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Multifactorization of Complete Graphs into Equal Copies of Stars and Cycles.

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Abstract: Let K_n (λ) denote a complete graph with n vertices with edge multiplicity λ. Factorization of a graph is a partition of the given graph into isomorphic spanning subgraphs. A (G,H) – Multifactorization of a graph is a factorization of given graph into G and H with at least one copy of G and H. In this paper, we studied about the (G,H) – Multifactorization of K_n (λ) with equal copies of G and H, where G is a Star- factor and H is Cycle – factor.

Keywords: *Complete Graphs. Multifactorization, Cycle and Stars 2000 Mathematics Subject Classification Number : 05C70,05C38.*

1. Introduction

Let G be a simple and finite graph. Let K_n denote a complete graph on n vertices. G(λ) denote a graph G with edge multiplicity λ. Let $S_n= K_{1, n-1}$ denote a star, C_n denote a cycle and P_n denote a path on n vertices respectively. Let K_n(λ) denote a complete graph on n vertices with edge multiplicity λ. Decomposition of a graph is a partition of the given graph into isomorphic subgraphs. Factorization of a graph is a partition of the given graph into isomorphic spanning subgraphs. A (G,H) – Multidecomposition of a graph is a decomposition of given graph into G and H with at least one copy of G and H. A (G,H) – Multifactorization of a graph is a factorization of given graph into G- factors and H- factors with at least one copy of G and H. The study of multi-decomposition was introduced by Atif Abeiuda and Mike Daven in 2003 [1]. Priyadharsini and Muthusamy extended the study of Multi-decomposition and Multifactorization for many cases [3]. More studies on multidecomposition can be found in book [6].

Hurd and Sarvate [5] studied a decomposition of a K_v(λ), (λ≥2) into H_3 graphs with equal copies. They gave a necessary and sufficient condition for the decomposition of K_v(λ) into equal copies of C_3 and P_3. Now we rewrite the above problem as (G,H) – Multi-decomposition of K_n(λ) with equal copies of G and H. The Authors [4] have settled the problem of (G,H). – Multi-factorization of K_n(λ) with equal copies of (C_n,P_n). In this paper we extend the above problem as (G,H) – Multifactorization of K_n(λ) with equal copies of ($K1, _{n-1}$, C_n).

To prove our results, we need the following results.

Walecki Construction [2]: (i)The graph K_{2n+1} has a Hamilton cycle decomposition.

(ii) The graph K_{2n} has a Hamilton Path decomposition.

Lemma: (i) For even m > 2, the graph $2K_{2m}$ has a C_{2m} decomposition.

(ii) The graph $2K_{2m+1}$ has a Hamilton path decomposition.

Lemma []: The graph $2K_m$ has a $K1,_{m-1}$ – factorization.

Corollary: The graph K_{2n} has a Hamilton path decomposition

($K_{1, n-1}$, C_n) – Multifactorization of K_n(λ) with equal copies of $K1, _{n-1}$ and C_n.

Lemma 1:[3]

For n ≥ 3, K_n (λ) has a ($K_{1, n-1}$, C_n) – multifactorization if and only if

(i)|E(K($_n$) (λ))| = r(n-1)+ sn , r,s > 0,

(ii) λ ≥ 3 and

(iii) r ≡ 0 (mod n).

The above theorem gave a necessary and sufficient conditions for multifactorization of $K1_{,n-1}$ and Cn.

Lemma 3.1 .

For n≥ 3, K_n (λ) has a ($K1,_{n-1}$,Cn)r – multifactoriazation into equal copies (r copies) if

(i) |E(K(n) (λ))| = r(n-1)+r n , r > 0

(ii) r ≡ 0 (mod n) and

(iii) λ ≡ 0 (mod 2(2n-1), for even n and

(iv) λ ≡ 0 (mod (2n-1), for odd n.

Proof:

(i) Assume ($K_{1,n-1}$,C_n) – multifactoriazation into equal copies (r copies). Then there exist an integer such that 'r' such that

K_n (λ) = r $K_{1, n-1}$+ r Cn

Hence |E(K(n) (λ))| = (n-1) r + (n)r , r > 0. ---(1)

If λ =1,2, then |E(K(n) (λ))| = (n-1) r+ (n) r, no 'r' satisfy the above equation.

Now to prove (ii), In (I) , rn is the number of edges in 'r' copies of C_n and it forms a spanning subgraph in K_n (λ). Therfore r copies of $K_{1,n-1}$ should also form a regular subgraph in K_n (λ), which is possible only if each vertex is a center vertex for the same number of copies of $K_{1,n-1}$. Hence r ≡ 0 (mod n).

By (i), |E(K(n) (λ))| = (n-1)r + (n)r,

λn(n-1) | 2 = (n-1) r + nr = (2n-1)r.

λ n(n-1) | 2 = n| 2 λ(n-1).,----(1)

Consider the Equation (1)

As (n-1,2n-1) = 1.

(2n-1) | λ n (n-1)/2.

For even n., (2n-1) | λ n/2

Which implies λ ≡ 0 (mod (2n-1) and λ should be even.

λ ≡ 0 (mod 2(2n-1),

For odd n, (2n-1) | λn/2

Which implies λ ≡ 0 (mod (2n-1)

Conversely, suppose conditions (i), (ii), (iii) and (iv) are satisfied.

Case(i): For odd n, λ ≡ 0 (mod (2n-1).

(2n-1) K_n = (n-1) $2K_n$+ K_n = [n(n-1) | 2] $K_{1,n-1}$ ⊕ [n(n-1) | 2] C_n

As every $2K_n$ graph is $K_{1,\, n-1}$ factorable and $2K_{2n}$ has C_{2n} factorization.

Case(ii): For even n, λ ≡ 0 (mod 2(2n-1).

2(2n-1)K_n= (n-1) $2K_n$+ n $2K_n$

As every $2K_n$ graph is $K_{1,\, n-1}$ factorable and $2K_n$ is C_n factorable.

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