International Journal of
Intelligent Systems and Applications in Engineering

ISSN:2147-6799 www.ijisae.org Original Research Paper

Stiffness Analysis of Above Knee Prosthesis

Mücahit Ege*[1], Serdar Küçük[2]

Accepted 3rd September 2016

Abstract: While a healthy human walks, his or her legs mutually perform good repeatability with high accuracy. This provides an esthetical movement and balance. People with above knee prosthesis want to perform walking as esthetical as a healthy human. Therefore, to achieve a healthy walking, the above knee prosthesis must provide a good stiffness performance. Especially stiffness values are required when adding a second axis movement to the ankle for eversion and inversion. In this paper, stiffness analysis of above-knee prosthesis is presented. The translational displacement of above knee prosthesis is obtained when the prosthesis is subjected to the external forces. Knowing stiffness values of the above knee prosthesis, designers can compute prosthesis parameters such as ergonomic structure, height, and weight and energy consumption.

Keywords: *Stiffness analysis, above knee prosthesis, joint stiffness, prosthesis, accuracy*.

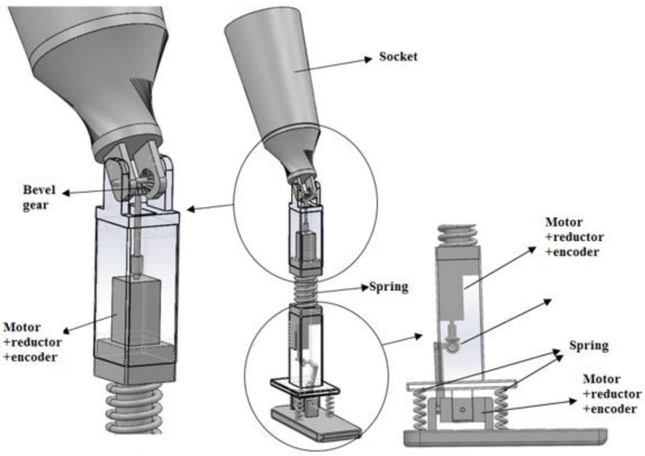
1. Introduction

It is required that serial robots have to make their tasks with a high accuracy, high precision and high stiffness [1]. The stiffness of robot manipulators generally provides to obtain the desired position and force commands with high accuracy [2, 3].
If the stiffness at the end point of robot manipulator is modified and identified accurately, it would be possible to compensate coupling and posing errors caused by the external forces [4].
(Dumas et al; 2010) introduced a method for the identification of the joint stiffness values of an industrial 6-DOF serial robot. In this method, it is aimed to evaluate joint stiffness values of any 6R serial robot using the model based on the Conservative Congruence Transformation (CCT).
(Pham et al; 2001) proposed a method for identification of the joint stiffness values using band pass filtering. This method requires closed-loop control based on the robot's dynamic model.
(Alici and Shirinzadeh; 2005) presented the enhanced stiffness modelling, identification, and characterization of robot manipulators using experimental data obtained from sensors.
Enhanced stiffness model is different from the conventional stiffness model which is derived firstly by (Mason and Salisbury; 1985). While inherent stiffness of system is used in the conventional model, enhanced model contains the stiffness component due to the change in the manipulator configuration and the external forces acting on the manipulator in addition to inherent stiffness of system. The conventional stiffness model is valid only when the manipulator is in quasi-static configuration with no loading, or when it has a constant Jacobian matrix throughout its workspace [4].
(Chen and Kao; 2000) reported that the conventional stiffness model derived by (Mason and Salisbury; 1985) is not valid. According to them, a model based on the CCT must be used as the generalized relationship between the joint stiffness matrices and Cartesian stiffness matrices due to preserving fundamental properties of the stiffness matrices [4].

(Ang and Andeen; 1995) presented variable passive compliance generated by means of topology of robot manipulators. As a conclusion they reported that a non-diagonal matrix is effective to prevent jamming and vibrations [4].
(Abele et al; 2007) presented two methods to obtain the Cartesian stiffness matrix of a 5R robot. They reported that second method is better than first because of considering the both joint and link stiffness. When load is applied, all deformations are considered such as links deformations and joint stiffness values.
This paper presents the stiffness analysis of above-knee prosthesis. The translational displacement of above knee prosthesis is obtained when the prosthesis is subjected to the external forces. Small displacements along x, y and z axes of the robot's end-effector are illustrated by figures. Knowing stiffness values of the above knee prosthesis, researches can design prosthesis as optimal structure, height, and weight and energy consumption.

2. Kinematic Analysis and Jacobian Matrix

2.1. Design of the Prosthesis



(A) (B) (C)

Figure 1. Proposed above knee prosthesis and its structure: knee joint(A); the entire prosthesis(B); ankle joint(C).

[1] *Istanbul Gedik University, Istanbul-34913, Turkey*
[2] *Kocaeli University, Istanbul-34913, Turkey*
** Corresponding Author: Email: mucahit.ege@gedik.edu.tr*
Note: This paper has been presented at the 3rd International Conference on Advanced Technology & Sciences (ICAT'16) held in Konya (Turkey), September 01-03, 2016.

The solid model of proposed above knee prosthesis is illustrated on (Figure.1). It has three joints. The first one is knee joint which is capable of one-axis movement and the second and third one compose of ankle joint which is capable of two-axis movement.

2.2. Kinematic Model of Above Knee Prosthesis

This section deals with kinematic model of above knee prosthesis. The coordinate systems attached to each joint of above knee prosthesis is presented in (Figure.1). The D-H parameters of the above knee prosthesis is illustrated in (Table.1).

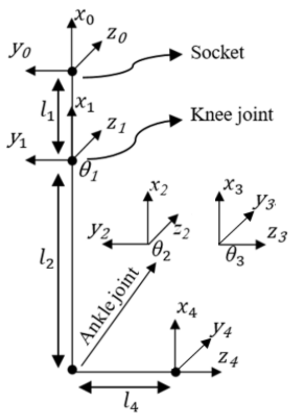


Figure 2. The coordinate systems attached to the joint

Table 1. D-H Parameters of Above Knee Prosthesıs

i (axis no)	α_{i-1} (degree)	a_{i-1} (mm)	d_i (mm)	θ_i (degree)
1	0	0	0	θ_1
2	0	l_1	0	θ_2
3	90	l_2	0	θ_3
4	0	0	l_4	0

The proposed above knee prosthesis has 3 rotational joints. The first one is knee joint which composes of one-axis movement. The second and third joints compose of ankle joint which performs two-axis movement. Ankle joint provides plantar flexion, dorsiflexion eversion and inversion movements.
The overall transformation matrix of above knee prosthesis can be written as

$$ {}^0_4T = {}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_4T \quad (1)$$

After determining D-H parameters, transformation matrices are obtained as follows.

$$ {}^0_4T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdots $$

$$ \cdots \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & l_2 \\ 0 & 0 & -1 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The multiplication of overall transformation matrices is obtained as

$$ {}^0_4T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

where

$$r_{11} = c\theta_3(c\theta_1 c\theta_2 - s\theta_1 s\theta_2)$$
$$r_{12} = -s\theta_3(c\theta_1 c\theta_2 - s\theta_1 s\theta_2)$$
$$r_{13} = c\theta_1 s\theta_3 + c\theta_2 s\theta_1$$
$$\boldsymbol{p_x} = l_2(c\theta_1 c\theta_2 - s\theta_1 s\theta_2) + l_4(c\theta_1 s\theta_2 + c\theta_2 s\theta_1) + l_1 c\theta_1$$
$$r_{21} = c\theta_3(c\theta_1 s\theta_2 + c\theta_2 s\theta_1)$$
$$r_{22} = -s\theta_3(c\theta_1 s\theta_2 + c\theta_2 s\theta_1)$$
$$r_{23} = s\theta_1 s\theta_2 - c\theta_1 c\theta_2$$
$$\boldsymbol{p_y} = l_2(c\theta_1 s\theta_2 + c\theta_2 s\theta_1) - l_4(c\theta_1 c\theta_2 - s\theta_1 s\theta_2) + l_1 s\theta_1$$
$$r_{31} = s\theta_3$$
$$r_{32} = c\theta_3$$
$$r_{33} = 0$$
$$\boldsymbol{p_z} = 0 \quad (4)$$

The Jacobian matrix of prosthesis is obtained as follows

$$ J = \begin{bmatrix} -l_1 s\theta_1 - l_2 s(\theta_1+\theta_2) + l_4 c(\theta_1+\theta_2) \\ l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) + l_4 s(\theta_1+\theta_2) & \cdots \\ 0 \end{bmatrix} $$

$$ \begin{matrix} -l_2 s(\theta_1+\theta_2) + l_4 c(\theta_1+\theta_2) & 0 \\ \cdots\ l_2 c(\theta_1+\theta_2) + l_4 s(\theta_1+\theta_2) & 0 \\ 0 & 1 \end{matrix}\Bigg] \quad (5)$$

3. Stiffness Modelling

The following relationship can be stated between actuated torques and the corresponding external forces and moments exerted on end-effector of the prosthesis can be expressed as:

$$\Gamma = J^T \omega \quad (6)$$

where Γ and F are the 3x1 vectors and $\Gamma = [\tau_1\ \tau_2\ \tau_3]^T$ is the actuator forces/torques needed to balance the external forces and $\omega = [f_x\ f_y\ f_z\ f_{x(\alpha)}\ f_{y(\beta)}\ f_{z(\gamma)}]^T$represents the corresponding external forces & moments exerted on end-effector of the prosthesis. There are two major changes happens in the manipulator because of its motion. First change happens angular position of the joints due to the torques/forces applied to the joints.

$$\Gamma = K_\theta \Delta_\theta \quad (7)$$

where $K_\theta = diag.[K_{\theta 1}\ K_{\theta 2}\ K_{\theta 3}]$ denotes joint stiffness matrix and $\Delta_\theta = [\delta_{\theta 1}\ \delta_{\theta 2}\ \delta_{\theta 3}]^T$ represents change in the positions of the joints. Second change happens on the end-effector of the manipulator due to the external force and moment applied to the end-effector of the manipulator.

$$\omega = K_x \Delta_x \tag{8}$$

where K_x illustrates the Cartesian stiffness matrix of the manipulator and $\Delta_\theta = [\delta_{\theta x}\ \delta_{\theta y}\ \delta_{\theta z}]^T$ denotes change in the end-effector of the manipulator.

The following important identity is obtained by applying partial diffraction to the (Equation.6) with respect to θ.

$$\frac{\partial \Gamma}{\partial \theta} = \left(\frac{\partial J^T}{\partial \theta}\right)\omega + J^T \frac{\partial \omega}{\partial X}\frac{\partial X}{\partial \theta} \tag{9}$$

(Equation.9) can be written as follows:

$$K_\theta = K_C + J^T K_x J \tag{10}$$

where K_C is the complementary stiffness matrix can be written for a 3DOF robotic manipulator as follows

$$K_C = \left[\frac{\partial J^T}{\partial \theta_1}\omega\ \frac{\partial J^T}{\partial \theta_2}\omega\ \frac{\partial J^T}{\partial \theta_3}\omega\right] \tag{11}$$

The stiffness matrix seen at the end-effector of the manipulator can be illustrated as

$$K_X = J^{-T}(K_\theta - K_C)J^{-1} \tag{12}$$

In order to find joint stiffness matrix, the Cartesian stiffness matrix can be simplified by ignoring K_C as follows:

$$K_X = J^{-T} K_\theta J^{-1} \tag{13}$$

(Equation.8) can be rewritten by substituting (Equation.12) in (Equation.14) as follows.

$$\omega = J^{-T} K_\theta J^{-1} \Delta_X \tag{14}$$

(Equation.14) can be rearranged as

$$\Delta_X = J K_\theta^{-1} J^T \omega \tag{15}$$

(Equation.15) can be rewritten as follows

$$\Delta_X = Ax \tag{16}$$

where x and A include 6x1 vector of joint compliances and 6x6 matrix having external forces/moments and elements of Jacobian matrices.

$$x = [1/k_{\theta_1}\ 1/k_{\theta_2}\ 1/k_{\theta_3}]^T \tag{17}$$

4. Stiffness Analysis

Cartesian stiffness matrix can be obtained by using (Equation.11) as follows

$$K_c = \begin{bmatrix} (-l_1 c\theta_1 - d_3 s\theta_1)f_x + (-l_1 s\theta_1 + d_3 c\theta_1)f_y \\ 0 \\ c\theta_1 f_x + s\theta_1 f_y \end{bmatrix} \cdots$$

$$\cdots \begin{matrix} c\theta_1 f_x + s\theta_1 f_y \\ 0 \\ 0 \end{matrix}\Bigg] \tag{18}$$

A prosthesis end effector is forced to track a trajectory from its zero position $(\theta_1 = \theta_2 = \theta_3 = 0)$ to final position $(\theta_1 = 150,\ \theta_2 = 15,\ \theta_3 = 10)$ to identify joint stiffness values. The travel time of robot trajectory is planned as 3 seconds at 100Hz frequency. The joint stiffness values $(K_{\theta 1}, K_{\theta 2}$ and $K_{\theta 3})$ along the trajectory are identified. Since trajectory frequency 100 Hz, 300 sample of joint stiffness values are obtained. The arithmetic averages of these sample values gives joint stiffness values. In this manipulator (Equation.16) can be written for prosthesis to find joint compliance values

$$\Delta_X = \begin{bmatrix} j_{11} & j_{12} & 0 \\ j_{21} & j_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1/k_{\theta 1} & 0 & 0 \\ 0 & 1/k_{\theta 2} & 0 \\ 0 & 0 & 1/k_{\theta 3} \end{bmatrix}\cdots$$

$$\cdots\begin{bmatrix} j_{11} & j_{21} & 0 \\ j_{12} & j_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} \tag{19}$$

where Jacobean parameters;

$$\begin{aligned} j_{11} &= -l_1 s\theta_1 - l_2 s(\theta_1+\theta_2) - l_4 c(\theta_1+\theta_2) \\ j_{12} &= -l_2 s(\theta_1+\theta_2) + l_4 c(\theta_1+\theta_2) \\ j_{21} &= l_1 s\theta_1 + l_2 c(\theta_1+\theta_2) + l_4 s(\theta_1+\theta_2) \\ j_{22} &= l_2 c(\theta_1+\theta_2) - l_3 s(\theta_1+\theta_2) \end{aligned} \tag{20}$$

This equation is reorganized to obtain (Equation.16)

$$\begin{bmatrix} \delta_{\theta x} \\ \delta_{\theta y} \\ \delta_{\theta x} \end{bmatrix} = \begin{bmatrix} j_{11}^2 f_x + j_{11}j_{21}f_y & 0 & j_{13}^2 f_x + j_{13}j_{23}f_y \\ j_{21}^2 f_y + j_{11}j_{21}f_x & 0 & j_{23}^2 f_y + j_{13}j_{23}f_x \\ 0 & f_x & 0 \end{bmatrix}\cdots$$

$$\cdots\begin{bmatrix} 1/k_{\theta_1} \\ 1/k_{\theta_2} \\ 1/k_3 \end{bmatrix} \tag{21}$$

5. Stiffness Verification

In order to compute the end effector displacements of proposed above knee prosthesis, the external forces are acted on anatomical position of human. Anatomical position is the erect position of the body with the face directed forward, the arms at the side, and the palms of the hands facing forward. It was used as a reference position for describing the relation of body parts to one to another [10].

Cartesian stiffness matrix is calculated by using (Equivalent.12). Typical stiffness values of $K_\theta = diag.[10^5, 10^5, 10^5]$ N.mm/rad are chosen for initial values. K_θ is constant, because it lets to the same joint stiffness values for it's different initial values [4].

In this study, in order to have displacement values of the end-effector, an experimental study is performed. The magnitudes of force vector are implemented from 0 Newton to 200 Newton with a step size of 10 Newton as shown in (Table.2).

Table 2. Stiffness verification results for prosthesis

Force Vector (F_x, F_y, F_z) (Newton)	Deflection Calculated		
	δ_x(mm)	δ_y(mm)	δ_z(mm)
0,0,0	0	0	0
10,10,10	0,00003655	0,00005472	0,00001
20,20,20	0,0000731	0,0001094	0,0002
30,30,30	0,0001097	0,0001641	0,0003
40,40,40	0,0001462	0,0002189	0,0004
50,50,50	0,0001828	0,0002736	0,0005
60,60,60	0,0002193	0,0003283	0,0006
70,70,70	0,0002559	0,000383	0,0007
80,80,80	0,0002924	0,0004377	0,0008
90,90,90	0,000329	0,0004924	0,0009
100,100,100	0,0003655	0,0005472	0,001
110,110,110	0,0004	0,0006	0,0011
120,120,120	0,0004	0,0007	0,0012
130,130,130	0,0005	0,0007	0,0013
140,140,140	0,0005	0,0008	0,0014
150,150,150	0,0005	0,0008	0,0015
160,160,160	0,0006	0,0009	0,0016
170,170,170	0,0006	0,0009	0,0017
180,180,180	0,0007	0,001	0,0018
190,190,190	0,0007	0,001	0,0019
200,200,200	0,0007	0,0011	0,002

(Figure.3) represents the deflection values along the x, y and z-axes of prosthesis end-effector under the forces exerted on the prosthesis as shown in (Table.2).
(Figure.3) represents the deflection values along the x, y and z-axes of prosthesis end-effector under the forces exerted on the prosthesis as shown in (Table.2).

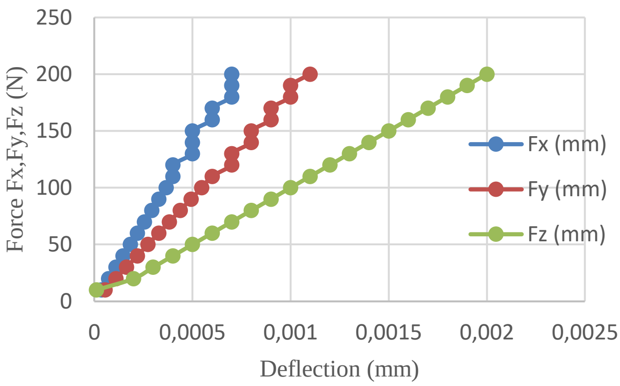


Figure 3. Force-deflection curve for F_x, F_y, F_z

In (Figure.4), (Figure.5) and (Figure.6), F_x, F_y *and* F_z forces are presented in separately.

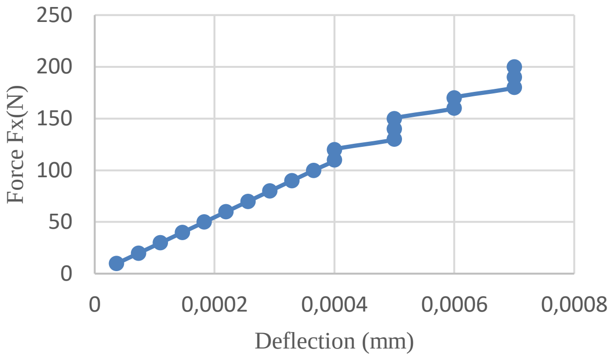


Figure 4. Force-deflection curve for F_x.

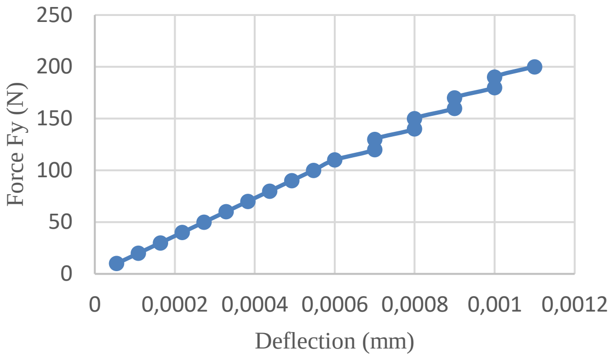


Figure 5. Force-deflection curve for F_y.

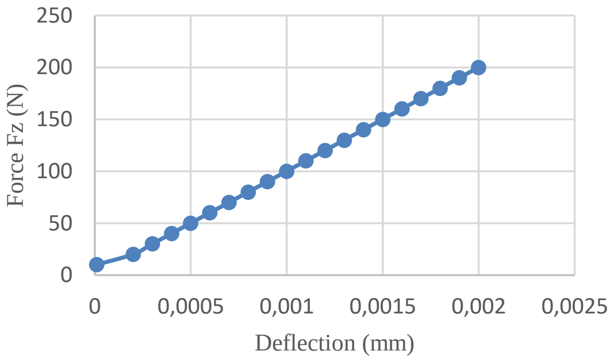


Figure 6. Force-deflection curve for F_z.

6. Conclusion

In this study, the stiffness analysis of the proposed above-knee prosthesis is presented. The translational displacement of above knee prosthesis is obtained when the prosthesis is subjected to the external forces from 0 Newton to 200 Newton with a step size 10 Newton. The computed displacements along x, y and z axes of the prosthesis's end-effector are illustrated by a table and figures. These results can be used to design and manufacture an above knee prosthesis. Results can also help researchers to choose the material which will be used.

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