_1, v_4, \dots, v_{6s+1}\}$, $W_5 = \{v_2, v_5, \dots, v_{6s+2}\}$ and $W_6 = \{v_3, v_6, \dots, v_{6s+3}\}$. Now $f_{\gamma_{hs}}(W_i) = 1$ for $i = 4$ to 6 so that $f_{\gamma_{hs}}(G) = 1$.

Case 3(a): Let $n = 6s - 1$ $(s \geq 1)$. Then $\gamma_{hs}(G) = 2s + 1$. In this case G has more than one γ_{hs}-sets. Let W_7 be a γ_{hs}-set of G. It is easily verified that no singleton subset of W_7 and so $f_{\gamma_{hs}}(G) \geq 2$. Let $W_7 = \{v_1, v_2\} \cup \{v_6, v_{12}, \dots, v_{6s}\} \cup \{v_7, v_{13}, \dots, v_{6s+1}\}$. Then W_7 is the unique γ_{hs}-set containing $\{v_2, v_6\}$ so that $f_{\gamma_{hs}}(G) = 2$. This is true for all γ_{hs}-sets of G. Therefore, $f_{\gamma_{hs}}(G) = 2$.

Case 3(b): Let $n = 6s + 1$ $(s \geq 1)$. Then $\gamma_{hs}(G) = 2s + 1$. In this case G has more than one γ_{hs}-sets. Let W_8 be a γ_{hs}-set of G. It is easily verified that there is no singleton subset of W_8 and so $f_{\gamma_{hs}}(G) \geq 2$. Let $W_8 = \{v_1, v_4, v_7, \dots, v_{6s+1}\}$. Then W_8 is the unique γ_{hs}-set containing $\{v_1, v_{6s+1}\}$ so that $f_{\gamma_{hs}}(G) = 2$. This is true for all γ_{hs}-sets of G. Therefore, $f_{\gamma_{hs}}(G) = 2$.

Case 4: $n = 6s + 2$ or $n = 6s + 4$, where $(s \geq 1)$.

Case 4(a): Let $n = 6s + 2$ $(s \geq 1)$. Then $\gamma_{hs}(G) = 2s + 2$. In this case G has more than one γ_{hs}-sets. Let W_9 be a γ_{hs}-set of G. It is easily verified that there is no singleton subset of W_9 and so $f_{\gamma_{hs}}(G) \geq 3$. Let $W_9 = \{v_1, v_2\} \cup \{v_3, v_6, \dots, v_{6s}\}$. Then W_9 is the unique γ_{hs}-set containing $\{v_1, v_2, v_3\}$ so that $f_{\gamma_{hs}}(G) = 3$. This is true for all γ_{hs}-sets of G. Therefore $f_{\gamma_{hs}}(G) = 3$.

Case 4(b): Let $n = 6s + 4$ $(s \geq 1)$. Then $\gamma_{hs}(G) = 2s + 2$. In this case G has more than one γ_{hs}-sets. Let W_{10} be a γ_{hs}-set of G. It is easily verified that there is no singleton subset of $W_{10}$and so $f_{\gamma_{hs}}(G) \geq 3$. Let $W_{10} = \{v_2, v_3\} \cup \{v_6, v_{12}, \dots, v_{6s}\} \cup \{v_7, v_{13}, \dots, v_{6s+1}\}$. Then W_{10} is the unique γ_{hs}-set containing $\{v_2, v_3, v_6\}$ so that $f_{\gamma_{hs}}(G) = 3$. This is true for all γ_{hs}-sets of G. Therefore, $f_{\gamma_{hs}}(G) = 3$.

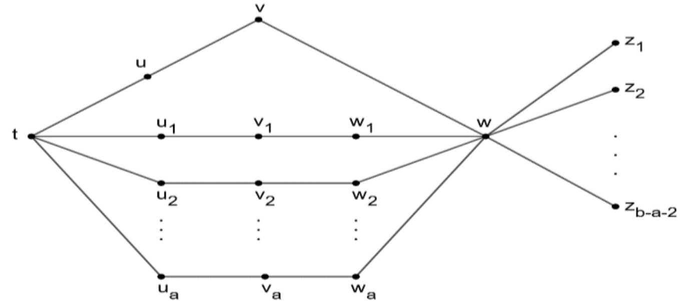
Theorem 2.15. For every pair of integers a and b with $0 \leq a \leq b$ and $b \geq a + 2$, there exists a connected graph G such that $f_{\gamma_{hs}}(G) = a$ and $\gamma_{hs}(G) = b$.

Proof: Let $P: t, u, v, w$ be a path of order four and $P_i: u_i, v_i, w_i$ $(1 \leq i \leq a)$ be a copy of path of order three. Let H be a graph obtained from P_i and P by introducing the edges tu_i and ww_i $(1 \leq i \leq a)$. Let G be the graph obtained from H by introducing the $b - a - 2$ new vertices $z_1, z_2, \dots, z_{b-a-2}$ and joining w with each z_i $(1 \leq i \leq b - a - 2)$. The graph G is shown in Figure 3.

First, we prove that $\gamma_{hs}(G) = b$. Let $Z = \{z_1, z_2, \dots, z_{b-a-2}\}$ be the end vertices of G. By Theorem 1.1, Z is a subset of every Steiner hop dominating set of G. Since $S(Z) \neq V(G)$, Z is not a Steiner hop dominating set of G. It is easily seen that t and w are Steiner hop dominating vertices of G. Let $Z_1 = Z \cup \{t, w\}$ be the set of all Steiner hop dominating vertices of G. Since $S(Z_1) \neq V(G)$, Z_1 is not a Steiner hop dominating set of G and so $\gamma_{hs}(G) \geq b - a - 2 + 2 = b - a$. Let $H_i = \{u_i, w_i\}$ $(1 \leq i \leq a)$. We prove that every γ_{hs}-set of G contains exactly one vertex from each H_i $(1 \leq i \leq a)$. On the contrary, suppose that there exists a γ_{hs}-set W such that W contains no element from each H_i $(1 \leq i \leq a)$. Then $W \subseteq V(G) - \{u_1, u_2, \dots, u_a, w_1, w_2, \dots, w_a\}$. Hence, it follows that $S(W) \neq V(G)$,which is a contradiction. Therefore, every γ_{hs}-set of G contains exactly one vertex from each H_i $(1 \leq i \leq a)$ and so $\gamma_{hs}(G) \geq b - a + a = b$. Let $W = Z_1 \cup \{v_1, v_2, \dots, v_a\}$. Then W is a Steiner set of G. Since $d(f, S) = 2$, where $f \in W$ and $S \subset V \setminus W$, W is a hop dominating set of G. Therefore, W

is a Steiner hop dominating set of G so that $\gamma_{hs}(G) = b$.

Next, we prove that $f_{\gamma_{hs}}(G) = a$. By Theorem 2.9., $f_{\gamma_{hs}}(G) \leq \gamma_{hs}(G) - |Z_1| = b - (b - a) = a$. Since every γ_{hs}-set of G contains exactly one vertex from each H_i $(1 \leq i \leq a)$ and Z_1 is a subset of every γ_{hs}-set of G. Every γ_{hs}-set of G is of the form $W_1 = Z_1 \cup \{c_1, c_2, \ldots, c_a\}$, where $c_1 \in H_i$ $(1 \leq i \leq a)$. Let T be any proper subset of W_1 with $|T| < a$. Then there exists H_j for some j such that $T \cap H_j = \varphi$, which shows that $f_{\gamma_{hs}}(G) = a$.



A graph G with $f_{\gamma_{hs}}(G) = a$ and $\gamma_{hs}(G) = b$ with $0 \leq a \leq b$ and $b \geq a + 2$.

Figure 3

3. Conclusion

In this paper, the forcing Steiner hop domination number for the standard graphs were determined and a realization result is given. This can be extended with other parameters of the graphs in further studies.

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